dc SQUID

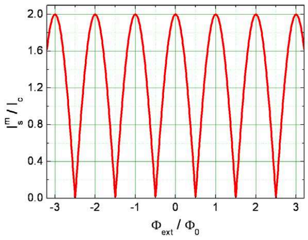
Negligible screening, zero voltage

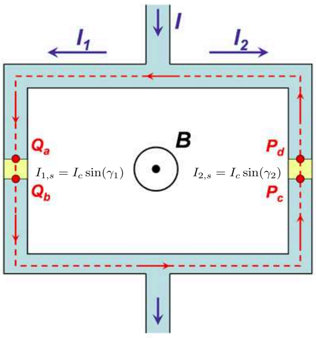
$$\beta_L = \frac{2LI_c}{\Phi_0} \ll 1$$

Circulating currents do not compensate the external flux

$$\Phi \simeq \Phi_a$$

$$I_{s,max} \simeq 2I_c \left|\cos\left(\pi\frac{\Phi_a}{\Phi_0}\right)\right|$$





Figures from:
R. Gross, A. Marx, Walther Meissner Institut
http://www.wmi.badw.de/teaching/Lecturenotes/

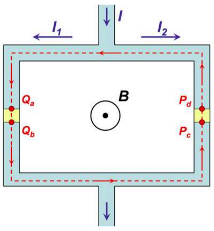
Nonzero screening, zero voltage

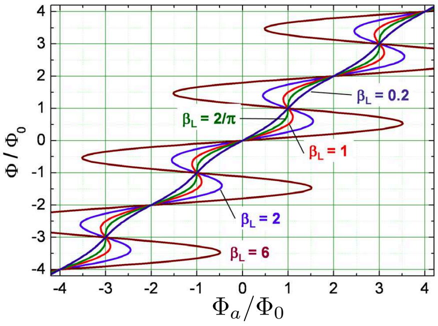
$$\beta_L = \frac{2LI_c}{\Phi_0}$$

Circulating currents compensate the external flux.
Strong screening: towards flux quantization in the loop.

$$\Phi \neq \Phi_a$$

$$\frac{\Phi_a}{\Phi_0} = \frac{\Phi}{\Phi_0} + \frac{\beta_L}{2} + \sin\left(\pi\frac{\Phi}{\Phi_0}\right)$$





Figures from:
R. Gross, A. Marx, Walther Meissner Institut
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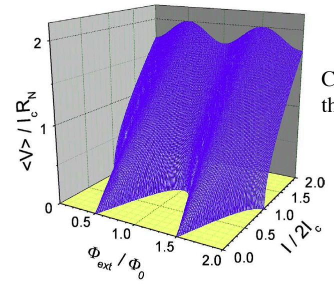
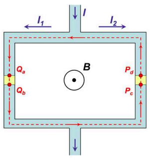
Negligible screening, nonzero voltage

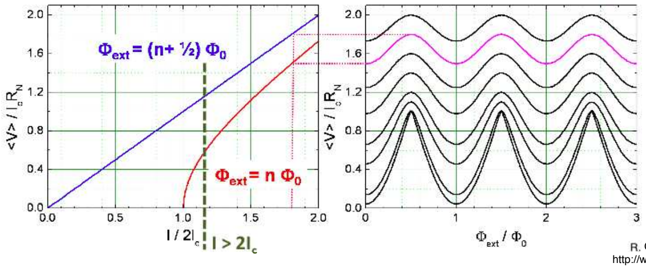


$$\beta_L = \frac{2LI_c}{\Phi_0} \ll 1$$

Circulating currents do not compensate the external flux $\Phi \simeq \Phi_a$



$$\langle V(t) \rangle = I_c R_N \sqrt{\left(\frac{I}{2I_c}\right)^2 - \cos^2\left(\pi\frac{\Phi_a}{\Phi_0}\right)}$$



Maximum modulation of the average voltage for *I=2I*$_c$.

Figures from:
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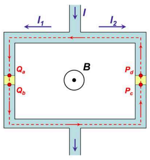
dc SQUID: transfer function



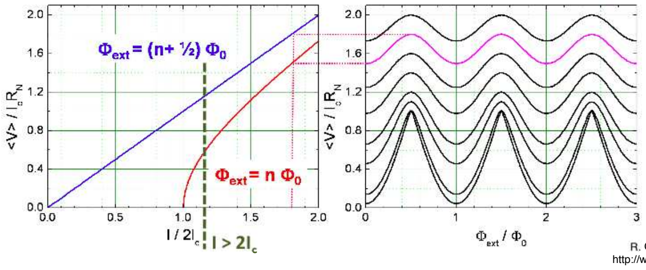
$$\langle V(t) \rangle = I_c R_N \sqrt{\left(\frac{I}{2I_c}\right)^2 - \cos^2\left(\pi\frac{\Phi_a}{\Phi_0}\right)}$$ Flux-to-voltage converter.

Transfer function: $$H = \left|\frac{\partial \langle V(t) \rangle}{\partial \Phi_a}\right|$$

Usual operational point: $$I \gtrsim 2I_c$$ $$\Phi_a \simeq (2n+1)\frac{\Phi_0}{4}$$ $$\beta \lesssim 1$$

One finds numerically

$$H \simeq \frac{I_c R_N}{\Phi_0/2} \simeq \frac{R_N}{L}$$



Figures from:
R. Gross, A. Marx, Walther Meissner Institut
http://www.wmi.badw.de/teaching/Lecturenotes/

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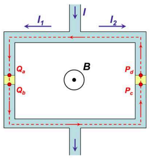
dc SQUID: parameters



$$\langle V(t) \rangle = I_c R_N \sqrt{\left(\frac{I}{2I_c}\right)^2 - \cos^2\left(\pi\frac{\Phi_a}{\Phi_0}\right)}$$ Flux-to-voltage converter.

Transfer function: $$H = \left|\frac{\partial \langle V(t) \rangle}{\partial \Phi_a}\right|$$

Usual operational point: $$I \gtrsim 2I_c$$ $$\Phi_a \simeq (2n+1)\frac{\Phi_0}{4}$$ $$\beta \lesssim 1$$

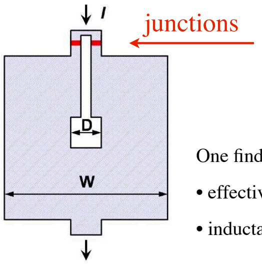
One finds numerically

$$H \simeq \frac{I_c R_N}{\Phi_0/2} \simeq \frac{R_N}{L}$$

Other requirements:
- large loop (large $\Delta\Phi$ with small ΔB). But then large L! –> small H and small β

Figures from:
R. Gross, A. Marx, Walther Meissner Institut
http://www.wmi.badw.de/teaching/Lecturenotes/

“washer” SQUIDs



“field focussing” device: the large superconducting area expels the flux, and focusses into the small hole.

One finds:

- effective area (where field is collected) scales with *W*: $A_{eff} \sim D \cdot W$
- inductance scale with *D*: $L \simeq 1.25\mu_0 D$

Caveats: *W* too large –> fluxons enters the superconducting area, and thermal activated motion leads to noise

Figures from:
R. Gross, A. Marx, Walther Meissner Institut
http://www.wmi.badw.de/teaching/Lecturenotes/

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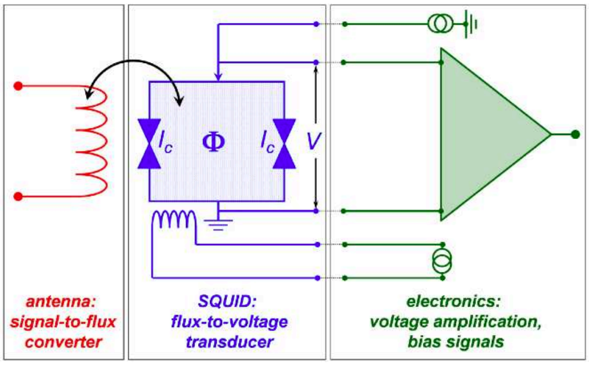
real dc SQUID



Figures from:
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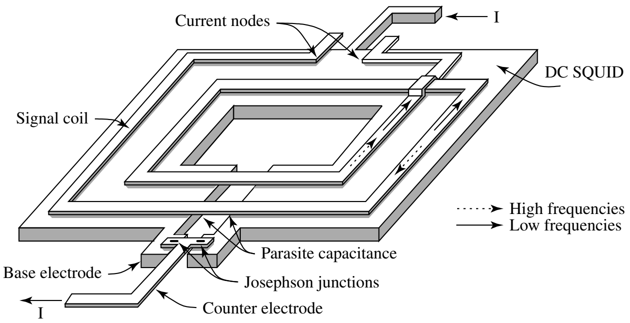
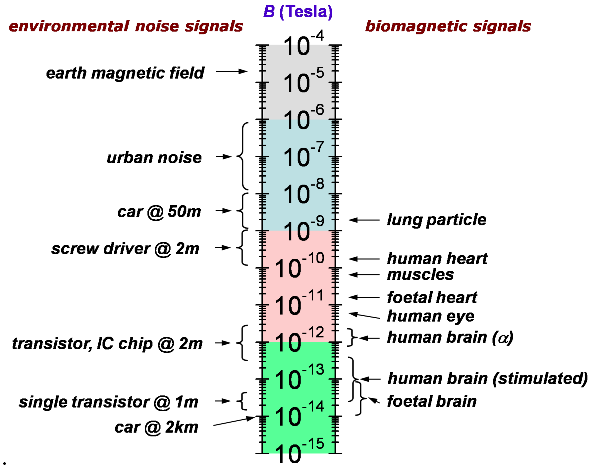
Coupling to the signal.



A spiral coil brings the signal over the SQUID area

Figures from:
Fossheim-Sudbo

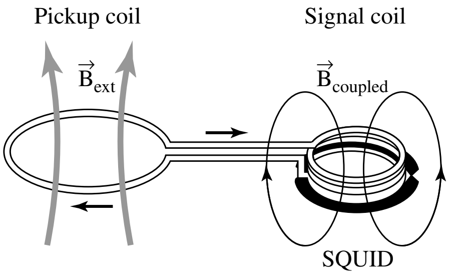
SQUIDs as magnetometers



Biomagnetism

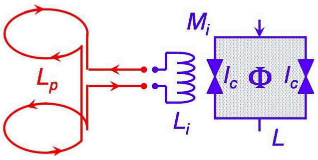
Figures from:
R. Gross, A. Marx, Walther Meissner Institut
http://www.wmi.badw.de/teaching/Lecturenotes/

Transformer configurations



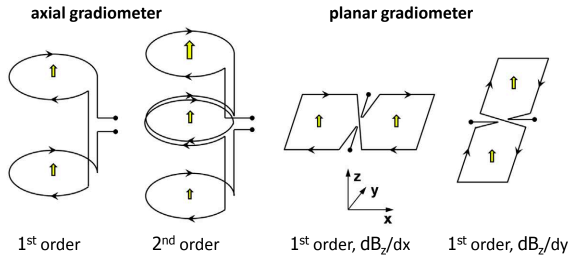
Figures from:
Fossheim Sudbo

Gradiometer configurations



Sensitive to spatial derivatives

$$\frac{\partial B}{\partial z}$$



Figures from:
R. Gross, A. Marx, Walther Meissner Institut
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