# Josephson effect: voltage standard



## Physics

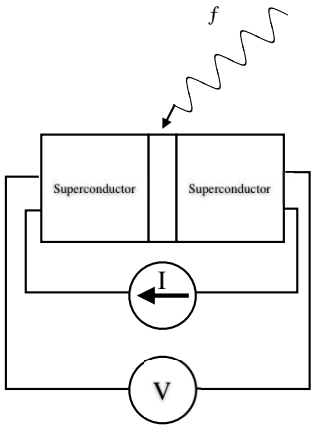
Across a JJ irradiated by an ac current of frequency $f=\omega/2\pi$ there are current spikes at voltages

$$V_n = n\frac{\hbar}{2e}\omega = n\Phi_0 f$$

$n = 1, 2, 3, ....$

Amplitude of the time-averaged supercurrent spikes ($J_n$ : Bessel function of order $n$):

$$|I_{s,n}| = I_c \left| J_n\left(\frac{\Phi_0 V_{rf}}{f}\right)\right|$$



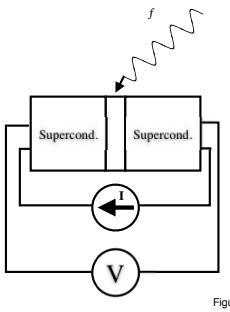



## Overdamped JJ

$$\beta_C = \frac{2\pi}{\Phi_0} I_o R_N^2 C \ll 1$$

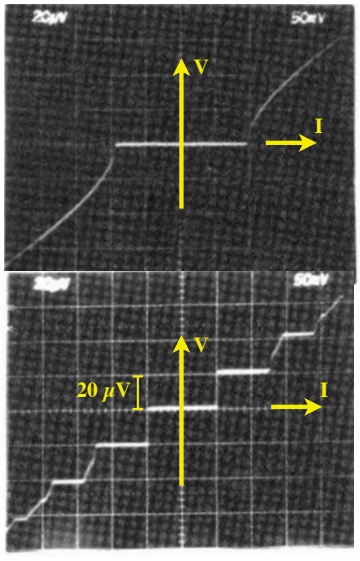
$$V_n = n\frac{\hbar}{2e}\omega = n\Phi_0 f \qquad n = 1, 2, 3, ....$$

$$|I_{s,n}| = I_c \left| J_n\left(\frac{\Phi_0 V_{rf}}{f}\right)\right|$$




No microwave irradiation

Voltage Steps by increasing current under microwave irradiation (here $f$=10 GHz)



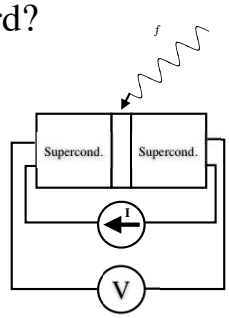

Figures from:
J. Niemeyer, in "Handbook of Applied Superconductivity", ed. by B. Seeber IOP Publishing, 1998

# A voltage standard?

Warning: 100 GHz —> V ≈ 200 $\mu$V (n=1): compare to 1 V or 10 V voltage standard.

Many ($10^{3\div4}$) junctions in series ?

Use high $n$ ?




$$V_n = n\frac{\hbar}{2e}\omega = n\Phi_0 f$$

$n = 1, 2, 3, ....$

$$|I_{s,n}| = I_c \left| J_n\left(\frac{\Phi_0 V_{rf}}{f}\right)\right|$$

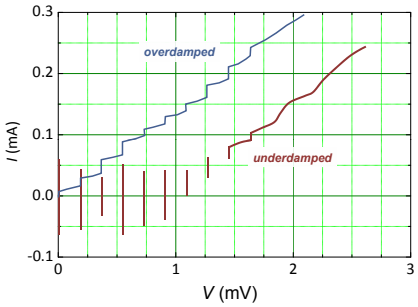


# Underdamped JJ

$$\beta_C = \frac{2\pi}{\Phi_0} I_o R_N^2 C \gg 1$$

Zero current steps:

(recall the washboard potential)




Experimental IVCs obtained for an underdamped and overdamped Nb Josephson junction under microwave radiation. The IVCs clearly show the constant voltage steps at $V_n$ (data from C.A. Hamilton, Rev. Sci. Instr. 71, 3611 (2000)).

Figure from:
R. Gross, A. Marx, Walther Meissner Institut
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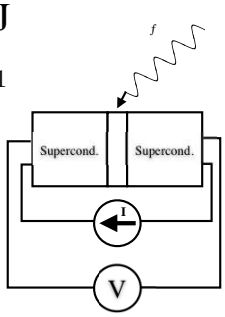



$$V_n = n\frac{\hbar}{2e}\omega = n\Phi_0 f$$

$n = 1, 2, 3, ....$

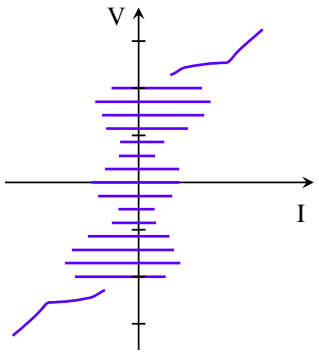
$$|I_{s,n}| = I_c \left| J_n\left(\frac{\Phi_0 V_{rf}}{f}\right)\right|$$

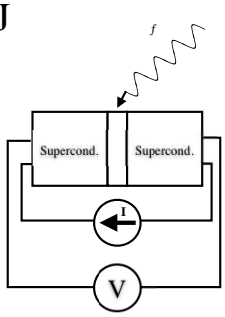


# Underdamped JJ

Zero current steps:

(recall the washboard potential)



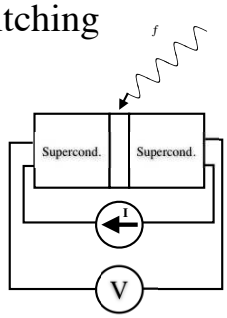




$$V_n = n\frac{\hbar}{2e}\omega = n\Phi_0 f$$

$n = 1, 2, 3, ....$

$$|I_{s,n}| = I_c \left| J_n\left(\frac{\Phi_0 V_{rf}}{f}\right)\right|$$

# Problem: unequal switching




Warning: 100 GHz —> V ≈ 200 $\mu$V (n=1): compare to 1 V or 10 V voltage standard.

Many ($10^{3\div4}$) junctions in series ?

Underdamped junctions –> zero current steps

But: stable zero current steps –> long-term (calibration) phase lock to the external microwave oscillator for a large series array.

$$V_n = n\frac{\hbar}{2e}\omega = n\Phi_0 f$$

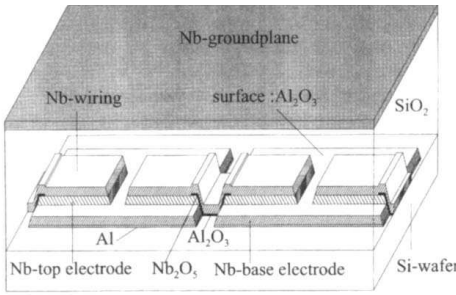
- reduction of external noise
- avoid a Josephson junctions regime where chaotic phenomena can occur.

Most stable when (numerical): $\omega = 2\pi f \gg \sqrt{\frac{2eI_c}{\hbar C}}$

Low critical current
High JJ capacitance
(underdamped JJ)



# Arrays of JJ




$$V_n = n\frac{\hbar}{2e}\omega = n\Phi_0 f$$

$n = 1, 2, 3, ....$

$$|I_{s,n}| = I_c \left| J_n\left(\frac{\Phi_0 V_{rf}}{f}\right)\right|$$

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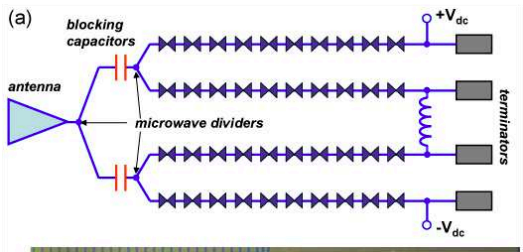
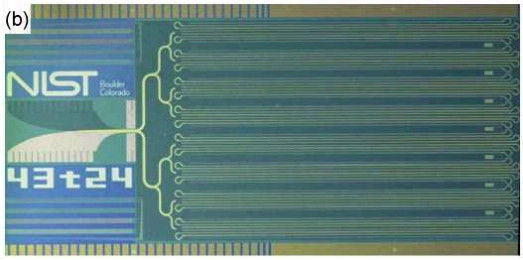
Many junctions in series

Problems:
- JJ homogeneity (over ~$10^3$ JJs): different junction parameters –> unequal switching –> chaos
- homogeneous irradiation



# Voltage standard



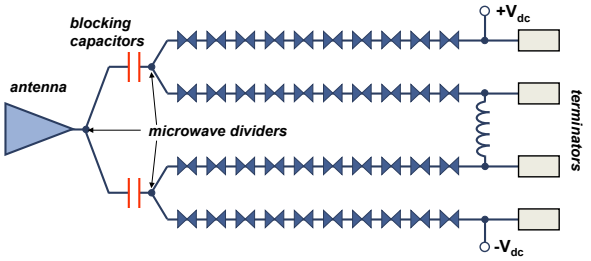




JJ in series, seen in parallel by the microwave stripline

Figure from:
R. Gross, A. Marx, Walther Meissner Institut
http://www.wmi.badw.de/teaching/Lecturenotes/

NIST:
20208 (16 X 1263 lines) JJ operates at 75 GHz

# Voltage standard

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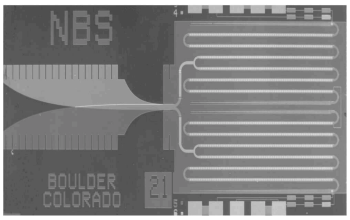
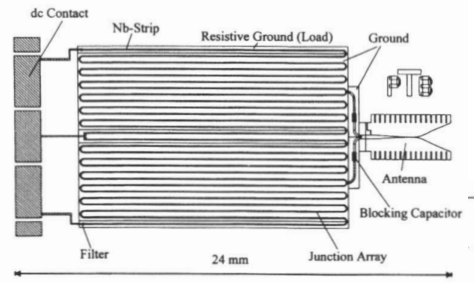


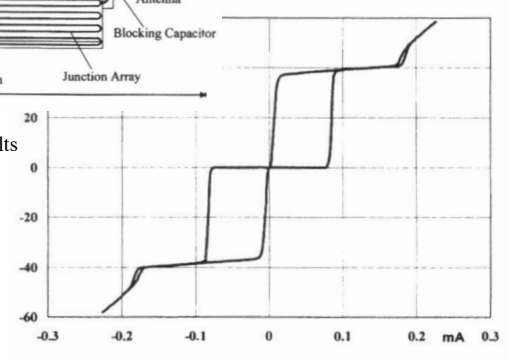

NIST: 3020 JJ series, 1V standard



# Voltage standard




Voltage standard. 14000 JJ.

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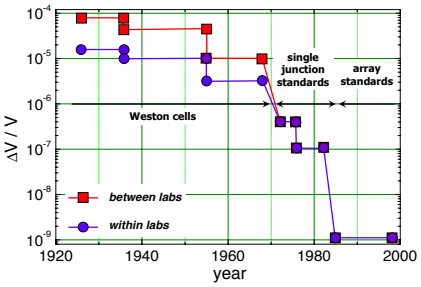
# Voltage standard




Note: relative accuracy between labs = within labs with JJ voltage standard (make use of fundamental constants!)

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However, note:

Direct proof of accuracy of a Josephson Voltage Standard is ***impossible***:

while frequency (which tunes the Shapiro steps) can be determined to $10^{-12}$, no other voltage source of sufficient accuracy exist!

The instrument under testing limits the accuracy of the calibration.