Effects of coherence on the degree of polarization in a Young interference pattern

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Recent predictions concerning the relationship between the degree of polarization at a typical point of a Young interference pattern and the degree of coherence of the electromagnetic field at the pinholes are tested by a simple experiment. In particular, it is shown that light that is completely unpolarized at the pinholes can become partially polarized across the fringe pattern.

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In recent years a good deal of research has been done concerning the coherence and the polarization properties of stochastic electromagnetic beams. In particular, it was found that in some cases there is an intimate relationship between the coherence properties of beams and their degree of polarization (see, for example, Refs. 1–9). However, up to now this relationship has not been studied in great detail.

In the present Letter we first carry further the theoretical analysis described in a recent paper.[10] Specifically, we study the effect of the degree of coherence of light at the pinholes on the degree of polarization of the light in a Young interference pattern, and we obtain curves that show such relationships for a particular class of stochastic beams. We then describe an experiment that verifies the theoretical predictions.

We consider a stochastic electromagnetic beam, with its axis along the z direction, whose cross-spectral density matrix[6] of the light at the pinholes plane is given by

$$\mathbf{W}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) = \begin{bmatrix} W_{xx}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) & W_{xy}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) \\ W_{yx}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) & W_{yy}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) \end{bmatrix}, \tag{1}$$

where

$$W_{xx}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) = s(\omega)\exp\left(-\frac{\vec{\rho}_1^{\,2}+\vec{\rho}_2^{\,2}}{4\sigma^2}\right)\exp\left[-\frac{(\vec{\rho}_1-\vec{\rho}_2)^2}{2\delta_x^2}\right],$$

$$W_{yy}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) = s(\omega)B\exp\left(-\frac{\vec{\rho}_1^{\,2}+\vec{\rho}_2^{\,2}}{4\sigma^2}\right) \times\exp\left[-\frac{(\vec{\rho}_1-\vec{\rho}_2)^2}{2\delta_y^2}\right],$$

$$W_{xy}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) = W_{yx}^{(0)}(\vec{\rho}_1,\vec{\rho}_2;\omega) = 0. \tag{2}$$

Here $\vec{\rho}_1$ and $\vec{\rho}_2$ are transverse position vectors (perpendicular to the z axis) of the two pinholes, $s(\omega)$ represents the spectral density at the origin $\vec{\rho}=0$, and the parameters B, σ, δ_x, and δ_y are independent of position but may depend on frequency ω.

The spectral degree of polarization of the field at the pinholes can be calculated from the general formula[11]

$$\mathcal{P}^{(0)}(\vec{\rho},\omega) = \sqrt{1-\frac{4\,\mathrm{Det}\,\mathbf{W}(\vec{\rho},\vec{\rho},\omega)}{[\mathrm{Tr}\mathbf{W}(\vec{\rho},\vec{\rho},\omega)]^2}}, \tag{3}$$

where Det stands for the determinant and Tr for the trace. On substituting from Eqs. (1) and (2) into Eq. (3), we readily find that

$$\mathcal{P}^{(0)} = \left|\frac{1-B}{1+B}\right|. \tag{4}$$

As for the spectral degree of coherence, we refer to the definition given in Ref. 6, i.e.,

$$\mu^{(0)}(\vec{\rho}_1,\vec{\rho}_2,\omega) = \frac{\mathrm{Tr}\mathbf{W}(\vec{\rho}_1,\vec{\rho}_2,\omega)}{\sqrt{\mathrm{Tr}\mathbf{W}(\vec{\rho}_1,\vec{\rho}_1,\omega)\mathrm{Tr}\mathbf{W}(\vec{\rho}_2,\vec{\rho}_2,\omega)}}. \tag{5}$$

It should be mentioned that a similar approach, in the space–time domain, was given in Ref. 2, where the trace of the correlation matrix was called the equivalent mutual intensity. It should also be mentioned that other definitions for the degree of coherence in the electromagnetic case have been proposed.[12,13] The definition given by Eq. (5) is directly related to the visibility of the interference fringes in a Young interferometer, as introduced by Zernike. For the light distribution characterized by the cross-spectral density matrix in Eqs. (2) it is [with $\vec{\rho}_1=-\vec{\rho}_2=(x,0)$]

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$$\mu^{(0)} = \frac{\exp(-2x^2/\delta_x^2) + B\exp(-2x^2/\delta_y^2)}{1+B}. \quad (6)$$

Following the same procedure as in Ref. 10, we obtain for the degree of polarization at a point at the center of the fringe pattern of the Young interferometer

$$\mathcal{P} = \frac{2\mathcal{P}^{(0)} + (1+\mathcal{P}^{(0)})\exp(-2x^2/\delta_x^2) - (1-\mathcal{P}^{(0)})\exp(-2x^2/\delta_y^2)}{2 + (1+\mathcal{P}^{(0)})\exp(-2x^2/\delta_x^2) + (1-\mathcal{P}^{(0)})\exp(-2x^2/\delta_y^2)}. \quad (7)$$

Figure 1 shows the behavior of the degree of polarization $\mathcal{P}$ at the center of the fringe pattern as a function of the degree of coherence of the light at the pinholes, calculated from Eqs. (6) and (7), for the case where $\mathcal{P}^{(0)}=0$, and for different values of the ratio δ_x/δ_y. These curves are very similar to those obtained in Ref. 10 for a somewhat different stochastic electromagnetic beam. Moreover, this figure also shows that the effect discussed in Ref. 10 (effect of coherence at the pinholes on the degree of polarization in the interference pattern) can be produced even with unpolarized light.

Next we will discuss an experiment to confirm these predictions. Using the setup shown in Fig. 2,

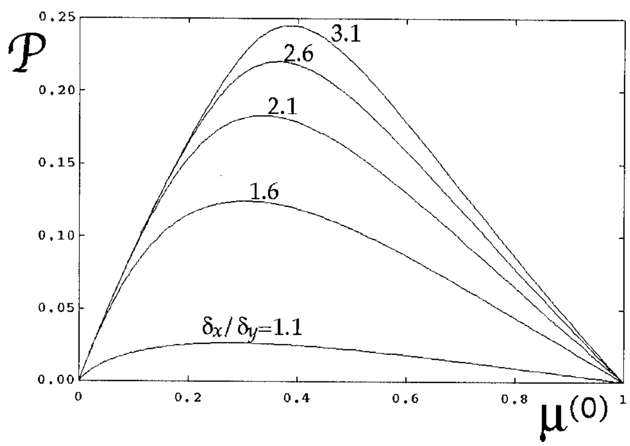

Fig. 1. Degree of polarization $\mathcal{P}$ at the center of the fringe pattern, as a function of the degree of coherence at the pinholes $\mu^{(0)}$, for the case where the degree of polarization at the pinholes is $\mathcal{P}^{(0)}=0$, for different values of the ratio δ_x/δ_y.

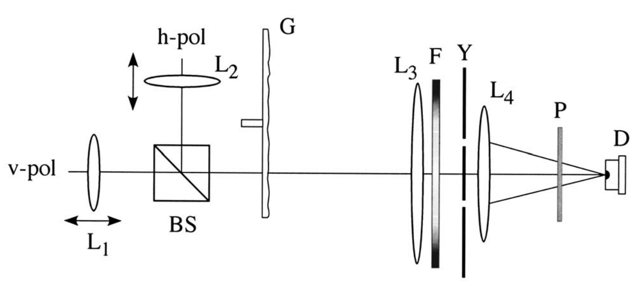

Fig. 2. Experimental setup. L_1, L_2, L_3, L_4, lenses; BS, beam splitter; G, rotating ground glass; F, Gaussian amplitude filter; Y, pinhole Young mask; P, linear polarizer; D, detector.

we synthesized the source described by Eq. (2). The arrangement is basically the same as that introduced in Ref. 14 for synthesizing the so-called Collett–Wolf source.[15] The main difference is that two different lasers with orthogonal linear polarization states are now used. In particular, with a suitable choice of the positions of lenses L_1 and L_2, the spot sizes obtained on the rotating ground glass (G) can be adjusted at will. Then, by virtue of the van Cittert–Zernike theorem,[11] it is possible to generate the elements W_{xx} and W_{yy} of the cross-spectral density matrix specified by Eqs. (2), once the effects of lens L_3 and of Gaussian amplitude filter F are taken into account. The powers of the two lasers have been chosen such that $\mathcal{P}^{(0)}=0$.

Figure 3 shows the values of the spectral degree of polarization on axis $\mathcal{P}$ (dots) as a function of the spectral degree of coherence at the pinholes $\mu^{(0)}$, measured for different values of the ratio δ_x/δ_y, together with the corresponding theoretical behavior predicted by Eq. (7), when $\mathcal{P}^{(0)}\to 0$.

The particular structure of the cross-spectral density matrix whose elements are given by Eq. (2) makes it possible to provide an intuitive interpretation of the results of the experiment. The basic question is: How can light that is completely unpolarized at the pinholes produce a partially polarized field at the axial point of the interference pattern? A key point is that, if $\delta_x \neq \delta_y$, the x- and the y-components of the fluctuating electric fields at the pinholes have different degrees of correlation. For example, suppose that $\delta_x > \delta_y$. Let us cover the pinhole mask with a linear polarizer whose axis can be set parallel either to the x-axis or to the y-axis. Since the x components of the field are more strongly correlated than the y components, the visibility of the fringes that are formed when the polarizer is parallel to the x-axis is higher than that pertaining to the fringes seen when the polarizer is parallel to the y-axis. For both fringe systems, the axial point corresponds to a position where constructive interference takes place. On the other hand, the maximum produced by the x-components is higher than that associated with the y-components. Accordingly, the overall spectral density at the axial point is the sum of two contributions with different weights. This, in turn, implies that light observed at the axial point is partially polarized. It should be stressed, however, that while the above interpretation can be used whenever the cross-spectral density

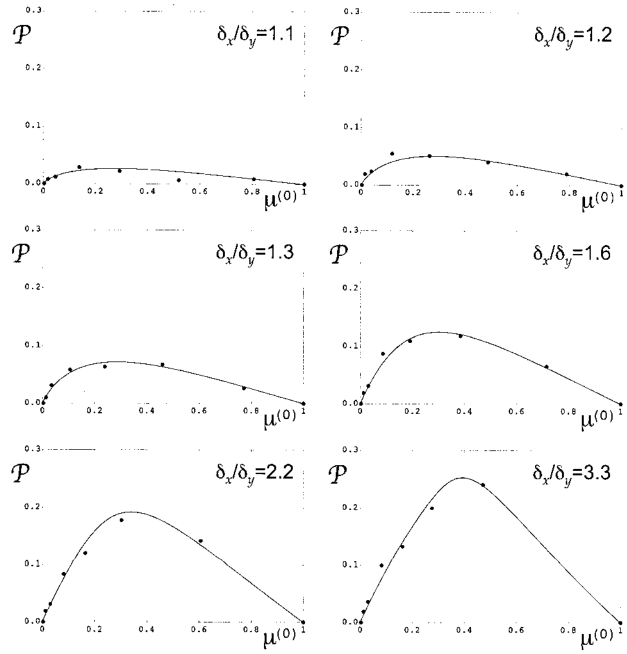

Fig. 3. Experimental values (dots) of the axial degree of polarization $\mathcal{P}$ of the light as functions of the spectral degree of coherence $\mu^{(0)}$ at the pinholes, measured for different values of the ratio δ_x/δ_y. The solid curves are theoretical values calculated from Eq. (7), with $\mathcal{P}^{(0)}=0$.

matrix is diagonal, beams with more general matrices may require more sophisticated interpretations. We further note that the results discussed in the present paper bear a relation with those first reported by James in his pioneering paper,[1] where polarization changes in free propagation, due to different correlation properties of the x- and y-components across the source, were theoretically predicted. Finally, it should be mentioned that a definition of the degree of coherence of the form specified by Eq. (4) was first proposed in Ref. 16, in a three-dimensional treatment.

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