Phase and Amplitude Retrieval in Ghost Diffraction from Field-Correlation Measurements

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We report the results of experiments about the inversion of ghost diffraction with pseudothermal light. A complete retrieval of the complex transmission function of planar transparencies, illuminated by spatially incoherent, quasimonochromatic light, is achieved. This is obtained by measuring the field (instead of the intensity) correlation function. In particular, the determination of the phase of the correlation function is made particularly easy and robust by the use of a suitably modified Young interferometer. The presented results refer to the cases of a clear slit and a phase step.

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Introduction.—Ghost diffraction and ghost imaging (GD and GI, respectively) are of current interest and continue to be the subject of several studies, both of theoretical and experimental nature (see, for instance, the recent review paper by Gatti *et al.* [1] and Refs. [2–6]).

Basically, a GD-based system consists of two physically correlated optical fields, propagating along two separated optical paths. In one of the two paths an object, typically a planar transparency, is inserted. The information about such an object is contained in the correlation function between the two propagated fields.

Much interest about GD and GI has been focused onto quantum features of phenomena related to imaging and diffraction [7–10], thus leading to the use of genuinely nonclassical light, such as that involving the quantum entanglement of photon pairs generated via spontaneous parametric down-conversion [11]. GD or GI patterns are then revealed by measuring the correlation function of intensity fluctuations, through suitable electronic circuitry. On the other hand, it has been shown that GD and GI patterns can be obtained using classical light [12,13]. This has been confirmed by some experiments [3–6] in which GD and GI tests were performed with pseudothermal and true thermal light, using intensity correlation techniques. It is to be noted, however, that such techniques give information only about the modulus of the field-correlation function, leaving its phase completely unknown [1].

The aim of this Letter is to show results of what we could call an inverse GD experiment. In particular, we address the problem of a *complete* retrieval of the transmission function of a planar transparency, by measuring both the modulus and the phase of the field-correlation function in a GD scheme under pseudothermal illumination. To our knowledge, this is the first experiment of this kind. Basically, we replace the intensity correlator by a classical Young interferometer, suitably modified in order for the nontrivial phase measurement to be made in an accurate and robust way [14].

Summary of the theoretical basis.—We consider two identical copies of a (spatially) incoherent field distribution at a certain initial transverse plane, say $z = 0$. Let $V_0(\mathbf{r})$ denote the optical field pertinent to a single realization. The cross-correlation function of the field at the plane $z = 0$ can be written as

$$\langle V_0^*(\mathbf{r}_1)V_0(\mathbf{r}_2)\rangle \propto I_0(\mathbf{r}_1)\delta(\mathbf{r}_2 - \mathbf{r}_1), \tag{1}$$

where $I_0(\mathbf{r})$ denotes the intensity profile. The two copies are mutually shifted by an arbitrary quantity, so that they freely propagate along two parallel mean directions. Furthermore, at the plane $z = 0$ one of the two copies impinges on a transparency with transmission function $\tau(\mathbf{r})$. The cross-correlation function of the fields across the plane $z = d > 0$, say $g(\mathbf{r}_1, \mathbf{r}_2)$, can be evaluated by considering the optical fields, say $V_d(\mathbf{r})$ and $U_d(\mathbf{r})$, generated by the free propagation of V_0 and τV_0, respectively, up to the plane $z = d$. In particular, within the paraxial approximation we have

$$\begin{aligned} V_d(\mathbf{r}) &\propto \int V_0(\mathbf{r}')K_d(\mathbf{r} - \mathbf{r}')d^2r, \\ U_d(\mathbf{r}) &\propto \int \tau(\mathbf{r}')V_0(\mathbf{r}')K_d(\mathbf{r} - \mathbf{r}')d^2r, \end{aligned} \tag{2}$$

where $K_d(\mathbf{r}) = \exp(ik\mathbf{r}^2/2d)$ denotes the Fresnel shift-invariant propagator, and unessential constant phase and amplitude factors have been omitted. On substituting from Eq. (1) into Eq. (2), after trivial algebra the cross-correlation function $g(\mathbf{r}_1, \mathbf{r}_2)$ turns out to be [1]

$$g(\mathbf{r}_1, \mathbf{r}_2) \propto \mathcal{F}\{\tau I_0\}\left(\frac{\mathbf{r}_2 - \mathbf{r}_1}{\lambda d}\right), \tag{3}$$

where λ denotes the wavelength, and $\mathcal{F}\{f\}(\mathbf{p})$ is the Fourier transform of the function $f(\mathbf{r})$, i.e.,

$$\mathcal{F}\{f\}(\mathbf{p}) = \int f(\mathbf{r})\exp(-i2\pi\mathbf{r}\cdot\mathbf{p})d^2p, \tag{4}$$

with $\mathbf{p}$ being the spatial frequency. Accordingly, the complete information (in modulus and phase) on τ is retrieved by inverse Fourier transforming g.

Description of the experimental setup.—The experimental setup, based on the scheme used by Ferri *et al.* [3] is sketched in Fig. 1. A light beam, emitted by a cw source (a Nd:Yag frequency doubled laser) operating at $\lambda = 532$ nm, is focused onto a rotating ground glass (G) by a $10\times$ microscope objective (O_1). The longitudinal position of the objective can be adjusted by means of a micrometric screw, so that the area of the light spot on G can be varied. Such a spot behaves like an incoherent light source whose linear dimensions, in our experiments, could range from some microns to several millimeters.

Light radiated from the synthesized incoherent source is collimated by the lens L_1 (10 cm focal length), and its transverse extension is then limited by an iris diaphragm (D, diameter about 3 mm) placed beyond the lens. The beam is eventually sent into a beam splitter cube (BS), that produces two identical replicas of the incident beam. Because of the reflection occurring at the internal surface of BS, the two emerging beams are mutually reversed, that is, one is the specular replica of the other. As we shall see in a moment, this represents a key feature for our experiments.

The two replicas propagate along parallel directions. One of the two replicas impinges onto the object, which consists of a planar transparency (S), while the other one is used as a reference field. Linear dimensions of the coherence area of the radiation across the object plane (Π_1), adjusted by means of the longitudinal position of the objective O_1, have to be much smaller than the feature size of the object itself [1].

As said above, the correlation function has to be measured at different pairs of points across an observation plane, say Π_2, far enough from the object plane Π_1. The distance d between Π_1 and Π_2 must be large enough that van Cittert–Zernike theorem can be applied [15,16]. In fact, for pseudothermal light, the correlation function across Π_1 has finite width, so that the distance d cannot be made arbitrarily small. In practice, the distance d must be chosen on taking into account both the characteristic size of the object and the coherence area of impinging radiation [17].

The field across the plane Π_2 is imaged, by the magnifying telescope M ($4\times$), onto the plane Π_3, where an opaque mask (Y) with two pinholes is placed. Such holes (diameter about 400 μm each, mutual distance 8 mm) form the Young interferometer that allows the correlation function of the field between two points in Π_3 to be measured. By means of the telescope, and by taking the value of D into account, the field coherence area of both fields is made larger than the size of the pinholes on the mask Y. The interference pattern obtained by superimposing the fields emerging from the two pinholes is then observed across the back focal plane (Π_4) of the lens L_4 (15 cm focal length), through the objective O_2 ($20\times$), which images the pattern onto a 256×240 CCD array. Interference patterns are eventually acquired by means of a Spiricon LBA-300 system.

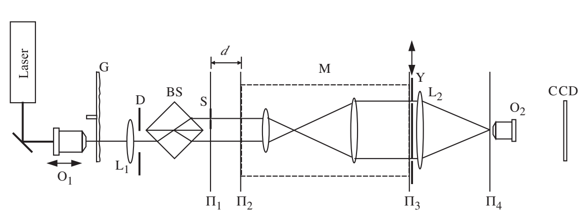

FIG. 1. Experimental setup

Differently from a classical Young interferometer, in which the correlation function is sampled by varying the mutual distance of the pinholes [18], in the present scheme such distance is kept fixed. In fact, owing to the specularity of the two replicas produced by the BS, it is sufficient to translate laterally the mask Y of an amount x to change by $2x$ the equivalent mutual distance between the two points. This means that the behavior of the correlation function can be detected without changing the distance between the pinholes, but simply by shifting the mask laterally. What is more important, the period of the fringes in the observation plane do not depend on $(x_2 - x_1)$, so that the determination of the phase of the correlation function can easily be achieved with good accuracy. A more detailed description of the interferometer and of its operation principles can be found in Ref. [14]. Finally, the inverse problem of retrieving the complex transmission function of the object S can be solved simply by Fourier transforming the measured correlation function. Such retrieval problem will be addressed in the next section, where experimental results will be presented for two practical cases.

Experimental results.—As a first test case, we use an object consisting of a one-dimensional vertical slit of width $w = 570 \pm 10$ μm. The linear size of the coherence area across Π_1 has been set to about 1/10 of the slit width, so that the minimum distance d has been evaluated as about 20 cm. Such a value was then chosen for d (20 ± 0.5 cm). Measures of the correlation function have been performed by detecting visibility and position of Young interference patterns [14,15],

$$I_{\mathrm{tot}}(\xi) = I_1 + I_2 + 2\sqrt{I_1 I_2}|g| \cos\left(\frac{2\pi d}{\lambda f}\xi + \phi\right), \tag{5}$$

where ξ is the transverse coordinate at the focal plane, I_1 and I_2 are the intensity values measured when the two holes are open separately, and $|g|$ and ϕ are modulus and phase of g, respectively. More precisely, for a typical position of the mask Y, we acquire the intensity patterns I_1, I_2, and I_{tot}. The acquisition time for each of these patterns is much longer than the time over which the transmission function of G varies in a significant way. Accordingly, each pattern could be assumed to be pro-

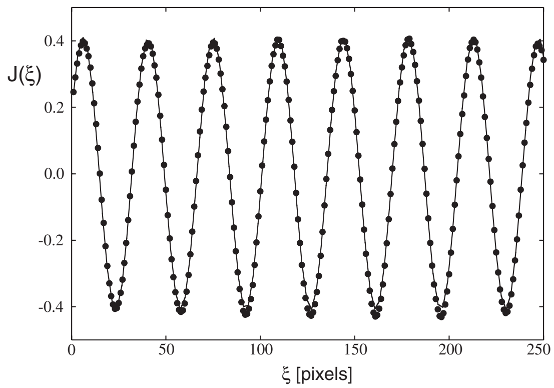

FIG. 2. 1D profile obtained by summing over all the lines of the matrix J (dots), together with the sinusoidal best fit (solid curve) obtained from Eq. (5).

duced by a pseudothermal source. From the above three intensity distributions the matrix $J = (I_{\rm tot} - I_1 - I_2)/(2\sqrt{I_1 I_2})$ was computed. Note that, as can be seen from Eq. (5), values of J are independent of the vertical coordinate, apart from noise effects. Averages over all the lines of J have then been used to reduce the noise. Figure 2 shows a typical behavior of the $J(\xi)$ (dots) together with the corresponding sinusoidal best fit. According to Eq. (5), parameters of the fit provide the values of $|g|$ and ϕ. The latter quantities are shown in Figs. 3(a) and 3(b), respectively, as functions of the mask displacement x on the Π_3 plane. Uncertainties of the values obtained by this procedure have been estimated as being of the order of 1% (for $|g|$) and 0.1 rad (for ϕ) for patterns with high-visibility. Such estimates increase up to a factor of about 3 for low-visibility patterns.

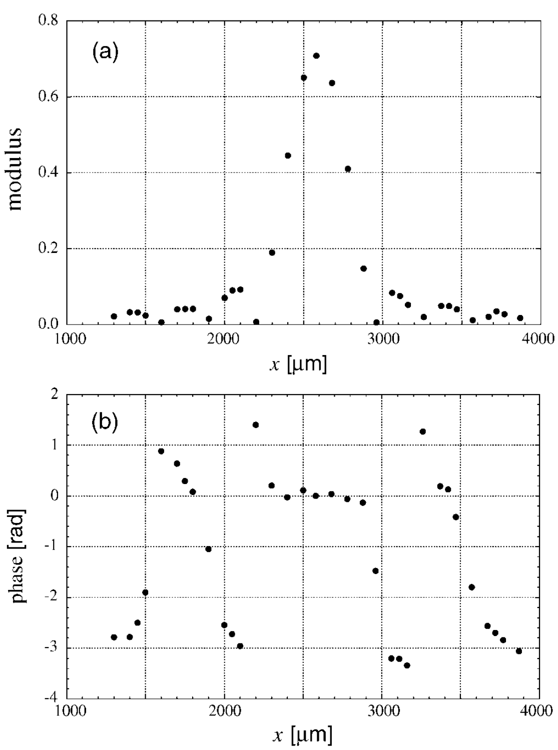

FIG. 3. Measured modulus (a) and phase (b) of the correlation function g for the case of the slit (dots).

Retrieved modulus and phase of the transmission function of the slit, obtained by inverse Fourier transforming the correlation function shown in Fig. 3, are reported in Fig. 4 as solid and dashed curves, respectively. Note that, the acquisition step was reduced where the fringe visibility varied more significantly, so that the samples are not evenly spaced in the spatial frequency domain. Accordingly, instead of using a discrete fast Fourier transform algorithm to retrieved the correlation function, we directly discretized the Fourier integral in Eq. (4) on using the samples. In the same figure the actual size of the slit is also shown by two vertical dotted lines. It can be noted that the retrieved modulus of the transmission function is significantly different from 0 only in correspondence of the actual slit. Furthermore, in the same region, the phase turns out to be practically constant, as was expected. It should be noted that the retrieved transmission function presents an oscillating behavior, which is reminiscent of the Gibbs phenomenon, related to the truncation of the Fourier spectrum, which also fixes the spatial resolution of the device.

In the second experiment the slit was replaced by a phase step, consisting of the edge of a microscope slide, vertically placed in the Π_1 plane. The phase step was measured by means of a Mach-Zehnder interferometer and turned out to be $\Delta\phi = 2.8 \pm 0.5\,(\mathrm{mod}2\pi)$ rad. Modulus and phase of the correlation function g, obtained by the same above measurement procedure, are plotted in Figs. 5(a) and 5(b), respectively. In particular [Fig. 5(b)], the phase of the correlation function displays a sudden change of π at the origin of the spatial frequency axis. This is a well-known phenomenon, [19] which occurs whenever the object exhibits a phase step $\Delta\phi$ ($\neq 0$) and turns out to be independent from $\Delta\phi$ itself. Because of the shift theorem for the Fourier transform, the phase of the correlation function varies linearly before and after the

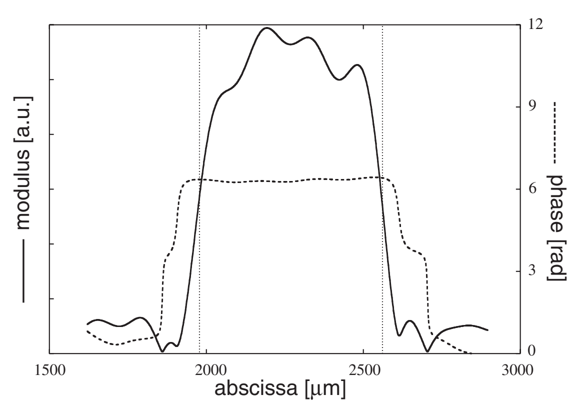

FIG. 4. Modulus (solid curve) and phase (dashed curve) of the retrieved transmission function of the slit. Dotted lines denote the actual extension of the aperture.

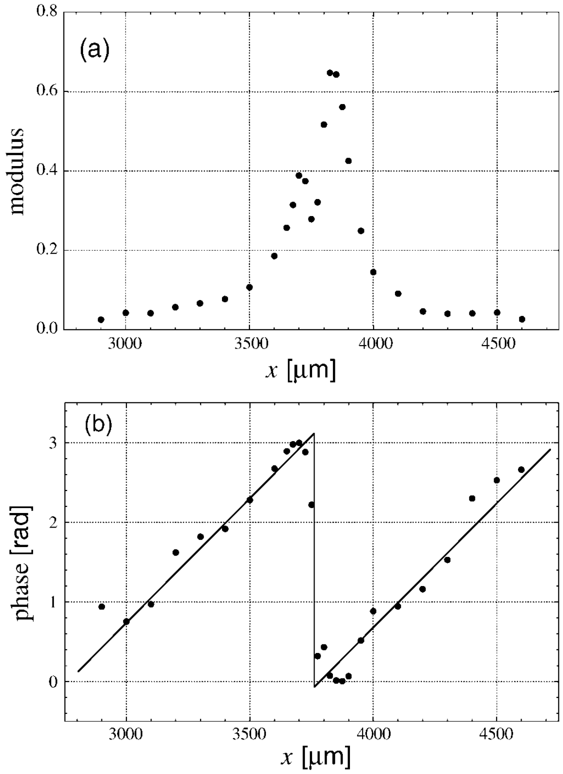

FIG. 5. Measured modulus (a) and phase (b) of the correlation function g for the case of the phase step (dots). Solid line in (b) represents the theoretical prediction.

origin of the spatial frequencies, unless the phase-step object is centered on the optical axis. The theoretical curve accounting for this phenomenon is drawn in Fig. 5(b) as a continuous line.

Figure 6 shows the behavior of the modulus (solid curve) and the phase (dashed curve) of the retrieved transmission function of the phase step, obtained by inverse Fourier transforming the complex data of Fig. 5. The vertical dotted line represents the position of the phase discontinuity. The presence of the dip in the retrieved modulus is related to the truncation of the Fourier spectrum of the discontinuous function τ. Nevertheless, the retrieved phase profile shows a sudden change with a phase step of about 2.7 rad, in good agreement with the measured value of $\Delta\phi$.

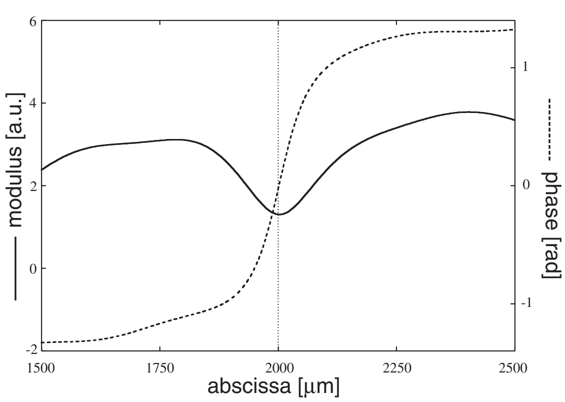

FIG. 6. Modulus (solid curve) and phase (dashed curve) of the retrieved transmission function of the phase step. Vertical dashed line represents the position of the phase discontinuity.

Conclusions.—In this Letter we have presented the first experimental results concerning the retrieval of planar complex transparencies by inversion of the GD patterns obtained under pseudothermal illumination. They put into evidence that, on exploiting some intriguing properties of the GD theory, inverse source problems can be successfully faced, even with complex transmission functions, using spatially incoherent light. This is to be contrasted to the ordinary behavior of optical systems which use spatially incoherent light, for which any information about the phase of the object is lost. This new possibility offered by GD devices is connected, of course, to the use of a suitable reference beam.

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