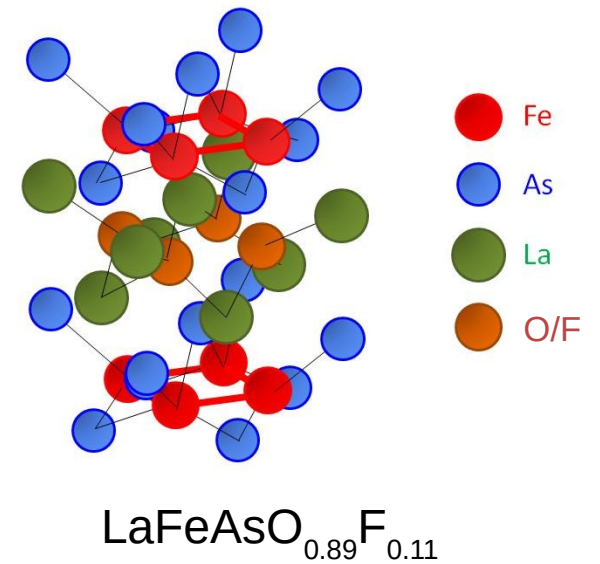
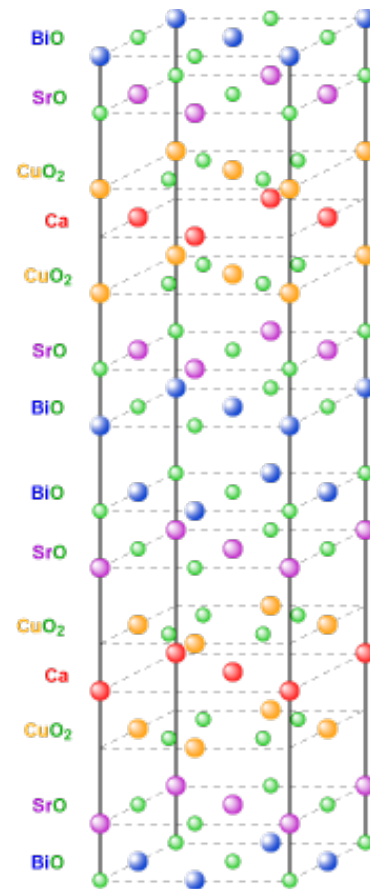
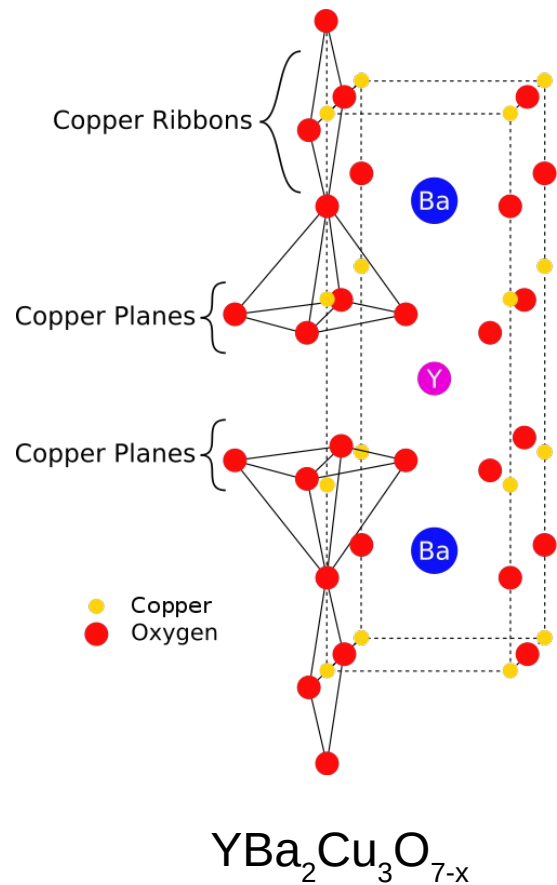
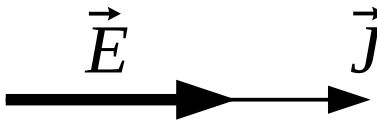


Superconduttori anisotropi



Richiami

- Materiale isotropo: vettore effetto (e.g. \vec{E}) // vettore causa (e.g. \vec{J}):

$$\vec{E} = \rho \vec{J}$$


scalare

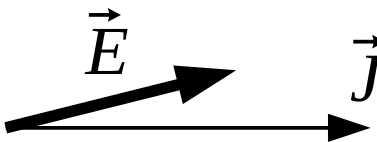
Altri esempi:

$$\vec{D} = \epsilon \vec{E}$$

$$\vec{q} = -k \vec{\nabla} T$$

...

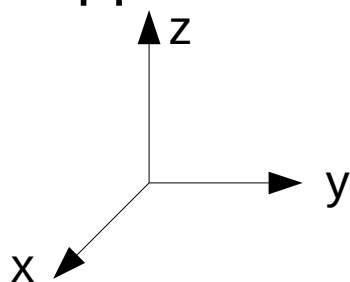
- Materiale anisotropo: in genere \vec{E} non è // a \vec{J}

$$\vec{E} = \bar{\rho} \vec{J}$$


tensore

- Rappresentazione

Se assi rif.=assi principali & no Hall



$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \rho_{11} & \rho_{21} & \rho_{31} \\ \rho_{12} & \rho_{22} & \rho_{32} \\ \rho_{13} & \rho_{23} & \rho_{33} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

matrice

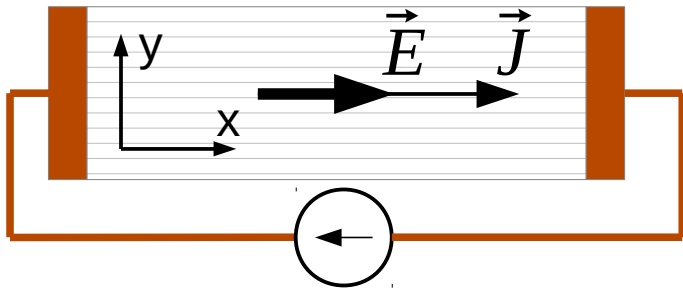
$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 & 0 \\ 0 & \rho_{22} & 0 \\ 0 & 0 & \rho_{33} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix}$$

matrice diagonale

Misure di resistenza

Caso isotropo: $\rho_{mis} = \frac{E}{J}$ ← misurato
 ← imposto

Caso anisotropo (2D): $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$

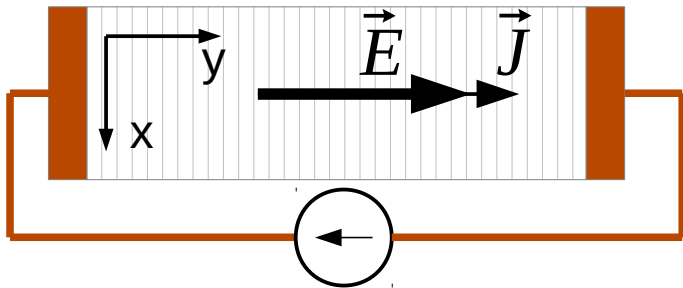


$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix} \begin{pmatrix} J \\ 0 \end{pmatrix} = \rho_{11} J \hat{x} = E_x \hat{x}$$

$$\Rightarrow \rho_{mis} = \rho_{11}$$

$$\vec{E} \parallel \vec{J}$$

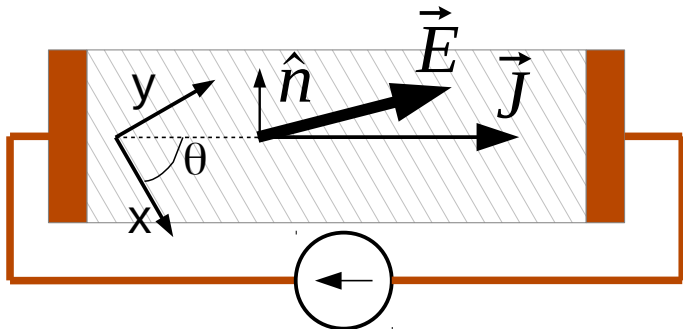
misure lungo assi principali: singoli elementi del tensore



$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix} \begin{pmatrix} 0 \\ J \end{pmatrix} = \rho_{22} J \hat{y} = E_y \hat{y}$$

$$\Rightarrow \rho_{mis} = \rho_{22}$$

$$\vec{E} \parallel \vec{J}$$



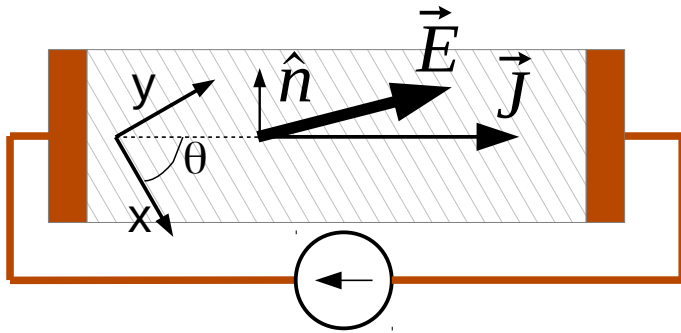
$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix} \begin{pmatrix} J \cos \theta \\ J \sin \theta \end{pmatrix} = J \begin{pmatrix} \rho_{11} \cos \theta \\ \rho_{22} \sin \theta \end{pmatrix} \Rightarrow \rho_{mis} = ?$$

$$\vec{E} \not\parallel \vec{J}$$

Misure di resistenza

Isotropia: $\rho_{mis} = \frac{E}{J}$

Anisotropia (2D): $\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$



$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{pmatrix} \begin{pmatrix} J \cos \theta \\ J \sin \theta \end{pmatrix} = J \begin{pmatrix} \rho_{11} \cos \theta \\ \rho_{22} \sin \theta \end{pmatrix} \rightarrow \rho_{mis} = ?$$

Misure lungo **due** direzioni: $\parallel \vec{J}$ e $\perp \vec{J}$

$$\parallel \vec{J} \quad \rho_{mis, \parallel} = \frac{E_{\parallel}}{J} = \frac{\vec{E} \cdot \hat{J}}{J} = \rho_{11} \cos^2 \theta + \rho_{22} \sin^2 \theta$$

oss.: dipendenza dall'angolo θ tra assi di anisotropia e stimolo

$$\perp \vec{J} \quad \text{i.e.} \quad \parallel \hat{n} \quad \rho_{mis, \perp} = \frac{E_{\perp}}{J} = \frac{\vec{E} \cdot \hat{n}}{J} = \sin \theta \cos \theta (\rho_{22} - \rho_{11})$$

$$\hat{n} = \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}$$

NB: in esperimenti reali, imporre e controllare l'effettivo percorso della corrente è tutt'altro che banale

Tensore massa efficace

- Modello di Drude

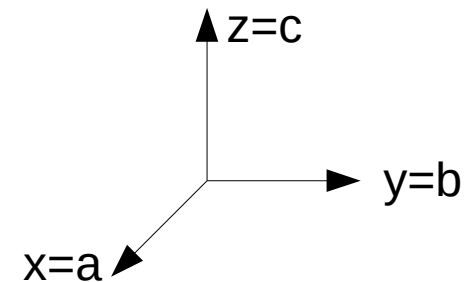
$$\rho = \frac{m}{n e^2 \tau}$$

- Materiale anisotropo, riferimento con assi principali:

$$\bar{\rho} = \frac{\bar{m}}{n e^2 \tau} \quad \longrightarrow \quad \bar{m} = \begin{pmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{pmatrix}$$

oss.: si assume che τ sia uno scalare

**tensore
di massa**



- Superconduttori con anisotropia uniassiale:

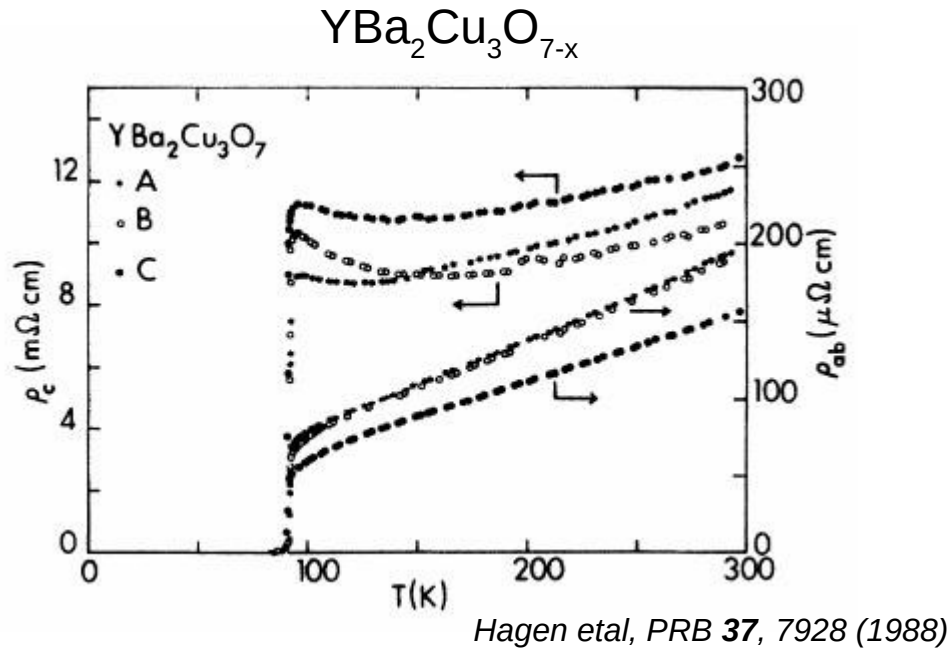
$$m_{11} \simeq m_{22} \ll m_{33}$$

$$\bar{m} = \begin{pmatrix} m_{11} & 0 & 0 \\ 0 & m_{11} & 0 \\ 0 & 0 & m_{33} \end{pmatrix} = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \gamma^2 \end{pmatrix}$$

$$\gamma^2 = \frac{m_{33}}{m_{11}} = \frac{m_c}{m_{ab}} > 1$$

**fattore di
anisotropia**

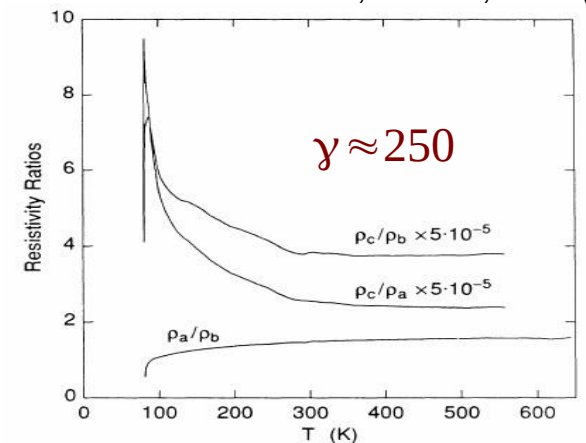
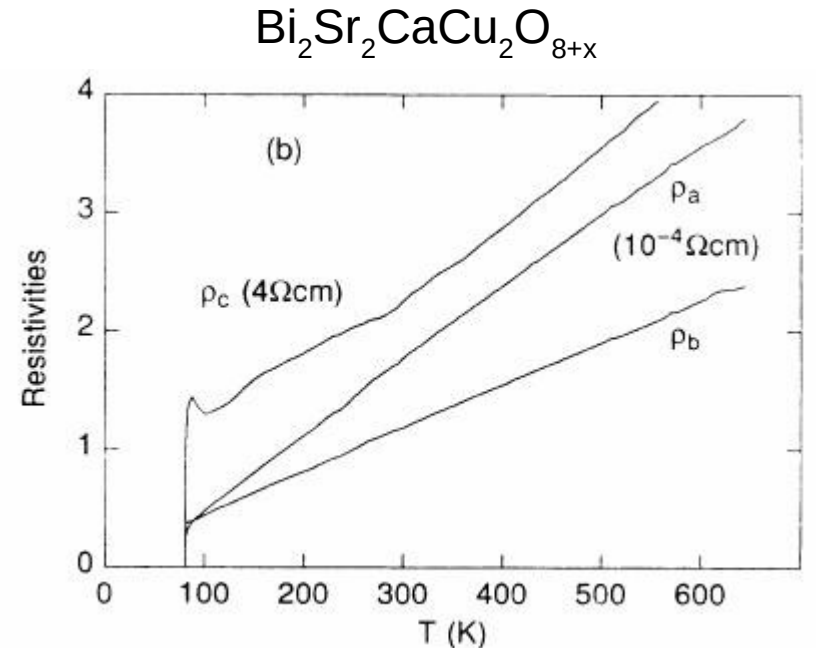
Resistenza anisotropia nei cuprati (stato normale)



$$\frac{\rho_c}{\rho_{ab}} = \gamma^2 \approx 60 - 80 (290 \text{ K})$$

$$\gamma \approx 7 - 9$$

Moderatamente anisotropo

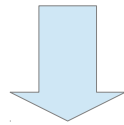


Fortemente anisotropo

Anisotropia nello stato misto: teoria GL

- GL (3D) anisotropa, con tensore \bar{m} . Il termine col gradiente diventa:

isotropo $\frac{1}{2m} \left| (-i\hbar \nabla + 2e \vec{A}) \psi(\vec{r}) \right|^2$



anisotropo $\frac{1}{2} (-i\hbar \nabla + 2e \vec{A})^* \psi^*(\vec{r}) \cdot [\bar{m}^{-1} (-i\hbar \nabla + 2e \vec{A}) \psi(\vec{r})] =$

$$= \frac{1}{2m_{ab}} \left| (-i\hbar \nabla_{xy} + 2e \vec{A}_{xy}) \psi(\vec{r}) \right|^2 + \frac{1}{2m_c} \left| \left(-i\hbar \frac{\partial}{\partial z} + 2e A_z \right) \psi(\vec{r}) \right|^2$$

Anisotropia di ξ e λ

- Lunghezza di coerenza: $\xi_i = \frac{\hbar}{\sqrt{2 m_i |\alpha|}} \propto \frac{1}{\sqrt{m_i}}$ con $i = ab, c$ $\Rightarrow \xi_{ab} > \xi_c$

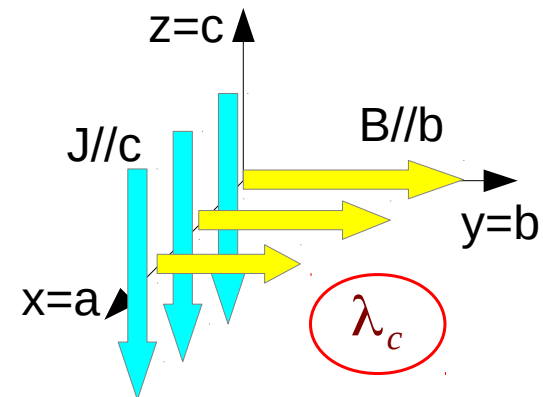
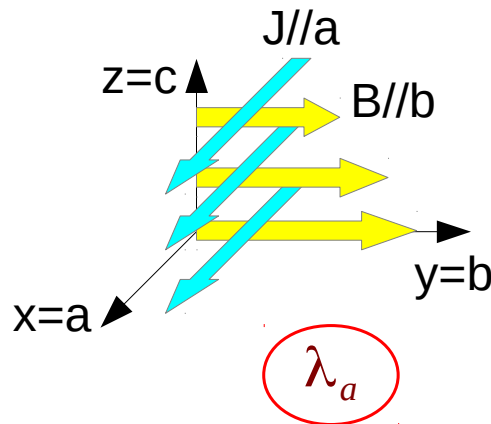
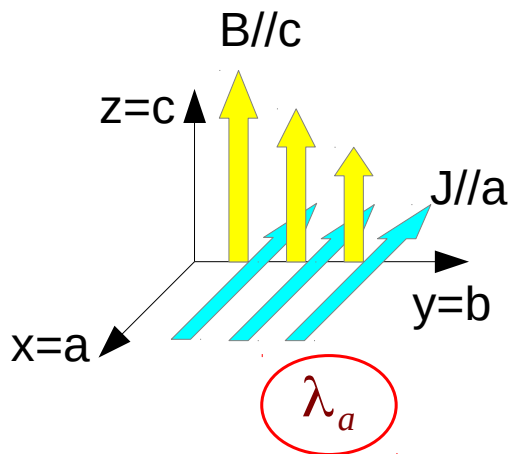
– ξ_i : variazione di ψ lungo l'asse i -mo

- Lunghezza di penetrazione di London:

$$\xi_i \lambda_i = \frac{\Phi_0}{2\sqrt{2\pi} B_c} \Rightarrow \lambda_i \propto \sqrt{m_i} \Rightarrow \lambda_{ab} < \lambda_c$$

effetto opposto
dell'anisotropia

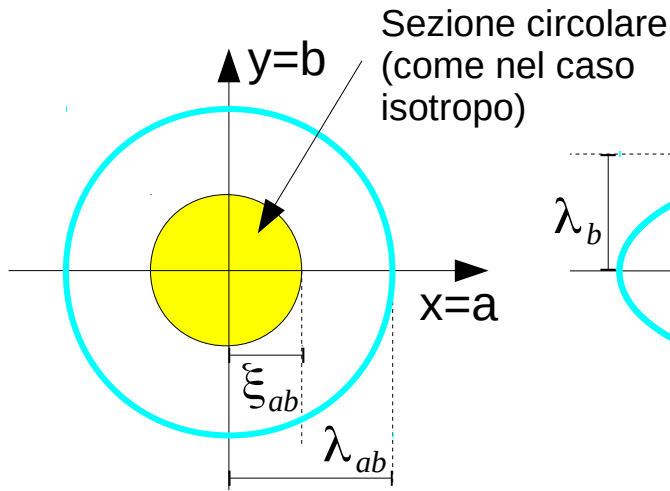
– λ_i : associata all'effetto di schermo dovuto a correnti lungo l'asse i -mo (\neq schermo di campi lungo l'asse i -mo)



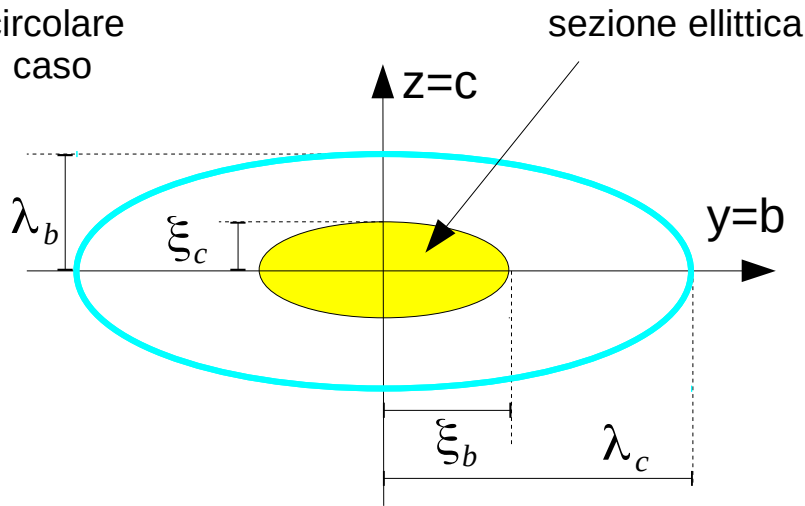
Effetti dell'anisotropia su flussoni e B_{c2}

θ : angolo tra B e asse z

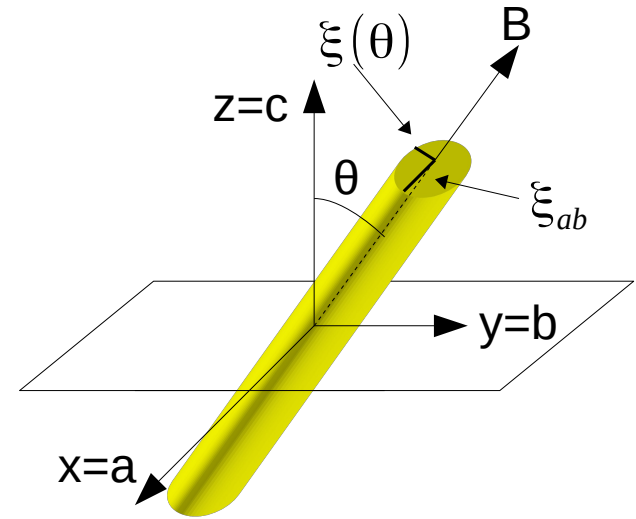
$B(0) // c$



$B(90^\circ) // a$



$B(\theta)$



$$B_{c2}(0) = \frac{\Phi_0}{2\pi \xi_{ab}^2}$$

area sezione del nucleo

$$B_{c2}(90^\circ) = \frac{\Phi_0}{2\pi \xi_{ab} \xi_c}$$

$$B_{c2}(90^\circ) = \frac{\Phi_0 \gamma}{2\pi \xi_{ab}^2}$$

$$B_{c2}(\theta) = \frac{\Phi_0}{2\pi \xi_{ab} \xi(\theta)}$$

$$\xi(\theta) = \xi_{ab} \sqrt{\cos^2 \theta + \gamma^{-2} \sin^2 \theta}$$

Anisotropia: campi critici B_{c1} e B_{c2}

Dipendenze di B_{c2} da θ e T

$$B_{c2}(\theta) = \frac{\Phi_0}{2\pi \xi_{ab}^2 \sqrt{\cos^2 \theta + \gamma^{-2} \sin^2 \theta}} \propto 1 - T/T_c$$

$\xi \propto \frac{1}{\sqrt{(1 - T/T_c)}}$

Espressione alternativa:

$$\left(\frac{B_{c2}(\theta) \cos \theta}{B_{c2\parallel c}} \right)^2 + \left(\frac{B_{c2}(\theta) \sin \theta}{B_{c2\parallel ab}} \right)^2 = 1$$

Ricapitolazione rapporti di anisotropia:

$$B_{c1} \propto 1/\lambda^2$$

$$\gamma = \left(\frac{m_c}{m_{ab}} \right)^{1/2} = \frac{\xi_{ab}}{\xi_c} = \frac{\lambda_c}{\lambda_{ab}} = \frac{B_{c2,\parallel ab}}{B_{c2,\parallel c}} = \frac{B_{c1,\parallel c}}{B_{c1,\parallel ab}}$$

Anisotropia nello stato misto: modello LD

- Modello di Lawrence-Doniach (2D): strati bidimensionali sovrapposti con accoppiamento Josephson tra strati adiacenti. Energia di Gibbs (termini in B omessi per brevità):

$$G = \sum_n \int dx dy \left[\alpha |\psi_n|^2 + \frac{1}{2} \beta |\psi_n|^4 + \frac{\hbar^2}{2 m_{ab}} \left(\left| \frac{\partial \psi_n}{\partial x} \right|^2 + \left| \frac{\partial \psi_n}{\partial y} \right|^2 \right) + \frac{\hbar^2}{2 m_c s^2} |\psi_n - \psi_{n-1}|^2 \right] + \dots$$

somma sugli N strati integrale (2D) sul singolo strato distanza tra strati accoppiamento Josephson

- Prendendo $\psi_n = |\psi_n| e^{i\varphi_n}$ con lo stesso $|\psi_n| \forall n$

$$\frac{\hbar^2}{2 m_c s^2} |\psi_n - \psi_{n-1}|^2 = \frac{\hbar^2}{m_c s^2} |\psi_n|^2 [1 - \cos(\varphi_n - \varphi_{n-1})]$$

Energia di accoppiamento Josephson con $E_j = \hbar^2 / m_c s^2 |\psi_n|^2$

- Limite LD \rightarrow GL per $s \ll \xi_c$ (scala di variazione di ψ lungo z):

$$\frac{\hbar^2}{2 m_c} \frac{|\psi_n - \psi_{n-1}|^2}{s^2} \rightarrow \frac{\hbar^2}{2 m_c} \left| \frac{\partial \psi}{\partial z} \right|^2$$

$\xi_c(T) \rightarrow \infty$ per $T \rightarrow T_c$

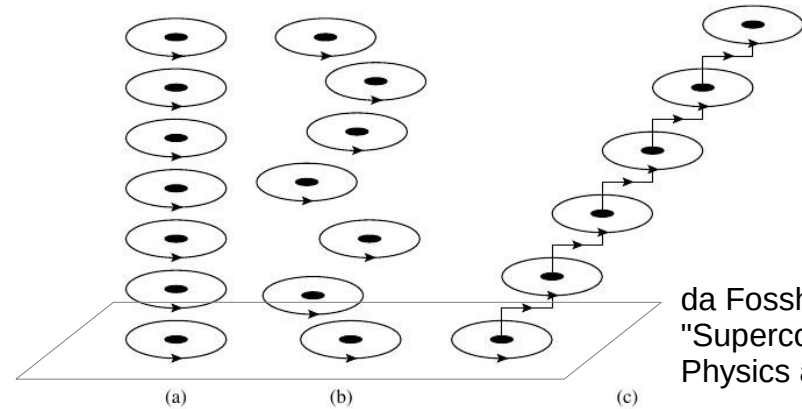
crossover da 2D a 3D a T^* t.c. $\xi_c(T^*) = s / \sqrt{2}$

YBCO: $T^*/T_c \sim 0.84$, 3D tra $T^* \sim 78$ K e $T_c = 92$ K
 BSCCO: $T^*/T_c = 0.999$, 3D per ~ 0.1 K sotto T_c

SC quasi-2D: flussoni e B_{c2}

strati disaccoppiati → flussone scomposto in oggetti 2D (“pancakes”) (pancake su piani adiacenti si attraggono)

- a) H perpendicolare ai piani (basse T)
- b) alte T: agitazione termica disallinea i pancake
- c) campo inclinato



da Fossheim e Sudbø
"Superconductivity -
Physics and Applications"

Dipendenza angolare di B_{c2} :

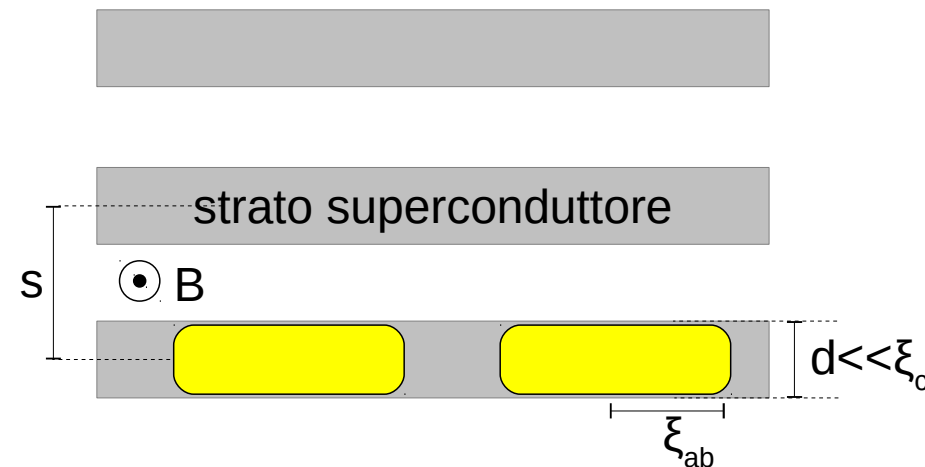
$$\frac{B_{c2}(\theta) |\cos \theta|}{B_{c2||c}} + \left(\frac{B_{c2}(\theta) \sin \theta}{B_{c2||ab}} \right)^2 = 1$$

cuspidate a $\theta=90^\circ$

con:

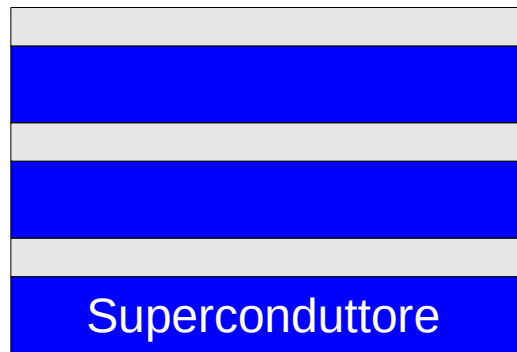
$$B_{c2||c} = \frac{\Phi_0}{2\pi \xi_{ab}^2(T)} \propto 1 - T/T_c \quad \text{come 3D}$$

$$B_{c2||ab} = \frac{\Phi_0}{2\pi \xi_{ab}(T) \left(\frac{d}{2\sqrt{3}} \right)} \propto \sqrt{1 - T/T_c}$$



Confronti con l'esperimento

Multistrati artificiali con
superconduttori a bassa T_c



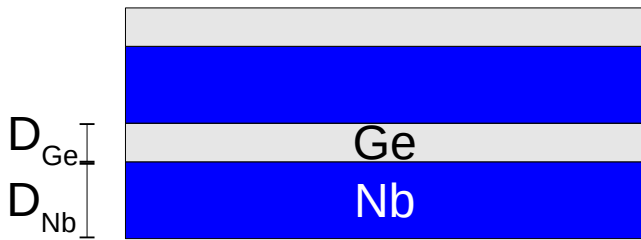
Controllo dei parametri strutturali

Crossover 3D - 2D

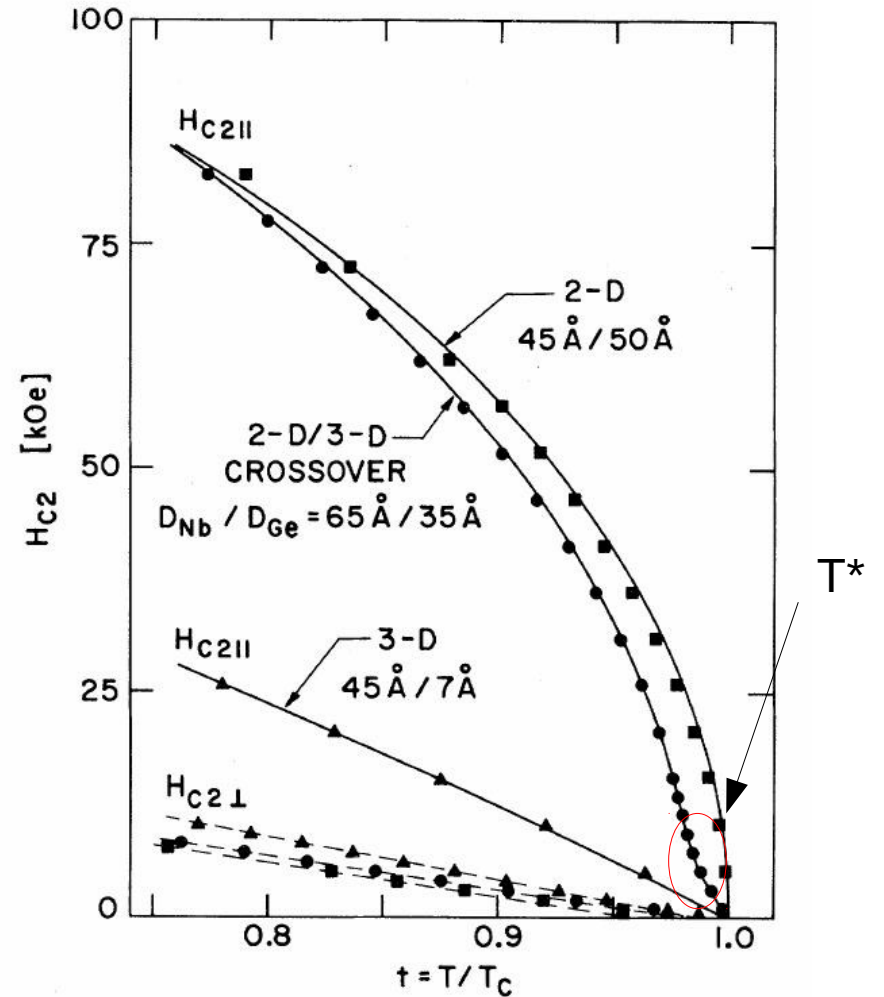
Multistrati: crossover 2D – 3D

Multistrato artificiale Nb/Ge

- Nb: (bulk: $T_c=9.25$ K, $\xi(0)=38$ Å)
- Ge: isolante (amorfo)



D_{Nb}/D_{Ge} (Å/Å)	T_c (K)	$\xi_z(0)$ (Å)
45/7	5.79	38.6
65/35	6.88	9.64
37/32	4.95	8.13
45/50	4.69	0



3D

$$H_{c2} \propto 1 - T/T_c$$

2D

$$H_{c2||c} \propto 1 - T/T_c$$

$$H_{c2||ab} \propto \sqrt{1 - T/T_c}$$

$$H_{c2||} = H_{c2||ab}$$

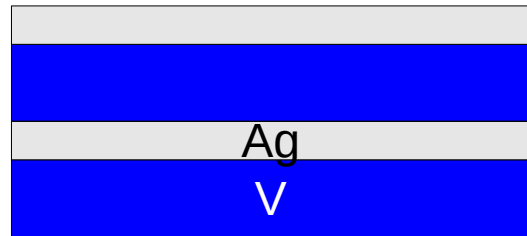
$$H_{c2\perp} = H_{c2||c}$$

Ruggiero et al, PRL 45, 1299 (1980)

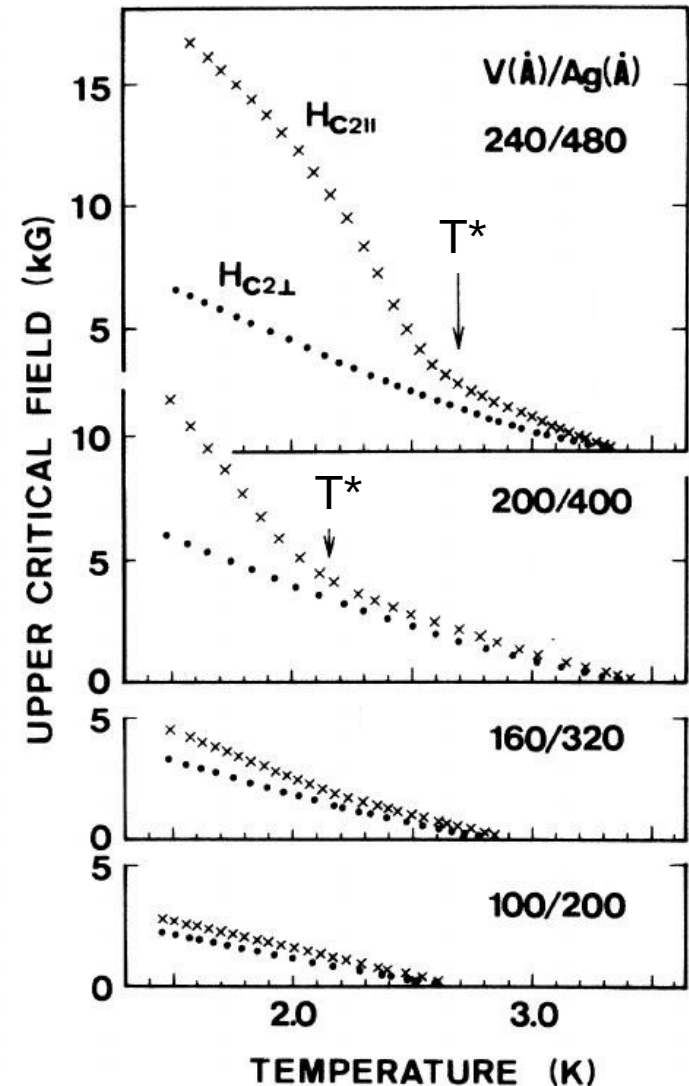
Multistrati: crossover 2D – 3D

Multistrato artificiale V/Ag

- V (bulk: $T_c = 5.4$ K)



Thickness of V/ thickness of Ag (Å)	t_A	ξ_{\perp} (Å)	T^*		
			ξ_{\parallel} (Å)	d_{\perp}^{Δ} (Å)	d_{\perp}^{\square} (Å)
240/480	0.79	274	431	640	687
200/400	0.62	250	310	584	627
160/320
150/50



Kanoda et al, PRB 33, 2052 (1986)

3D

$$B_{c2} \propto 1 - T/T_c$$

2D

$$B_{c2||c} \propto 1 - T/T_c$$

$$B_{c2||ab} \propto \sqrt{1 - T/T_c}$$

$$B_{c2||} = B_{c2||ab}$$

$$B_{c2\perp} = B_{c2||c}$$

Multistrati: crossover 2D – 3D

Multistrato artificiale Nb/Cu

Chun et al, PRB 29, 4915 (1984)

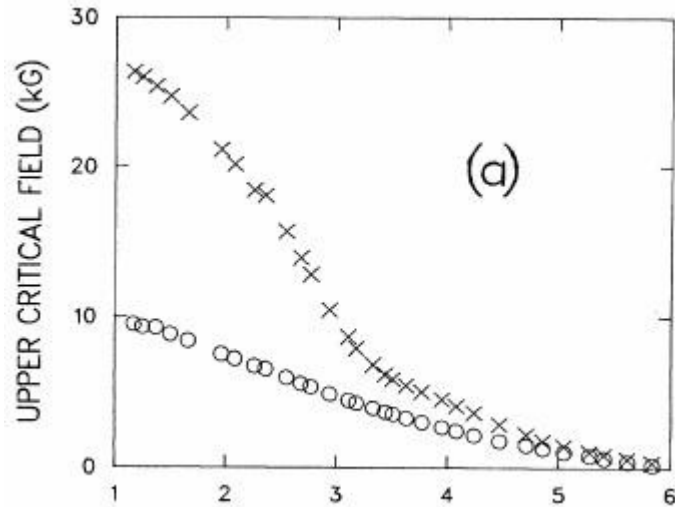


FIG. 3. (a) Upper critical fields for a Nb/Cu superlattice. $D_{\text{Nb}}=172 \text{ \AA}$ and $D_{\text{Cu}}=333 \text{ \AA}$ (1500 \AA of Cu as the outside layers); \times , $H_{c2\parallel}$; \circ , $H_{c2\perp}$.

3D

$$B_{c2} \propto 1 - T/T_c$$

2D

$$B_{c2\parallel c} \propto 1 - T/T_c$$

$$B_{c2\parallel ab} \propto \sqrt{1 - T/T_c}$$

$$\left(\frac{B_{c2}(\theta) \cos \theta}{B_{c2\parallel c}} \right)^2 + \left(\frac{B_{c2}(\theta) \sin \theta}{B_{c2\parallel ab}} \right)^2 = 1$$

$$\frac{B_{c2}(\theta) |\cos \theta|}{B_{c2\parallel c}} + \left(\frac{B_{c2}(\theta) \sin \theta}{B_{c2\parallel ab}} \right)^2 = 1$$

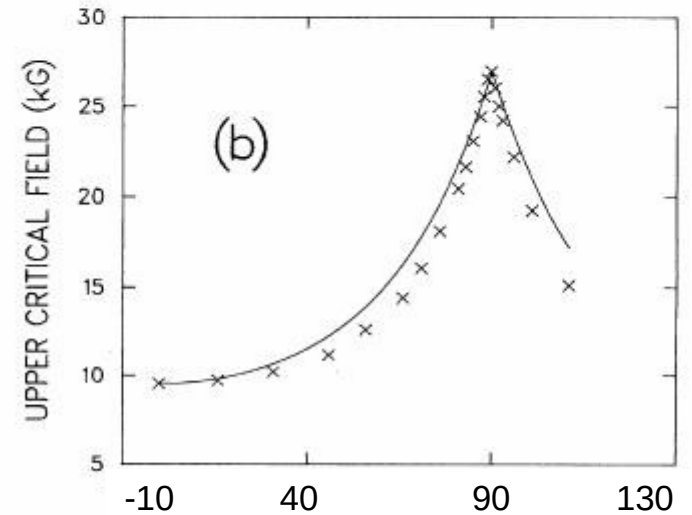
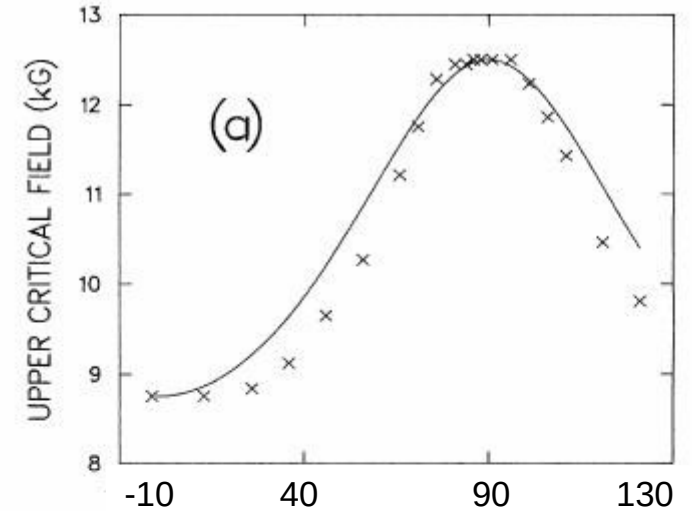


FIG. 6. Angular dependence of critical fields for a Nb/Cu superlattice showing (a) anisotropic 3D behavior ($D_{\text{Nb}}=23 \text{ \AA}$, $D_{\text{Cu}}=23 \text{ \AA}$, and $T=1.17 \text{ K}$), and (b) 2D behavior ($D_{\text{Nb}}=172 \text{ \AA}$, $D_{\text{Cu}}=333 \text{ \AA}$, and $T=1.17 \text{ K}$).

Approccio di scala

- Si dimostra che, nelle ipotesi:

- $\kappa \gg 1$ (cuprati: $\kappa \sim 100$)
- $B \gg B_{c1}$ (cuprati: $B_{c1} \sim 10 - 100 G$)

l'energia di Gibbs e tutte le grandezze termodinamiche dipendono da θ e B solo attraverso:

$$B/B_{c2}(\theta)$$

- Se si pone $B_{c2}(\theta) = B_{c2||c} f(\theta)$, data una generica quantità fisica Q :

$$Q(B, \theta; T) = Q\left(\frac{B}{B_{c2||c} f(\theta)}; T\right) \quad \text{Studio dell'anisotropia tramite la sola } f(\theta)$$

- proprietà osservata sperimentalmente anche in grandezze relative al trasporto elettrico:

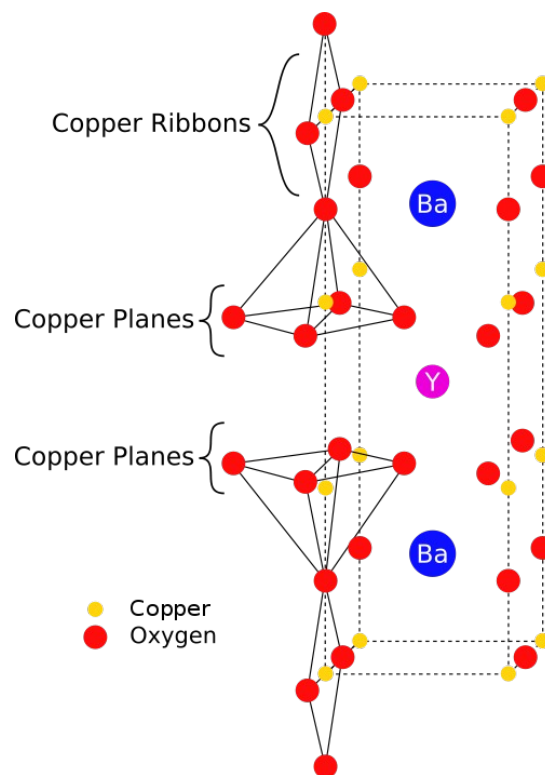
- resistività d.c. $\rho(B, \theta; T)$
- corrente critica $J_c(B, \theta; T)$
- impedenza superficiale $Z_s(B, \theta; T)$



Il pinning può rendere inapplicabili le proprietà di scala

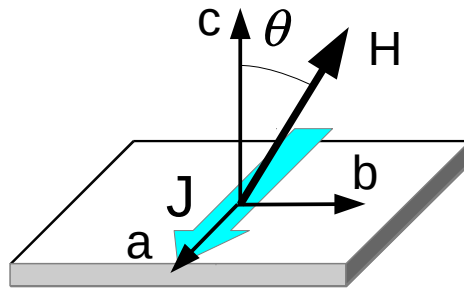
Confronti con l'esperimento

Misure angolari di
trasporto elettrico
su $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$



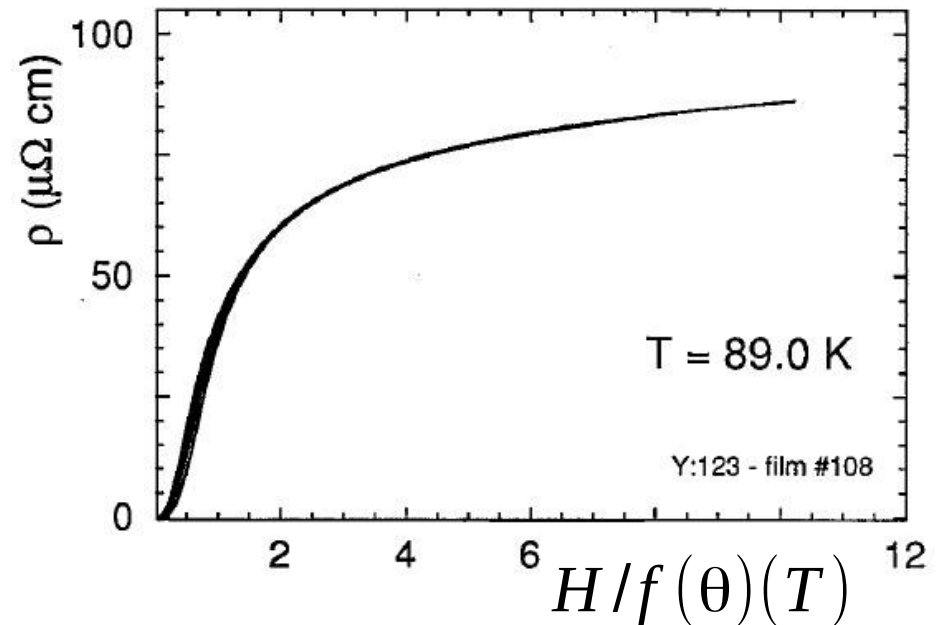
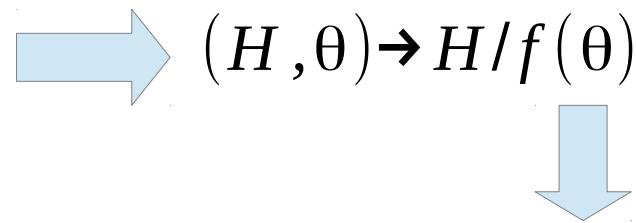
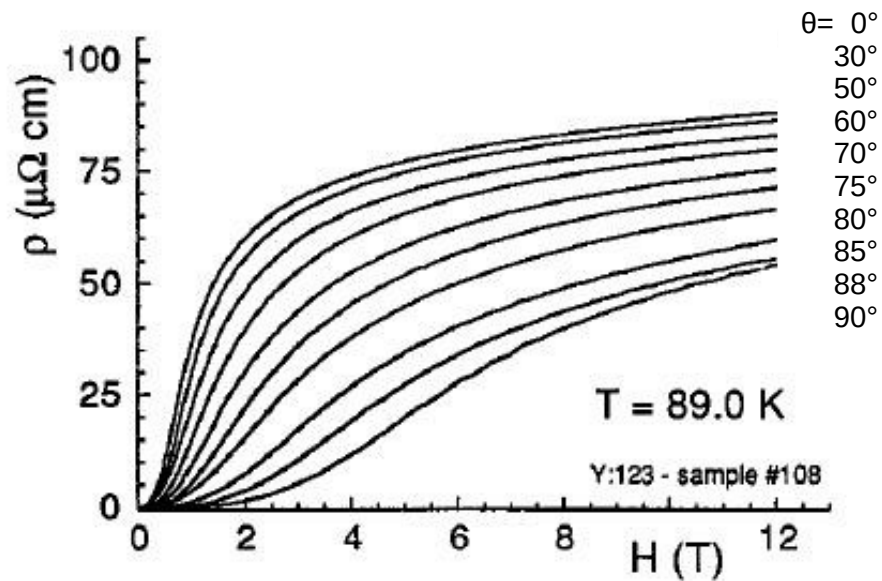
Proprietà di scala: $\rho(H, \theta)$ in d.c.

Misure su film di YBCO



$$\vec{J} \perp \vec{H} \quad \forall \theta$$

configurazione
"maximum Lorentz force"

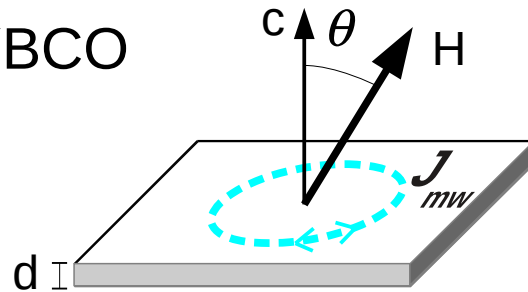


$f(\theta)$ determinata in modo tale da far sovrapporre tutte le curve

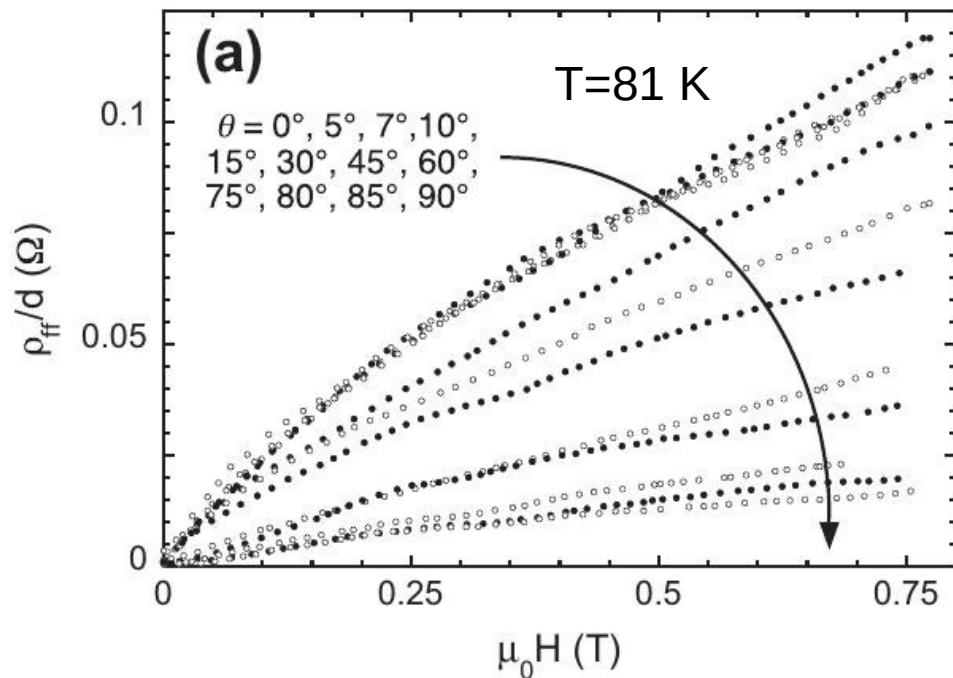
Proprietà di scala: $\rho_{ff}(H, \theta)$ a microonde

Misure di impedenza superficiale su film di YBCO

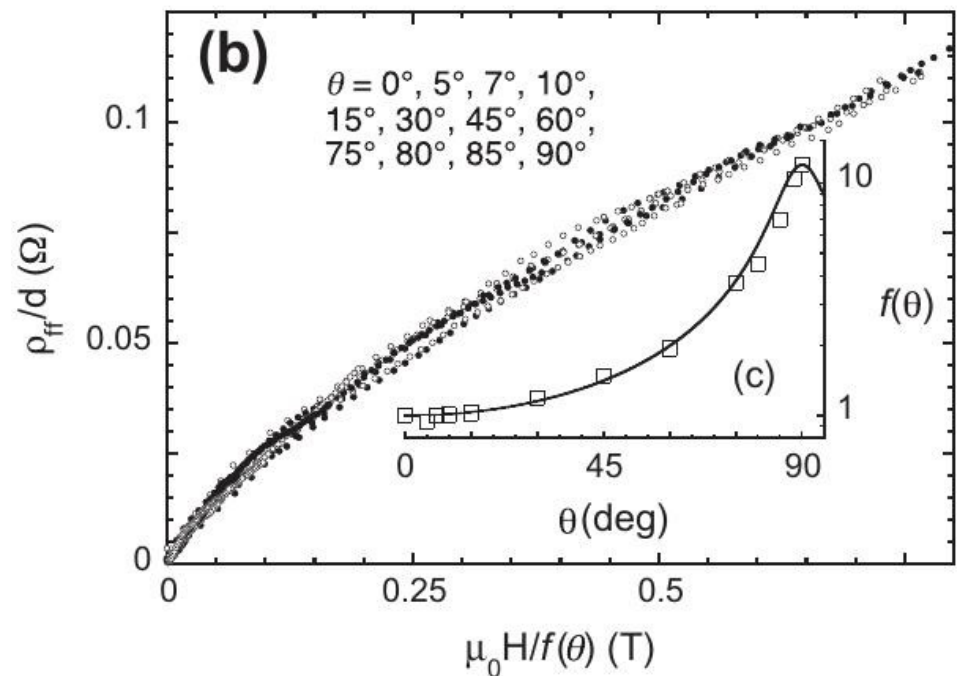
$$Z_s d = \tilde{\rho} = \rho_1 + i\rho_2 \quad \rightarrow \quad \rho_{ff}$$



correnti circolari



$$(H, \theta) \rightarrow H/f(\theta)$$

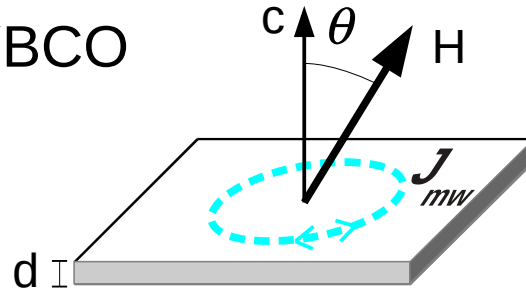


$f(\theta)$ determinata in modo tale da far sovrapporre tutte le curve

Proprietà di scala: resistività complessa (H, θ) a microonde

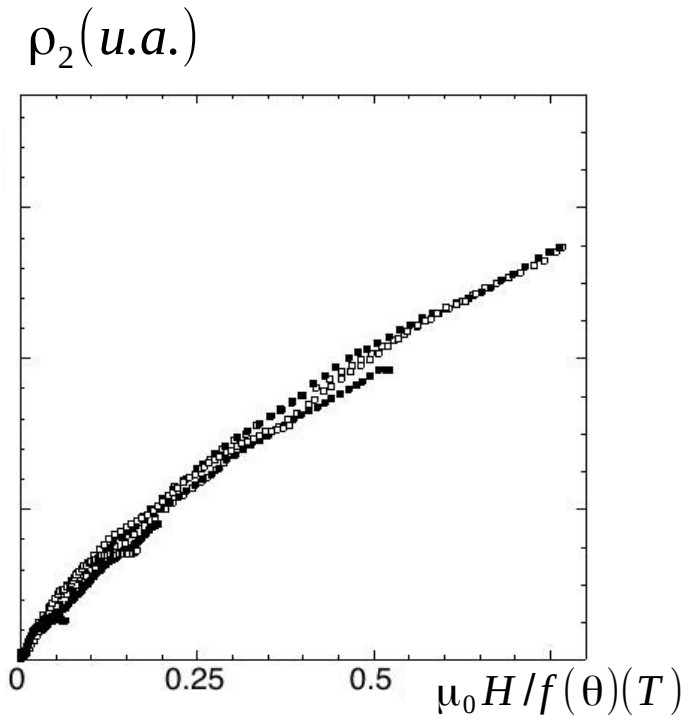
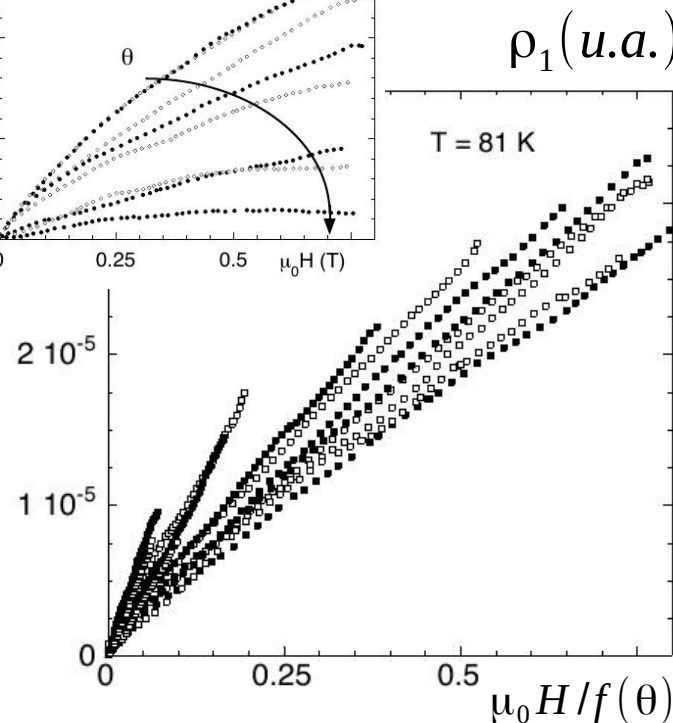
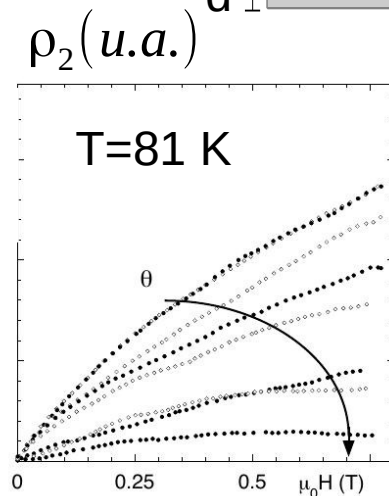
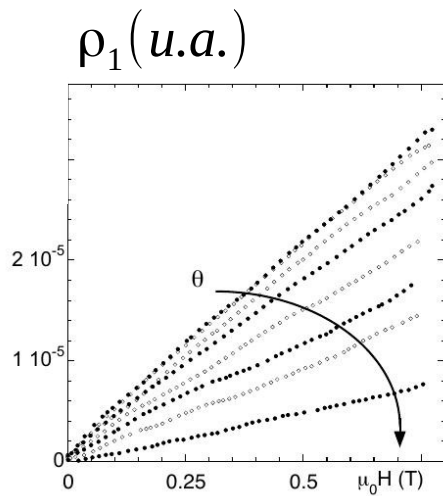
Misure di impedenza superficiale su film di YBCO

$$Z_s d = \tilde{\rho} = \rho_1 + i\rho_2$$



$f(\theta)$ determinata a partire da $\rho_{ff}(H, \theta)$

$$(H, \theta) \rightarrow H/f(\theta)$$

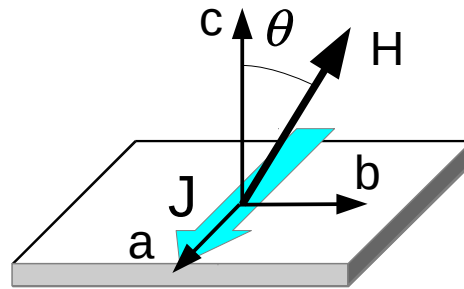


NB: ρ_1 e ρ_2 sono determinate anche dal pinning.

Proprietà di scala inapplicabile: sono presenti centri di pinning estesi

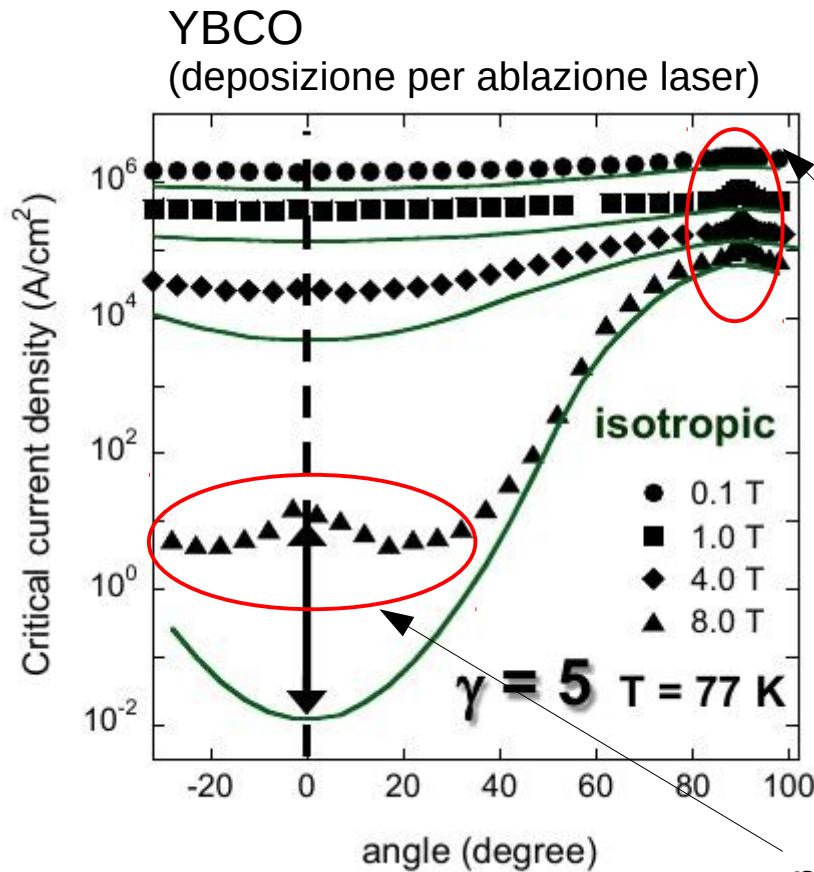
Proprietà di scala: $J_c(H, \theta)$

Misure su film di YBCO



$$\vec{J} \perp \vec{H} \quad \forall \theta$$

configurazione
"maximum Lorentz force"



pinning
"intrinseco"
dovuto ai
piani

pinning da
difetti estesi
lungo l'asse c

