Synthesis of electromagnetic Schell-model sources

M. Santarsiero,[1,*] R. Borghi,[2] and V. Ramírez-Sánchez[3]

[1]*Università Roma Tre, Dipartimento di Fisica, and CNISM, Via della Vasca Navale 84, I-00146 Rome, Italy*
[2]*Università Roma Tre, Dipartimento di Elettronica Applicata, and CNISM, Via della Vasca Navale 84, I-00146 Rome, Italy*
[3]*Universidad Complutense de Madrid, Departamento de Óptica, Facultad de Ciencias Físicas, Ciudad Universitaria, E-28040, Madrid, Spain*
**Corresponding author: santarsiero@fisuniroma3.it*

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A procedure for the synthesis of the most general electromagnetic Schell-model light source is proposed. It makes use of the generalized van Cittert–Zernike theorem to produce the electromagnetic source starting from a primary spatially incoherent source, characterized by a suitable position-dependent polarization matrix. By resorting to the spectral decomposition of the polarization matrix, it is shown how such an incoherent source can be synthesized by using a Mach–Zehnder interferometer, with suitable amplitude transmittances placed in its arms, fed by two mutually uncorrelated laser beams. Examples are given for the case of electromagnetic Gaussian Schell-model sources. © 2009 Optical Society of America

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1. INTRODUCTION

Schell-model (SM) sources were introduced in the scalar theory of coherence [1,2] as a fundamental tool to describe partially coherent light sources having shift-invariant degree of spatial coherence. They found application in the study of the radiation emitted by several natural sources [3,4] and, since they can always be synthesized in the laboratory starting from spatially incoherent sources [5], they soon became a typical tool for experimentally validating several results of the scalar theory of coherence [6–14].

The generalization of SM sources to the vectorial case has been considered in recent times, and, as well as their scalar counterparts, electromagnetic SM (ESM) sources [15] are playing an important role in the development of modern coherence theory. In particular, great attention has been devoted to the so-called electromagnetic Gaussian SM (EGSM) sources, which have been introduced [16–18] within the framework of the electromagnetic theory of coherence as the natural generalization to the vectorial case of scalar GSM sources [2,19]. Since their introduction, EGSM sources have proved to be useful in revealing interesting and intriguing new coherence and polarization features in the study of random electromagnetic beams [20–33].

On the other hand, synthesis of ESM sources is still far from being a common task in optics laboratories, and, until very recent times, experimental procedures had been proposed only for some specific classes of ESM sources [34,35]. The first experimental arrangement was presented some years ago [34] for the synthesis of a particular type of EGSM source. These ESGM sources were generated by exploiting the van Cittert–Zernike (vCZ, henceforth) theorem for electromagnetic sources [36], starting from two spatially incoherent primary sources that had suitable intensity profiles and were linearly polarized along orthogonal axes and that eventually superimposed through a Mach–Zehnder interferometer (MZI, henceforth). Later, a different method for generating EGSM beams, which does not use the vCZ theorem, was proposed [35]. According to it, the source is synthesized by superimposing two spatially coherent beams with orthogonal polarizations, their phases modulated by two mutually correlated liquid-crystal spatial light modulators and their intensities modulated by two Gaussian-shaped amplitude filters. Even in this case, however, obtainable sources constitute a subset of the most general class of EGSM sources.

In a recent paper, however, a procedure that represents the extension to the vectorial case of the well-established technique used in the scalar case has been suggested for the synthesis of the most general ESM source [37]. It has been shown, in fact, that any ESM source can be synthesized by using the vCZ theorem, starting from a primary planar spatially incoherent source that is characterized by a suitable position-dependent polarization matrix.

Controlling the intensity distribution and the polarization state of a spatially incoherent source is a task that is far from trivial from an experimental point of view. In the present paper we propose a possible arrangement for producing such a source, to be used in the synthesis of a general ESM source. The technique exploits the fact that any polarization matrix can be synthesized starting from two perfectly polarized fields having orthogonal polarizations, incoherently superimposed with suitable powers. The analytical properties of such fields suggest the use of an experimental setup based on a MZI, endowed with amplitude transmittances placed into its arms, and anisotropic optical elements. As we shall see, in many cases of practical interest only conventional optical elements are needed.

The paper is arranged as follows. In Section 2, ESM sources are described in the framework of the so-called beam-coherence–polarization (BCP) matrix, and realizability conditions for such sources are given. In Sections 3 and 4 the general synthesis scheme and the procedure to generate the primary incoherent source are given. As an example, the proposed method is applied to the particular case of EGSM sources in Section 5. Finally, the main conclusions are summarized in Section 6.

2. ELECTROMAGNETIC SCHELL-MODEL SOURCES

We shall describe an ESM source by means of the BCP matrix [38,39]. Let us recall that this is a 2×2 matrix, whose elements are defined through the relations

$$J_{\alpha\beta}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\langle E_\alpha(\boldsymbol{r}_1,t)E^*_\beta(\boldsymbol{r}_2,t)\rangle;\quad (\alpha,\beta=x,y), \tag{1}$$

where $E_\alpha(\boldsymbol{r},t)$ $(\alpha=x,y)$ is the analytic signal associated with the electric field component along the α axis of a quasi-monochromatic wave at position $\boldsymbol{r}$ and time t. The angular brackets denote time averages. The extension to a space–frequency approach, where the BCP matrix is replaced by the cross-spectral density tensor [2,3,15], is straightforward and will not be considered here.

The most general ESM source is characterized by a BCP matrix whose elements are of the following form [2,15]:

$$J_{\alpha\beta}(\boldsymbol{r}_1,\boldsymbol{r}_2)=s_\alpha(\boldsymbol{r}_1)s_\beta(\boldsymbol{r}_2)j_{\alpha\beta}(\boldsymbol{r}_1-\boldsymbol{r}_2). \tag{2}$$

Its diagonal elements ($J_{\alpha\alpha}$, $\alpha=x,y$) correspond to the mutual intensities that would characterize the source if the β component of the field $(\beta\neq\alpha)$ were eliminated, for example, by a linear polarizer. In the present case, they represent scalar SM sources and the functions $j_{\alpha\alpha}$ are the corresponding degrees of spatial coherence [2], while $s_\alpha(\boldsymbol{r})$ $(\alpha=x,y)$, defined as

$$s_\alpha(\boldsymbol{r})=\sqrt{J_{\alpha\alpha}(\boldsymbol{r},\boldsymbol{r})}, \tag{3}$$

are nonnegative functions. The analytical forms of such degrees of spatial coherence cannot be chosen at will, because they have to ensure that the functions $J_{\alpha\alpha}$ represent nonnegative definite kernels, as required for spatial correlation functions [2]. It can be proved, however, that the nonnegativeness condition is satisfied if, and only if, $j_{\alpha\alpha}$ have nonnegative Fourier transforms, i.e. [8],

$$\tilde{j}_{\alpha\alpha}(\boldsymbol{\nu})\geqslant 0\quad(\alpha=x,y) \tag{4}$$

for any $\boldsymbol{\nu}$, where the tilde denotes Fourier transformation and $\boldsymbol{\nu}$ is the vector position across the Fourier plane.

Functions J_{xy} and J_{yx}, on the other hand, account for the correlations that exist at two distinct points between the x and the y component of the field. In particular, since $J_{xy}(\boldsymbol{r}_1,\boldsymbol{r}_2)=J^*_{yx}(\boldsymbol{r}_2,\boldsymbol{r}_1)$, Eqs. (2) and (3) lead to

$$j_{yx}(\boldsymbol{r}_1-\boldsymbol{r}_2)=j^*_{xy}(\boldsymbol{r}_2-\boldsymbol{r}_1), \tag{5}$$

or, equivalently, in the Fourier domain,

$$\tilde{j}_{yx}(\boldsymbol{\nu})=\tilde{j}^*_{xy}(\boldsymbol{\nu}). \tag{6}$$

As far as the functional form of j_{xy} is concerned, it cannot be arbitrary either, because of the nonnegativity constraint that must be fulfilled by the BCP matrix [39]. In [37] it has been proved, however, that the necessary and sufficient condition for the BCP matrix of a typical ESM source to be *bona fide* is

$$|\tilde{j}_{yx}(\boldsymbol{\nu})|\leqslant\sqrt{\tilde{j}_{xx}(\boldsymbol{\nu})\tilde{j}_{yy}(\boldsymbol{\nu})} \tag{7}$$

for any $\boldsymbol{\nu}$. As we shall see in the next section, the proposed scheme for the synthesis of the ESM source takes Eq. (7) into account automatically.

3. SYNTHESIS SCHEME

The synthesis scheme proposed in [37] for generating EGSM sources basically consists of a Fourier transforming optical system, followed by an amplitude filter (see Fig. 1). The transmission function of the filter, say, $t_\alpha(\boldsymbol{r})$, may be different for the two polarization components. This could be implemented, for instance, by splitting the two polarization components by means of a polarizing beam splitter, filtering them with two different amplitude masks, and eventually recombining them. Of course, such a procedure is not necessary if $t_x(\boldsymbol{r})=t_y(\boldsymbol{r})$, in which case a single mask is sufficient.

The impulse response of the system in Fig. 1 is given by

$$H_\alpha(\boldsymbol{r};\boldsymbol{u})=t_\alpha(\boldsymbol{r})\exp\left(\mathrm{i}\frac{2\pi}{\lambda f}\boldsymbol{r}\cdot\boldsymbol{u}\right), \tag{8}$$

where $\boldsymbol{r}$ and $\boldsymbol{u}$ are two-dimensional spatial vectors across the output (Π_o) and input (Π_i) planes, respectively, λ is the radiation wavelength, and f is the focal length of the lens.

Let a spatially incoherent quasi-monochromatic planar source be placed across the plane Π_i and let $\hat{P}(\boldsymbol{u})$ denote the position-dependent polarization matrix of the field radiated from such source, at the source plane. Starting from the definition in Eq. (1), each element of the BCP matrix of the field at the output plane of the system can be evaluated as

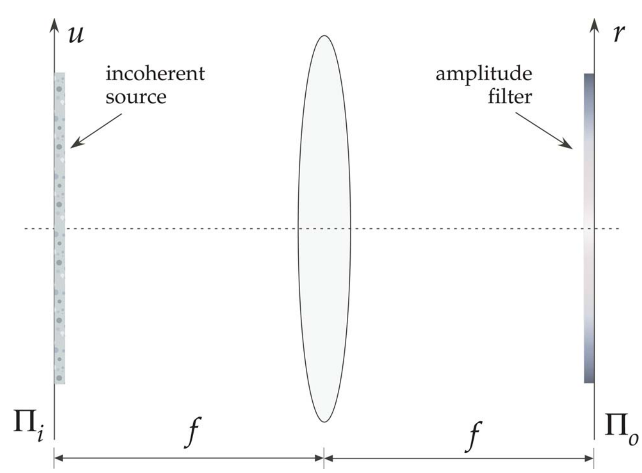

Fig. 1. (Color online) Fourier transforming optical system for the synthesis of electromagnetic Schell-model sources.

$$J_{\alpha\beta}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \int P_{\alpha\beta}(\boldsymbol{u}) H_\alpha(\boldsymbol{r}_1,\boldsymbol{u}) H^*_\beta(\boldsymbol{r}_2,\boldsymbol{u}) \mathrm{d}^2 u, \tag{9}$$

where $P_{\alpha\beta}$ are the elements of $\hat{P}$. It should be noted that, apart from curvature factors, an optical system of this kind corresponds to a free propagation in Fraunhofer conditions, so that Eq. (9) reduces to the vCZ theorem for electromagnetic sources [36].

On using Eqs. (8) and (9), together with the definition in Eq. (3), it is not difficult to show that the ESM source in Eq. (2) can be generated as the output of the optical system in Fig. 1, starting from a primary, spatially incoherent source characterized by a position-dependent polarization matrix, say, $\hat{P}(\boldsymbol{u})$, whose elements are given by

$$P_{\alpha\beta}(\boldsymbol{u}) = \frac{1}{\lambda^2 f^2} \tilde{j}_{\alpha\beta}\left(\frac{\boldsymbol{u}}{\lambda f}\right), \tag{10}$$

and using an amplitude mask across the plane Π_o of the form

$$t_\alpha(\boldsymbol{r}) = s_\alpha(\boldsymbol{r}). \tag{11}$$

It is interesting to note that the requirement that the incoherent source across Π_i be physically realizable is automatically satisfied if the condition in Eq. (7) is met. In fact, in order for the incoherent source to be realizable, the matrix $\hat{P}(\boldsymbol{u})$ must be a *bona fide* polarization matrix; i.e., it must be Hermitian, positive semidefinite, and with nonnegative diagonal elements at any source point. Hermiticity of $\hat{P}(\boldsymbol{u})$ follows directly from Eqs. (10) and (6), while the positivity of its diagonal elements comes from Eq. (4). On the other hand, semipositivity of $\hat{P}$ implies that its determinant must be nonnegative, i.e.,

$$P_{xx}(\boldsymbol{u}) P_{yy}(\boldsymbol{u}) \geqslant |P_{xy}(\boldsymbol{u})|^2 \tag{12}$$

for any $\boldsymbol{u}$. Taking Eq. (10) into account, the latter condition reduces to the following one:

$$\tilde{j}_{xx}\left(\frac{\boldsymbol{u}}{\lambda f}\right) \tilde{j}_{yy}\left(\frac{\boldsymbol{u}}{\lambda f}\right) \geqslant \left| \tilde{j}_{xy}\left(\frac{\boldsymbol{u}}{\lambda f}\right)\right|^2, \tag{13}$$

which is equivalent to that in Eq. (7). This means, in turn, that *any* ESM source can be realized through a synthesis scheme of this kind [37].

4. SYNTHESIS OF THE INCOHERENT SOURCE

It is easily realized that a critical point of the above procedure is represented by the synthesis of the primary incoherent source, whose position-dependent polarization matrix is defined in Eq. (10). In some simple cases this may not represent a serious problem [34], but in general one might need to control both the intensity and the polarization state of the source point by point, which is a rather challenging experimental problem. As we are going to show, however, this problem can be solved in the most general case by noting that the polarization matrix can always be expressed through its spectral decomposition [40], which in the present case reads

$$\hat{P} = \mu_+ \boldsymbol{U}_+ \boldsymbol{U}_+^\dagger + \mu_- \boldsymbol{U}_- \boldsymbol{U}_-^\dagger. \tag{14}$$

Here, $\mu_\pm$ are the two eigenvalues of $\hat{P}$, $\boldsymbol{U}_\pm$ are the corresponding eigenvectors, represented by column vectors, and the dagger denotes Hermitian conjugation. Of course, for a *bona fide* polarization matrix the eigenvalues are nonnegative and the eigenvectors are (or can be chosen as) orthonormal.

Physically, the expansion in Eq. (14) states that any polarization matrix can be synthesized starting from two perfectly polarized fields having orthogonal polarizations, represented by the Jones vectors $\boldsymbol{U}_\pm$, incoherently superimposed with powers given by $\mu_\pm$.

Eigenvalues and eigenvectors of $\hat{P}$ can be readily evaluated and turn out to be

$$\mu_\pm = \frac{1}{2}[(P_{xx} + P_{yy}) \pm \sqrt{(P_{xx} - P_{yy})^2 + 4|P_{xy}|^2}], \tag{15}$$

and

$$\boldsymbol{U}_+ = \frac{1}{\sqrt{1+\eta^2}} \begin{bmatrix} \eta e^{\mathrm{i}\phi} \\ 1 \end{bmatrix},$$

$$\boldsymbol{U}_- = \frac{1}{\sqrt{1+\eta^2}} \begin{bmatrix} -e^{\mathrm{i}\phi} \\ \eta \end{bmatrix}, \tag{16}$$

respectively, where

$$\eta = \frac{(P_{xx} - P_{yy}) + \sqrt{(P_{xx} - P_{yy})^2 + 4|P_{xy}|^2}}{2|P_{xy}|}, \tag{17}$$

and ϕ is the argument of the complex number P_{xy}. In the above expressions we omitted, for brevity, the explicit dependence on $\boldsymbol{u}$, but it should be kept in mind that the eigenvalues, the quantity η, and the phase ϕ depend in general on the position across the source. Furthermore, it should be stressed that η is nonnegative for any value of $\boldsymbol{u}$, as is evident from Eq. (17), but it may assume any value, including, as limiting values, zero and infinity, corresponding to modes polarized along x or y.

The symmetry of the mode structure evidenced in Eq. (16) suggests a practical way for producing the spatially incoherent source to be used in the synthesis of the ESM source. The scheme is depicted in Fig. 2. Basically, it relies on a MZI fed by two independent collimated monochromatic laser beams ($\ell_\pm$), linearly polarized at $\pi/4$ with respect to the horizontal, entering the interferometer from two orthogonal directions.

Consider first Fig. 2(a). The laser beam ℓ_+, after being expanded and collimated, passes through a transparency characterized by the real transmission function $\tau_+ = A\sqrt{\mu_+}$, placed at the transverse plane Π_+. The parameter A is an arbitrary amplitude factor and can be chosen at will, for instance, to set to 1 the maximum value of τ_+.

The polarizing beam splitter (PBS) splits the initial polarization state in such a way that the reflected component (y polarized) is sent to the upper branch of the MZI, while the transmitted component (x polarized) goes through the lower branch. The lens L_+ images the field emerging from Π_+ onto the planes Π_ℓ and Π_u, in the lower

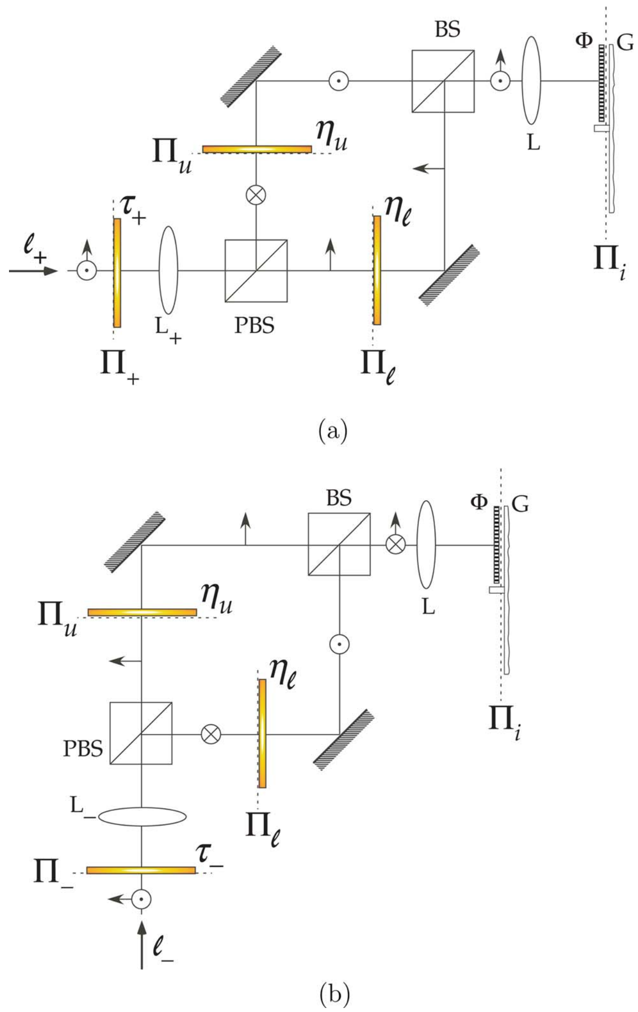

Fig. 2. (Color online) A MZI fed by two independent laser beams: ℓ_+ (a) and ℓ_- (b).

and in the upper path, respectively. At planes Π_ℓ and Π_u two transparencies, having transmission functions given by $\eta_\ell = \eta/\sqrt{1+\eta^2}$ and $\eta_u = 1/\sqrt{1+\eta^2}$, respectively, are placed, so that the x component of the field is multiplied by η_ℓ and the y component by η_u. Note that both such functions take values between 0 and 1, so that the corresponding transparencies consist in amplitude-only masks. The collecting, nonpolarizing beam splitter BS eventually provides the superposition of the images of the fields emerging from Π_ℓ and Π_u onto the plane Π_i, through the lens L. We shall assume, without loss of generality, that the optical lengths of the two arms are identical (or differ by an integer number of wavelengths).

At the plane Π_i, the phase difference ϕ between the x and the y component of the mode is introduced by means of a suitable anisotropic optical element (denoted by Φ in Fig. 2). The latter may consist, for instance, of a plate made of a birefringent material having locally varying thickness. The radiation emerging from the plate is eventually sent onto a rotating ground glass (G), which gives rise, across Π_i, to a spatially incoherent secondary source with polarization matrix given by the first term on the right-hand side of Eq. (14).

From an experimental point of view, since the phase element Φ and the rotating ground glass G should be placed at the same plane, a converging lens could be used to image on G the field emerging from Φ, or vice versa. It is important to stress, however, that for the synthesis of significant classes of ESM sources, namely, those for which the argument of P_{xy} is independent of the spatial coordinate $\boldsymbol{u}$, the element Φ consists of a conventional retardation plate that is used to introduce a spatially uniform phase delay between the two polarization components. In such cases, the synthesis of the source is performed using only conventional optical elements and amplitude transparencies. An example of such sources will be presented in the following section.

As far as the second term on the right-hand side of Eq. (14) is concerned, it is not difficult to realize that its synthesis can be achieved by using another laser beam, ℓ_-, polarized at 45°, feeding the same interferometer from the other face of the PBS, as depicted in Fig. 2(b). The lens L_- images the field emerging from a transparency, located across Π_- and characterized by the field transmission function $\tau_- = A\sqrt{\mu_-}$. The role of the PBS is somewhat reversed, since the reflected component (polarized along y) passes through the lower arm and is imaged onto the transparency η_ℓ, while the transmitted one (polarized along x) passes through the upper arm of the interferometer and is modulated by η_u. As for the previous case, the two components are recombined by the BS and imaged onto the plane Π_i by the lens L, but now they are in phase opposition because, within the interferometer, the y component of the electric field undergoes more reflections than in the previous case (see Fig. 2). Again, the phase difference ϕ between the orthogonal components of the field is given to the mode through the element Φ.

If the interferometer is fed by ℓ_+ and ℓ_- at the same time, a spatially incoherent source with locally varying polarization matrix $\hat{P}$ is produced across the plane Π_i. Such source can be used as the input of the optical system in Fig. 1 to produce ESM sources with assigned coherence–polarization properties. In the next section, results obtained here will be applied to the case of EGSM sources.

5. EGSM SOURCES

The most general EGSM source is characterized by the following BCP matrix [15]:

$$J_{\alpha\beta}(\boldsymbol{r}_1,\boldsymbol{r}_2) = I_{\alpha\beta}\exp\left(-\frac{r_1^2}{4\sigma_\alpha^2}\right)\exp\left(-\frac{r_2^2}{4\sigma_\beta^2}\right)\exp\left[-\frac{(\boldsymbol{r}_2-\boldsymbol{r}_1)^2}{2\delta_{\alpha\beta}^2}\right], \quad (18)$$

where σ_α, $\delta_{\alpha\beta}$ ($\alpha,\beta = x,y$) are real and positive parameters. Furthermore, $I_{\alpha\alpha} > 0$, while $I_{xy}(=I_{yx}^*)$ can be complex.

The number of independent *real* parameters is nine, but the values of such parameters can be chosen at will only within some specific ranges, which are specified by imposing the nonnegative definiteness of the BCP matrix [17,37,41]. In fact, the condition in Eq. (13), ensuring nonnegativeness of the BCP matrix, in this case reads

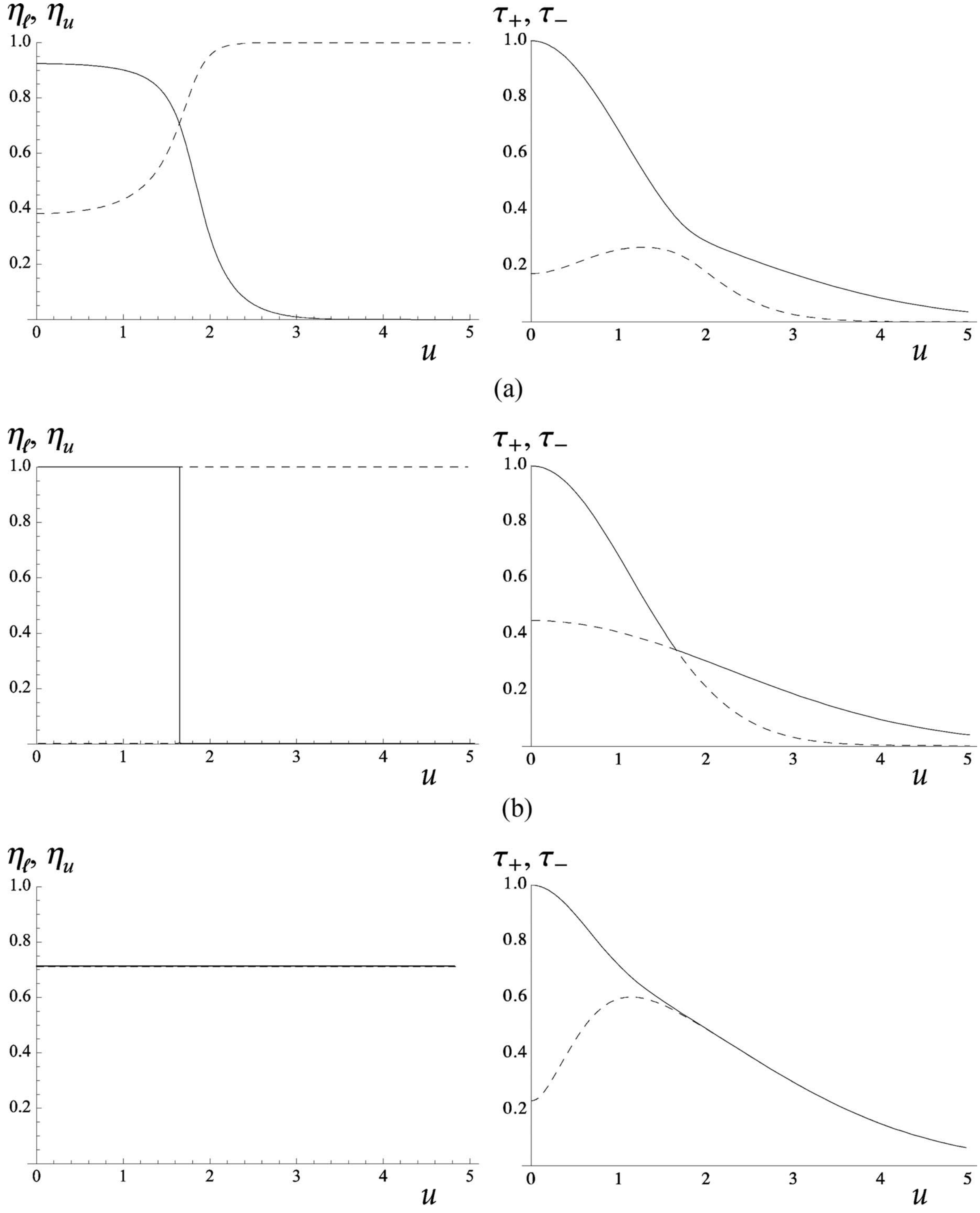

Fig. 3. Transmission functions (solid curves, η_ℓ and τ_+; dashed curves, η_u and τ_-) to be used in the synthesis procedure for three different choices of the source parameters: (a) $I_{xx}=1$, $I_{yy}=0.8$, $I_{xy}=0.4$, $\delta_{xx}=0.2$, $\delta_{yy}=0.1$, $\delta_{xy}=0.2$; (b) $I_{xx}=1$, $I_{yy}=0.8$, $I_{xy}=0$, $\delta_{xx}=0.2$, $\delta_{yy}=0.1$, $\delta_{xy}=0.2$; (c) $I_{xx}=1$, $I_{yy}=1$, $I_{xy}=0.1$, $\delta_{xx}=0.1$, $\delta_{yy}=0.1$, $\delta_{xy}=0.3$.

$$I_{xx}I_{yy}\delta_{xx}^2\delta_{yy}^2 \exp\left[-\frac{2\pi^2(\delta_{xx}^2+\delta_{yy}^2)}{\lambda^2 f^2}u^2\right] \geqslant |I_{xy}|^2\delta_{xy}^4 \exp\left(-\frac{4\pi^2\delta_{xy}^2}{\lambda^2 f^2}u^2\right), \qquad (19)$$

for any u. In particular, since the functions on the left- and right-hand sides of Eq. (19) decrease monotonically, on considering such inequality for the limiting cases $u\to 0$ and $u\to\infty$, the following fork inequality is derived:

$$\frac{\delta_{xx}^2+\delta_{yy}^2}{2} \leqslant \delta_{xy}^2 \leqslant \delta_{xx}\delta_{yy}\frac{\sqrt{I_{xx}I_{yy}}}{|I_{xy}|}. \tag{20}$$

From the results presented in the previous section, it follows that the EGSM source in Eq. (18) can be generated as the output of the optical system in Fig. 1, starting from a primary, spatially incoherent source characterized by a position-dependent polarization matrix whose elements are given by

$$P_{\alpha\beta}(\boldsymbol{u}) = \frac{2\pi I_{\alpha\beta}\delta_{\alpha\beta}^2}{\lambda^2 f^2}\exp\left(-\frac{2\pi^2\delta_{\alpha\beta}^2}{\lambda^2 f^2}u^2\right), \tag{21}$$

with a transmission mask at the plane Π_o chosen as

$$t_\alpha(\boldsymbol{r}) = \exp\left(-\frac{r^2}{4\sigma_\alpha^2}\right). \tag{22}$$

In this case, the phase of the upper off-diagonal term of the polarization matrix coincides with the argument of the complex number I_{xy}, and therefore it is independent of the spatial coordinate. Therefore, the phase modulator Φ present in the experimental setup of Section 4 reduces to a spatially uniform retardation plate. The synthesis of any EGSM can then be achieved using only amplitude transparencies and conventional optical elements. In the following, for simplicity, we assume I_{xy} to be real, so that we can get rid of the above retardation plate.

Numerical examples, shown in Fig. 3, refer to three different choices of the parameters corresponding to three typical cases of EGSM sources. Spatial coordinates and distances have been normalized in such a way that $\sqrt{\lambda f}=1$, i.e., $u/\sqrt{\lambda f}\to u$ and $\delta_{\alpha\beta}/\sqrt{\lambda f}\to\delta_{\alpha\beta}$ $(\alpha,\beta=x,y)$.

While the first case [Fig. 3(a)] corresponds to a rather generic situation, in the second one [Fig. 3(b)] there is no correlation between the orthogonal components of the field ($I_{xy}=0$), and in the third one [Fig. 3(c)] the orthogonal components have the same amplitude ($I_{xx}=I_{yy}$) and the same width of the degree of coherence ($\delta_{xx}=\delta_{yy}$).

The last example, in particular, corresponds the case studied in [34], where an EGSM source was synthesized on superimposing, through a MZI, two spatially incoherent sources with suitable intensity profiles, linearly polarized along x and y, respectively. Such profiles represented the sum and the difference, respectively, of two Gaussian functions having different widths and peak values. The EGSM was then produced after free propagation from the resulting spatially incoherent source, through the vCZ theorem. The final form of the BCP matrix of the source was then obtained by carrying out a rotation of $\pi/4$ of the polarization direction of the field.

It is worth showing how the setup presented here encompasses, as a particular case, that proposed and implemented in [34]. In fact, on choosing $\delta_{xx}=\delta_{yy}$ and $I_{xx}=I_{yy}$, the diagonal elements of $\hat{P}$ turn out to be coincident, i.e., $P_{xx}=P_{yy}$. Therefore, from Eqs. (15) and (17), we have

$$\eta = 1, \tag{23}$$

showing that the polarization of both modes is uniform across the source plane. In particular, since I_{xy} is assumed real, i.e., $\phi=0$, the two modes turn out to be linearly polarized at $\pm\pi/4$ [see Eq. (16)]. Having $\eta=1$ also implies that the two transparencies within the arms of the interferometer (η_u and η_ℓ) are identical and have uniform transmittance, so that they can be removed. Furthermore, we have

$$\mu_\pm = P_{xx} \pm |P_{xy}|, \tag{24}$$

i.e., the eigenvalues are given by the sum and the difference, respectively, of two Gaussian functions [see Eq. (21)]. These are exactly the intensity profiles of the two uncorrelated fields that overlap on the plane Π_i, as was the case for the experimental procedure of [34].

6. CONCLUSIONS

An experimental arrangement has been presented for the synthesis of the most general ESM quasi-monochromatic source. It is based on the use of the vCZ theorem for electromagnetic sources, starting from a spatially incoherent source characterized by a suitable position-dependent polarization matrix. The synthesis of incoherent sources of this kind can be realized by superimposing, on a rotating ground glass, two completely polarized, mutually incoherent light fields, whose shape, polarization, and power content are obtained from the spectral decomposition of the polarization matrix of the source. Such fields are physically realized by means of a MZI, with suitable amplitude transmittances placed in its arms, fed by two independent, linearly polarized laser beams entering the interferometer from two orthogonal directions.

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