Modal expansion for J_0-correlated electromagnetic sources

F. Gori,[1,*] M. Santarsiero,[1] and R. Borghi[2]

[1]Dipartimento di Fisica, Università Roma Tre, and CNISM, Via della Vasca Navale 84, I-00146 Rome, Italy
[2]Dipartimento di Elettronica Applicata, Università Roma Tre, and CNISM, Via della Vasca Navale 84, I-00146 Rome, Italy
*Corresponding author: gori@uniroma3.it

Received April 2, 2008; revised May 23, 2008; accepted May 23, 2008; posted July 11, 2008 (Doc. ID 94606); published August 8, 2008

We introduce a class of sources that turn out to be J_0 correlated in a scalar description but exhibit varied correlation properties when examined in electromagnetic terms. We prove that a modal expansion can be built explicitly for any of these sources and give an example to illustrate the richness of their behavior.

OCIS codes: 030.1640, 260.5430.

The so-called J_0-correlated Schell-model sources [1] have played a significant role in the scalar theory of coherence [2–12]. Basically, a J_0-correlated Schell-model scalar source is described, across its plane, by a cross-spectral density (CSD) [13], say $W(\mathbf{r}_1,\mathbf{r}_2)$, of the form

$$W(\mathbf{r}_1,\mathbf{r}_2)=\tau(r_1)\tau^*(r_2)J_0(\beta|\mathbf{r}_1-\mathbf{r}_2|), \tag{1}$$

where $J_0(\cdot)$ denotes the Bessel function of the first kind and zero order and β is a parameter having the dimensions of the inverse of a length. Furthermore, the squared modulus of the circularly symmetric function $\tau(r)$, which can be synthesized through a suitable filter, gives the intensity profile across the source plane. For such sources, it was shown that the modal expansion [13] can be found for any (well-behaving) intensity profile [1].

In recent times, there has been growing interest in the electromagnetic treatment of optical sources, where coherence and polarization properties are intertwined [14–32]. Obviously, finding the modes of an electromagnetic source [23] is even more difficult than in the scalar case, because one passes from one integral equation to a pair of coupled integral equations. As far as we know, explicit expressions for the vectorial modes have been found for only a few cases [23,24,31].

In this Letter we are interested in sources that would appear as J_0 correlated in a scalar treatment while exhibiting a richer structure when polarization is taken into account. We will show that, under suitable hypotheses, the vectorial modal expansion can be found for a large class of cases. For explaining our procedure, we will first come back to the modal expansion for the scalar case using an approach slightly different from the original one [1].

The starting point is that the shift-invariant part of the correlation function of any Schell-model source can be synthesized by using a suitable spatially incoherent source (primary source)[33]. In particular, for synthesizing the spectral degree of coherence [13] of the J_0-correlated source in Eq. (1), a very thin ring-shaped uniform intensity distribution can be used [2]. In fact, it is well known that the Fourier transform (FT) of a deltalike uniform annulus is a J_0 function. We now wonder: Can a modal expansion be found for the CSD along the annulus? This would solve the problem of the modal expansion across the synthesized source at its root. In fact, the modes across the synthesized source could be obtained by letting those pertain to the annulus propagate.

To examine this question in a slightly more general form, let us write the CSD across the spatially incoherent annulus, say $W_{\text{inc}}(\varphi_1,\varphi_2)$, in the form

$$W_{\text{inc}}(\varphi_1,\varphi_2)=I_{\text{inc}}(\varphi_1)\delta(\varphi_1-\varphi_2), \tag{2}$$

where $I_{\text{inc}}(\varphi)$ is related to the intensity distribution on the ring and $\delta(\cdot)$ denotes the Dirac delta function. Only the dependence on the angular coordinates is shown, being understood that the source is an infinitely thin annulus. It should be noted that Eq. (2) gives a nonnegative definite kernel (and, hence, a genuine CSD), provided only that $I_{\text{inc}}(\varphi)$ is a nonnegative function. At the same time, is must be realized that, owing to the presence of the angular delta function, W_{inc} does not belong to the class of the Hilbert–Schmidt kernels on the space $\{\varphi_1,\varphi_2\}\in[0,2\pi]\times[0,2\pi]$. As a consequence, Mercer's theorem [13] cannot be invoked and, in the general case, a modal expansion for W_{inc} simply does not exist. This can be seen by inserting Eq. (2) into the Fredholm integral equation [13]

$$\int_0^{2\pi} W_{\text{inc}}(\varphi_1,\varphi_2)\Phi(\varphi_2)\mathrm{d}\varphi_2=\Lambda\Phi(\varphi_1), \tag{3}$$

with Λ being the eigenvalue and $\Phi(\varphi)$ the corresponding eigenfunction. Using Eq. (2), we find

$$I_{\text{inc}}(\varphi)\Phi(\varphi)=\Lambda\Phi(\varphi). \tag{4}$$

The only case in which this equation can be satisfied by a well-behaving eigenfunction (banning deltalike structures for Φ) is when $I_{\text{inc}}(\varphi)=\Lambda$, i.e., when the intensity distribution is uniform over the ring. In this case, Λ behaves as an eigenvalue with infinite degeneracy, because any function can be thought of as an eigenfunction. A Mercerlike expansion can be built by using a complete set of orthonormal functions. The most simple choice is to use the basis of harmonic exponentials on the ring. This leads to the expansion

$$W_{\text{inc}}(\varphi_1,\varphi_2)=\Lambda\sum_{n=-\infty}^{+\infty}\Psi_n(\varphi_1)\Psi_n^*(\varphi_2), \tag{5}$$

with $\Psi_n(\varphi)=\exp(in\varphi)/\sqrt{2\pi}$. The modal expansion of the CSD at the plane of the synthesized source [see Eq. (1)] is obtained by Fourier transforming each mode and by taking into account the action of the filter. The result [1] corresponds in expressing J_0 in Eq. (1) by means of Graf's formula [34].

Let us now pass to electromagnetic sources. In this case, too, we shall start from a primary ring-shaped spatially incoherent source. The natural extension of Eq. (2) is

$$\hat{W}_{\text{inc}}(\varphi_1,\varphi_2)=\hat{I}_{\text{inc}}(\varphi_1)\delta(\varphi_1-\varphi_2), \tag{6}$$

where now $\hat{W}_{\text{inc}}(\varphi_1,\varphi_2)$ denotes the correlation tensor [15,32] and $\hat{I}_{\text{inc}}(\varphi)$ is a 2×2 polarization matrix [32], which specifies the polarization state along the ring. Let us now recall that the optical intensity is proportional to the trace of $\hat{I}_{\text{inc}}(\varphi)$. Because of our previous results, whenever such a trace is uniform along the ring, the source synthesized in the far zone will appear, in scalar terms, as a J_0-correlated source.

According to the modal theory for electromagnetic sources [23,24], the following coupled Fredholm integral equations have to be solved:

$$\int_0^{2\pi}\hat{W}_{\text{inc}}(\varphi_1,\varphi_2)\mathbf{\Psi}(\varphi_2)\mathrm{d}\varphi_2=\Lambda\mathbf{\Psi}(\varphi_1), \tag{7}$$

where now $\mathbf{\Psi}(\varphi)$ represents the Jones vector [35]:

$$\mathbf{\Psi}(\varphi)=\begin{bmatrix}\Psi_x(\varphi)\\ \Psi_y(\varphi)\end{bmatrix}, \tag{8}$$

which specifies the φ-dependent polarization state associated with the mode. By substituting Eq. (6) into Eq. (7), the Fredholm integral equation leads to the following linear system:

$$\hat{I}_{\text{inc}}(\varphi)\mathbf{\Psi}(\varphi)=\Lambda\mathbf{\Psi}(\varphi), \tag{9}$$

which corresponds to the eigensystem for the matrix $\hat{I}_{\text{inc}}(\varphi)$.

Because of the properties of the polarization matrix [32], for any fixed value of φ, the above system admits two nonnegative eigenvalues, say $\gamma_i(\varphi)$ $(i=1,2)$, with the corresponding (normalized) Jones eigenvectors $\mathbf{\Phi}_i(\varphi)$, which satisfy the orthonormality condition

$$\mathbf{\Phi}_i^\dagger(\varphi)\mathbf{\Phi}_j(\varphi)=\delta_{i,j}. \tag{10}$$

Accordingly, the polarization states associated with the two eigenvectors are mutually orthogonal. Furthermore, on denoting by $T(\varphi)$ the trace of the matrix $\hat{I}_{\text{inc}}(\varphi)$, which is proportional to the total intensity at a typical point on the ring, and by $P(\varphi)$ the degree of polarization [32], it is easily seen that $\gamma_{1,2}(\varphi)=T(\varphi)\times[1\pm P(\varphi)]/2$.

As in the scalar case, the kernel defined via Eq. (6) does not admit, in general, a Mercerlike modal expansion. There is, however, a significant exception. This is when the two eigenvalues γ_i turn out to be independent of the angular variable φ. This happens if both T and P are, in turn, independent of φ. From now on, we will assume this to be the case. Note that, even though P has the same value at any point of the ring, the polarization state may change at will from one point to another.

Under our hypotheses, the 2×2 polarization matrix $\hat{I}_{\text{inc}}$ can be decomposed as

$$\hat{I}_{\text{inc}}(\varphi)=\sum_{i=1,2}\gamma_i\mathbf{\Phi}_i(\varphi)\mathbf{\Phi}_i^\dagger(\varphi), \tag{11}$$

which, once substituted into Eq. (6), leads to the following expansion of $\hat{W}_{\text{inc}}(\varphi_1,\varphi_2)$:

$$\hat{W}_{\text{inc}}(\varphi_1,\varphi_2)=\sum_{i=1,2}\gamma_i\sum_{n=-\infty}^{+\infty}\mathbf{\Psi}_{i,n}(\varphi_1)\mathbf{\Psi}_{i,n}^\dagger(\varphi_2), \tag{12}$$

where

$$\mathbf{\Psi}_{i,n}(\varphi)=\frac{1}{\sqrt{2\pi}}\mathbf{\Phi}_i(\varphi)\exp(in\varphi) \tag{13}$$

are mutually orthonormal vector functions. Then, Eq. (12) can be clearly interpreted as a Mercerlike expansion of the correlation matrix of the incoherent annulus; the eigenvalues of the latter (Λ_i) just coincide with those (γ_i) of the polarization matrix $\hat{I}_{\text{inc}}$, and the vector modes are given by Eq. (13).

As an example of application, consider the following correlation matrix on the circle:

$$\hat{W}_{\text{inc}}(\varphi_1,\varphi_2)=\begin{bmatrix}1+\cos^2\varphi_1 & \sin\varphi_1\cos\varphi_1\\ \sin\varphi_1\cos\varphi_1 & 1+\sin^2\varphi_1\end{bmatrix}\delta(\varphi_1-\varphi_2), \tag{14}$$

where an inessential proportionality factor has been omitted. According to a well-known decomposition [35], the above polarization matrix can be thought of as arising from the superposition of a perfectly unpolarized field and of a perfectly polarized one, endowed with radial polarization. It is at once seen that, at any angle φ, we have $T=3$ and $\mathrm{Det}\{\hat{I}_{\text{inc}}(\varphi)\}=2$, so that $P=1/3$.

On applying the paraxial van Cittert–Zernike theorem for electromagnetic sources [17,18], it is found that the elements of the correlation matrix across the synthesized source ($\hat{W}$) are

$$\begin{cases} W_{xx}(\mathbf{r}_1,\mathbf{r}_2) = \tau(r_1)\tau^*(r_2)\Big[J_0(\beta r_{12}) - \frac{1}{3}J_2(\beta r_{12})\cos(2\vartheta_{12})\Big] \\ W_{xy}(\mathbf{r}_1,\mathbf{r}_2) = \frac{1}{3}\tau(r_1)\tau^*(r_2)J_2(\beta r_{12})\sin(2\vartheta_{12}) \\ W_{yy}(\mathbf{r}_1,\mathbf{r}_2) = \tau(r_1)\tau^*(r_2)\Big[J_0(\beta r_{12}) + \frac{1}{3}J_2(\beta r_{12})\cos(2\vartheta_{12})\Big] \end{cases}, \tag{15}$$

where $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$, ϑ_{12} is the polar angle of $\mathbf{r}_{12}$, and $J_2(\cdot)$ denotes the Bessel function of the first kind and order 2. Predictions of the electromagnetic example could be tested by using suitable anisotropic elements. For instance, a linear polarizer aligned along any axis across the source plane would reveal that the correlation between the corresponding components of the electric field at two points does not present the same behavior as that pertinent to a scalar J_0-correlated source. In particular, such correlation does not depend only on the distance between the two points but even on the inclination of the joining line. It is also worth noting that, on letting $\mathbf{r}_1=\mathbf{r}_2$, at a typical point of the source there is no correlation between the x and the y components of the electric field, whereas some correlation may exist when $\mathbf{r}_1\neq\mathbf{r}_2$.

As far as the modal structure of the source in Eq. (15) is concerned, it is easily found that the two eigenvalues turn out to be $\Lambda_1=2$ and $\Lambda_2=1$ while the corresponding Jones eigenvectors are obtained by Fourier transforming Eq. (13) with $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ linearly polarized along the radial and the azimuthal directions, respectively. It is interesting to note that the modes across the plane of the synthesized source can be expressed in terms of a linear combination of radial and azimuthal vector modes of the type introduced some years ago by Greene and Hall [36] in the study of Bessel–Gauss beam solutions of the vector wave equation.

References

1. F. Gori, G. Guattari, and C. Padovani, Opt. Commun. **64**, 311 (1987).
2. J. Turunen, A. Vasara, and A. T. Friberg, J. Opt. Soc. Am. A **8**, 282 (1991).
3. R. Simon, K. Sundar, and N. Mukunda, J. Opt. Soc. Am. A **10**, 2008 (1993).
4. M. W. Kowarz and G. S. Agarwal, J. Opt. Soc. Am. A **12**, 1324 (1995).
5. C. Palma and G. Cincotti, IEEE J. Quantum Electron. **33**, 1032 (1997).
6. R. Borghi, IEEE J. Quantum Electron. **35**, 849 (1999).
7. S. A. Ponomarenko, J. Opt. Soc. Am. A **18**, 150 (2001).
8. G. Gbur and T. D. Visser, Opt. Lett. **28**, 1627 (2003).
9. J. Garnier, J.-P. Ayanides, and O. Morice, J. Opt. Soc. Am. A **20**, 1409 (2003).
10. L. Wang and B. Lü, Optik (Jena) **117**, 167 (2006).
11. L. Rao, X. Zheng, Z. Wang, and P. Yei, Opt. Commun. **281**, 1358 (2007).
12. T. van Dijk, G. Gbur, and T. D. Visser, J. Opt. Soc. Am. A **25**, 575 (2008).
13. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge U. Press, 1995).
14. D. F. V. James, J. Opt. Soc. Am. A **11**, 1641 (1994).
15. F. Gori, M. Santarsiero, S. Vicalvi, R. Borghi, and G. Guattari, J. Opt. A **7**, 941 (1998).
16. S. R. Seshadri, J. Opt. Soc. Am. A **16**, 1373 (1999).
17. F. Gori, M. Santarsiero, R. Borghi, and G. Piquero, Opt. Lett. **25**, 1291 (2000).
18. M. A. Alonso, O. Korotkova, and E. Wolf, J. Mod. Opt. **53**, 969 (2006).
19. F. Gori, M. Santarsiero, G. Piquero, R. Borghi, A. Mondello, and R. Simon, J. Opt. A **3**, 1 (2001).
20. L. Pan and B. Lü, Optik (Jena) **113**, 459 (2002).
21. J. Tervo, T. Setälä, and A. T. Friberg, Opt. Express **11**, 1137 (2003).
22. E. Wolf, Phys. Lett. A **312**, 263 (2003).
23. F. Gori, M. Santarsiero, R. Simon, G. Piquero, R. Borghi, and G. Guattari, J. Opt. Soc. Am. A **20**, 78 (2003).
24. J. Tervo, T. Setälä, and A. T. Friberg, J. Opt. Soc. Am. A **21**, 2205 (2004).
25. R. Martínez-Herrero, G. Piquero, and P. Mejías, J. Opt. A **6**, S67 (2004).
26. P. Réfrégier and F. Goudail, Opt. Express **13**, 6051 (2005).
27. H. Roychowdhury and O. Korotkova, Opt. Commun. **249**, 379 (2005).
28. T. Shirai, Opt. Commun. **256**, 197 (2005).
29. F. Gori, M. Santarsiero, R. Borghi, and E. Wolf, Opt. Lett. **31**, 688 (2006).
30. A. Luis, J. Opt. Soc. Am. A **23**, 2855 (2006).
31. F. Gori, M. Santarsiero, and R. Borghi, Opt. Lett. **31**, 858 (2006).
32. E. Wolf, *Introduction to the Theory of Coherence and Polarization of Light* (Cambridge U. Press, 2007).
33. P. De Santis, F. Gori, G. Guattari, and C. Palma, J. Opt. Soc. Am. A **3**, 1258 (1986).
34. M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions* (Dover, 1972).
35. C. Brosseau, *Fundamentals of Polarized Light* (Wiley, 1998).
36. P. Greene and D. Hall, J. Opt. Soc. Am. A **15**, 3020 (1998).