Experimental support to the BCS theory.

Tables and figures from:
W. Buckel, R. Kleiner
"Superconductivity - Fundamentals and Applications", 2nd Ed.
Wiley, 2004
C. Enss, S. Hunklinger
"Low-Temperature Physics", Springer

Isotope effect

If the weak attractive interaction is driven by phonons (lattice distortions -> vibrations), superconductivity depends upon *lattice* properties (not only *electronic* properties). If electrons within δE at E_F are involved in the pairing process, one finds

$$T_c = 1.13\frac{\delta E}{k_B}e^{-1/N_n(E_F)V}$$

Identifying δE with the Debye frequency, since $\omega_D \propto M^{-1/2}$, $\quad T_c \propto M^{-1/2}$

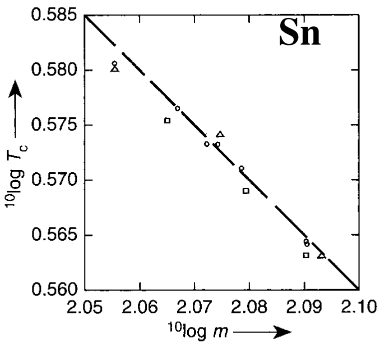
Isotope production -> experimental verification in superconductive elements

Table 3.1 Isotope effect in mercury [13].

Average atomic mass	199.7	200.7	202.0	203.4
Transition temperature T_c in K	4.161	4.150	4.143	4.126

Isotope effect

$$T_c \propto M^{-1/2}$$



dashed line: exponent -1/2

Isotope effect

More often one has departure from the simple law $T_c \propto M^{-1/2}$.

$$T_c \propto M^{-\beta}$$

Table 3.2 Isotope effect.

Element	Hg	Sn	Pb	Cd	Tl	Mo	Os	Ru
Isotope exponent β^{*}	0.50	0.47	0.48	0.5	0.5	0.33	0.2	0.0

*) The exponent β is obtained from experiment by fitting to the relation $T_c \propto M^{-\beta}$. The values shown are taken from R. D. Parks, "Superconductivity", Marcel Dekker, New York, 1969, p. 126.

Deviations are taken into account by more refined theories, possibly inserting additional parameters

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The energy gap

Theory:

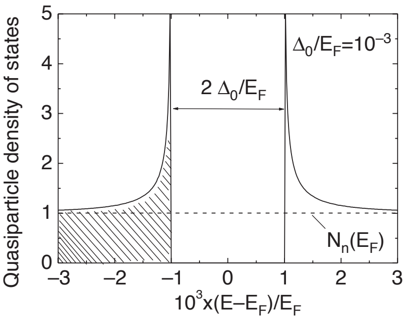

Fig. 3.7 Normalized density of states $N_s(E)/N_n(E_F)$ of the quasiparticles in a superconductor according to the BCS theory. The ratio Δ_0/E_F was taken as 10^{-3}. At the temperature $T = 0$ all states below the Fermi energy are occupied (hatched region).

Quasiparticles density of states, normalized to the density of states at E_F. A gap appears: no available states for quasiparticles: forbidden energy range.

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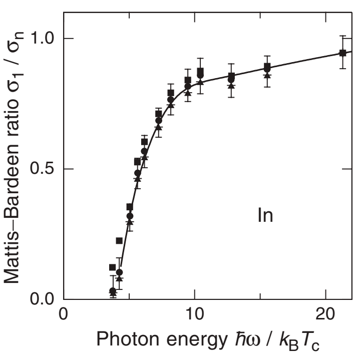
The energy gap

Experiment: measure the absorption of electromagnetic waves (quanta).

$\hbar\omega < 2\Delta \Longrightarrow$ no absorption

$\hbar\omega > 2\Delta \Longrightarrow$ Cooper pair breaking, finite conductivity



1 meV ≈ 240 GHz

Fig. 10.33. Frequency dependence of the Mattis–Bardeen ratio σ_1/σ_n of a thin indium film measured at 1.4 K. This ratio is proportional to the film absorption [487]

The energy gap

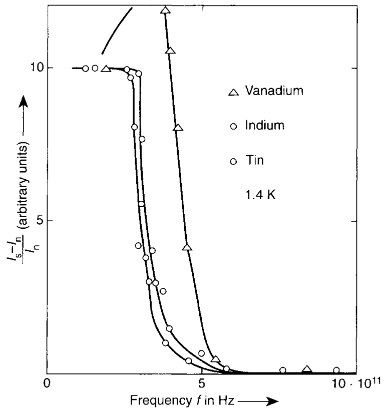


Fig. 3.9 Absorption of electromagnetic waves with frequency f in superconductors. Transition temperatures T_c are: V, 5.3 K; In, 3.42 K; Sn, 3.72 K. Measurement temperature is 1.4 K. In vanadium for $f < 2\Delta_0/h$ a frequency dependence is observed, the possible origin of which cannot be discussed here (from [20]).

Experiment: measure the reflection of electromagnetic waves (quanta) in the superconducting (I_s) and normal (I_N) state.
When $I_s - I_n = 0 \implies$ no apparent difference between normal and superconducting state: broken Cooper pairs.

1 meV ≈ 240 GHz

Energy gap and T_c

Theoretical prediction: $\Delta = 1.75\ k_B\ T_c$ (weak coupling)

Elements

Table 3.3 Energy gap $2\Delta_0$ in units of $k_B T_c$ for some superconducting elements. The numbers in brackets indicate the energy gap in meV. The values are taken from R. D. Parks, Superconductivity, p. 14, p.16 and D. H. Douglas Jr., L. M. Falicov, Progress of Low Temperature Physics, vol. 4 97 (1964), North-Holland, Amsterdam. For special details see Superconductivity Data, no. 19–1 (1982), Fachinformationszentrum Karlsruhe GmbH.

Element	Method			
	T_c in Kelvin	Tunnel junctions	Ultrasound	Light absorption
Sn	3.72	3.5 ± 0.1 (1.15)	–	3.5
In	3.4	3.5 ± 0.1 (1.05)	3.5 ± 0.2	3.9 ± 0.3
Tl	2.39	3.6 ± 0.1 (0.75)	–	–
Ta	4.29	3.5 ± 0.1 (1.30)	3.5 ± 0.1	3.0
Nb	9.2	3.6 (2.90)	4.0 ± 0.1	2.8 ± 0.3
Hg	4.15	4.6 ± 0.1 (1.65)	–	4.6 ± 0.2
Pb	7.2	4.3 ± 0.05 (2.70)		4.4 ± 0.1

Energy gap and T_c

Theoretical prediction: $\Delta = 1.75\ k_B\ T_c$ (weak coupling)

Compounds

Table 3.4 Energy gap $2\Delta_0$ for selected superconducting compounds (s-wave Cooper pairing). Experimental methods: tunneling effect, optical methods, nuclear spin resonance, specific heat, etc. Many data can also be found in the monograph [M14].

Material	T_c in K	$2\Delta_0$ in meV	$2\Delta_0/k_B T_c$	Reference	cf. Section
Nb_3Sn	18	6.55	4.2	[21]	2.3.1
NbN	13	4.6	4.1	[22]	2.3.1
MgB_2	40	3.6–15	1.1–4.5	[18]	2.3.2
Rb_3C_{60}	29.5	10–13	4.0–5.1	[17]	2.4
$ErRh_4B_4$	8.5	2.7–3	3.8–4.2	[23]	2.5
$PbMo_6S_3$	12	4–5	4–5	[24]	2.5
YNi_2B_2C	15.5	4.7	3.5	[25]	2.5
$NbSe_2$	7	2.2	3.7	[26]	2.7
$BaPb_{0.75}Bi_{0.25}O_3$	11.5	3.5	3.5	[27]	2.8.2
$Ba_{0.6}K_{0.4}BiO_3$	25–30	8	3.5	[28]	2.8.2