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Focal shift of focused flat-topped beams

R. Borghi [a], M. Santarsiero [b], S. Vicalvi [c]

[a] *Dipartimento di Ingegneria Elettronica, Università Roma Tre, and INFM, Via della Vasca Navale 84, I-00146 Rome, Italy*
[b] *Dipartimento di Fisica, Università Roma Tre, and INFM, Via della Vasca Navale 84, I-00146 Rome, Italy*
[c] *Dipartimento di Fisica, Università La Sapienza, Piazzale Aldo Moro 2, I-00185 Rome, Italy*

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Abstract

The phenomenon of focal shift is studied for the case of focused coherent beams showing a flat-topped transverse profile. The model describing such beams is that of flattened Gaussian beams, which, due to their peculiar analytical expression, are particularly fit for the study of paraxial propagation of flattened beams. Focal shifts for the fundamental Gaussian mode and for the field produced by diffraction of a converging spherical wave by a circular aperture are shown to be obtainable by this model as particular cases.

1. Introduction

When a light beam is focused through a converging lens, the axial coordinate where the intensity takes its maximum value does not coincide, in general, with the position of the focus as predicted by geometrical optics. Such phenomenon is currently known as *focal shift*. It was first observed dealing with microwaves [1,2], while studying diffraction of a spherical wave by a circular aperture, and later it was also predicted for focused Gaussian beams [3,4].

In 1981, Li and Wolf [5] showed that the actual position of the focus is always shifted toward the aperture, with respect to the position predicted by geometrical optics, and that the amount of such a shift depends on the Fresnel number (N_F) of the aperture, when viewed from the geometrical focus. In particular, the shift increases with decreasing N_F and becomes significant for values of N_F less than 10.

The interest on focused light fields is presently still very high, due to their theoretical and application aspects [6–8]. In particular, focal shifts, which have been studied also for other classes of focused beams (apertured Gaussian, annular, Bessel–Gauss beams, aberrated waves) [9–12], have shown to be of great relevance in the study and design of laser cavities. In fact, while for imaging optical systems Fresnel numbers are generally very high (of the order of a few thousand) and then the focal shift can be neglected in most applications, this is not the case for laser cavities, which may be characterized by Fresnel numbers of the order of the unity [13].

In this work we show results pertinent to the evaluation of the focal shift of focused flattened beams, i.e., beams that present a flat-topped profile at a given plane orthogonal to the propagation axis. Beams of this kind are encountered, for instance, in the study of optical resonators with variable reflectivity [13–16] or graded-phase mirrors [17].

The model which will be used to describe such beams is that of the so-called axially symmetric flattened Gaussian beams (FGB) [18]. They are characterized by two parameters, the order N and the waist size w_0, related to the flatness of the field profile at the waist plane and to its width, respectively. The most useful property of FGBs is that they can be expressed as a *finite* sum of Laguerre–Gaussian modes [13] and therefore their paraxial propagation features can be studied in an exact, sometimes very simple way [18–22]. For this reason the FGB approach should be preferred to other models, such as super-Gaussian beams, whose paraxial propagation can be dealt with only through numerical techniques [23,24]. A further approach of interest has been recently proposed by Sheppard [25].

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2. Preliminaries

We define a FGB as a beam specified by the following field distribution at its waist plane [18,20]:

$$U_N(r,0) = A_o \exp\left[-\frac{(N+1)r^2}{w_0^2}\right] \times \sum_{n=0}^{N} \frac{1}{n!}\left[\frac{(N+1)r^2}{w_0^2}\right]^n, \qquad (1)$$

where A_o is a constant factor, w_0 is a positive parameter, and N is an integer greater or equal to zero, which is the order parameter of the FGB, alluded to in the Introduction. Here, a reference frame with cylindrical coordinates (r,ψ,z), having the z-axis and the $z=0$ plane coincident with the propagation axis and the waist plane of the beam, respectively, has been introduced. Since we consider an axially symmetric beam, all the quantities pertinent to the field are supposed to be evaluated across a meridional plane. This allows us to consider them as functions of r and z alone.

In Fig. 1 curves of $U_N(r,0)/A_o$ versus r/w_0 are shown for several values of N. It can be seen that the curve is Gaussian for $N=0$, becomes flatter and flatter with increasing N, and tends to the function circ(r/w_0) when N goes to infinity.

The propagation of a FGB through a general paraxial optical system was studied in Ref. [20] and the particular case of a focused FGB was extensively treated in Ref. [21]. In the following we report some results, pertinent to the latter case, which will be used to evaluate the focal shift.

Following the classical approach of Lommel [26], we introduce the dimensionless coordinates

$$v = F\frac{r}{w_0}, \quad u = F\frac{z-f}{f}, \qquad (2)$$

where the parameter

$$F = kw_0^2/f \qquad (3)$$

coincides with 2π times the Fresnel number [26] pertinent to a circular aperture of radius w_0.

Using these new variables, the expressions of the propagated field and intensity take very simple forms in the limit of large Fresnel numbers. Indeed, the on-axis propagated intensity takes the form [21]

$$I_N^{(\infty)}(u) = (A_o F)^2 \frac{\left|[1+iu/2(N+1)]^{-(N+1)} - 1\right|^2}{u^2}, \qquad (4)$$

where the superscript (∞) has been used to remind that such an expression holds for high Fresnel numbers only. It should be stressed that, in this limit, once N has been chosen the intensity profile is completely specified (apart from the overall factor F^2) because the function in Eq. (4) does not depend either on the numerical aperture of the system or on the Fresnel number. Moreover, the intensity distribution is symmetric with respect to the position of the geometrical focus. In Fig. 2, the on-axis intensity is shown for several values of the order N.

In the particular cases of a fundamental Gaussian beam $(N=0)$ and of a spherical wave diffracted by a circular

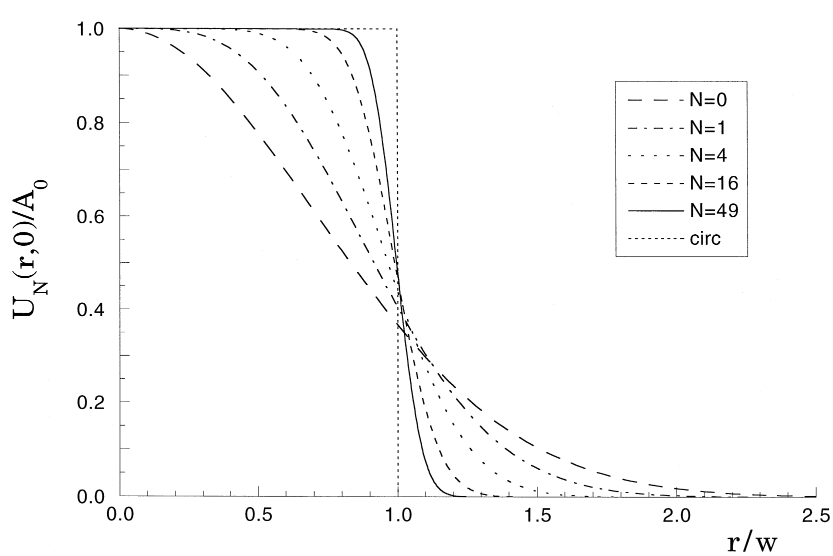

Fig. 1. Flattened Gaussian profiles for some values of N.

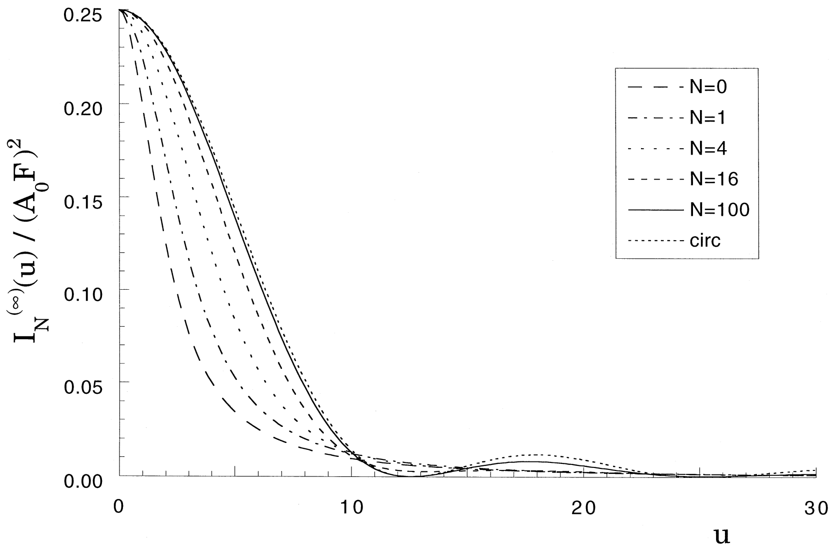

Fig. 2. On-axis intensity for some values of N in the limit of high Fresnel numbers.

aperture ($N \to \infty$), it is easy to show that Eq. (4) takes the forms

$$I_0^{(\infty)}(u) = \left(\frac{A_o F}{2}\right)^2 \frac{1}{1 + u^2/4}, \tag{5}$$

$$I_\infty^{(\infty)}(u) = \left(\frac{A_o F}{2}\right)^2 \frac{\sin^2(u/4)}{(u/4)^2}, \tag{6}$$

respectively.

In the next section we will see how the expression of the on-axis intensity can be generalized to the case of any Fresnel number and show how this more general expression gives account of the presence of a focal shift.

3. Focal shift of FGBs

The propagated field in the more general case of an arbitrary Fresnel number can be easily deduced starting

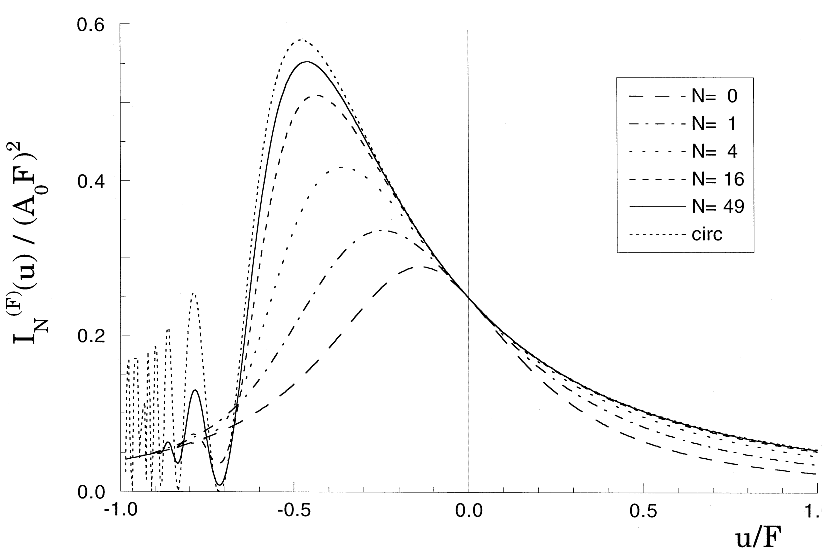

Fig. 3. On-axis intensity for $F = 5$ and some values of N.

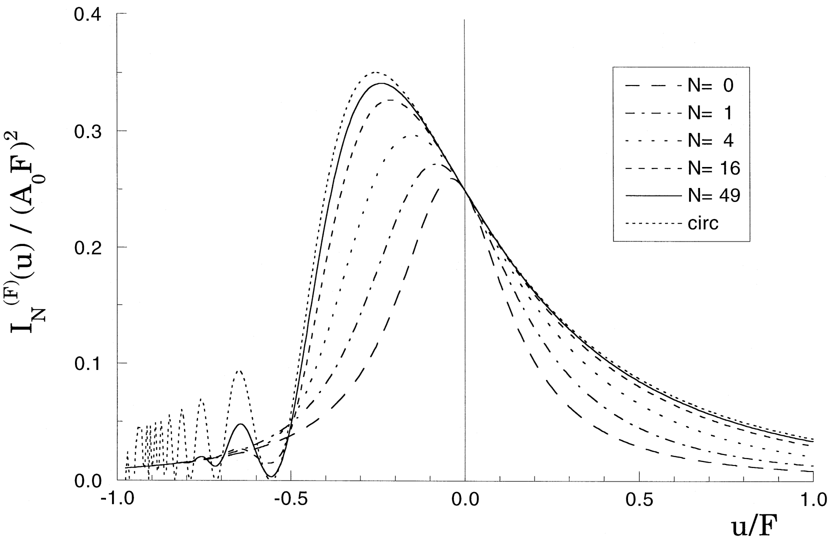

Fig. 4. On-axis intensity for $F = 10$ and some values of N.

from the results given in the previous section, by means of a suitable (non-linear) scaling of the coordinates [27].

In fact, by introducing the variable

$$u_F = \frac{u}{1 + u/F}, \tag{7}$$

the on-axis intensity for any value of F, denoted by $I_N^{(F)}(u)$, turns out to be [27]

$$I_N^{(F)}(u) = \left(1 - \frac{u_F}{F}\right)^2 I_N^{(\infty)}(u_F), \tag{8}$$

where $I_N^{(\infty)}$ is given in Eq. (4).

To show the effect of small Fresnel numbers on the axial intensity, in Figs. 3 and 4 plots of $I_N^{(F)}(u)$ are reported for two different values of F, versus the quantity u/F. The latter coincides with the relative coordinate $(z - f)/f$ [see Eq. (2)]. By further increasing F, the curves become more and more similar to those of Fig. 2.

It can be noted that the value of the axial intensity at $z = f$ is independent of N, as was proved in Ref. [21], and is given by $I_0 = (A_0 F/2)^2$, while its maximum value, say I_M, as well as the corresponding coordinate, z_M, depends on both N and F.

The value of z_M can be found by taking the derivative of the function in Eq. (8) and searching for its zeros. More

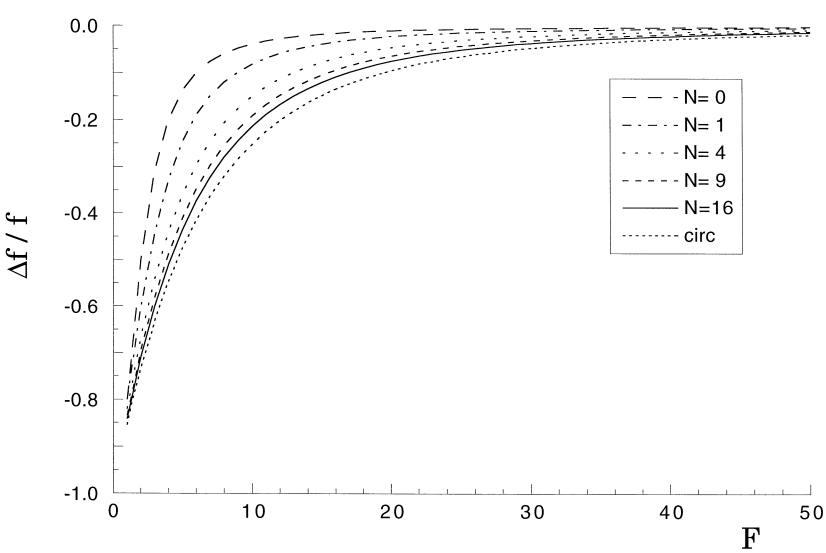

Fig. 5. Relative focal shift versus F for some values of N.

precisely, the condition for a maximum of the on-axis intensity is

$$\frac{\mathrm{d}}{\mathrm{d}u} I_N^{(F)}(u) = 0, \tag{9}$$

where Eq. (8) must be used. Since

$$\frac{\mathrm{d}}{\mathrm{d}u} = \frac{\mathrm{d}u_F}{\mathrm{d}u} \frac{\mathrm{d}}{\mathrm{d}u_F}, \tag{10}$$

and, from Eqs. (7) and (2), $\mathrm{d}u_F/\mathrm{d}u$ is a positive quantity if $z>0$ (as in our case), then the point of maximum intensity on the axis must be a root of the following equation:

$$\frac{\mathrm{d}I_N^{(F)}}{\mathrm{d}u_F} = -\frac{2}{F}\left(1-\frac{u_F}{F}\right) I_N^{(\infty)}(u_F) + \left(1-\frac{u_F}{F}\right)^2 \frac{\mathrm{d}}{\mathrm{d}u_F} I_N^{(\infty)}(u_F) = 0. \tag{11}$$

Finally, by introducing the function

$$G_N(u_F) = I_N^{(\infty)}(u_F)\left[\frac{\mathrm{d}}{\mathrm{d}u_F} I_N^{(\infty)}(u_F)\right]^{-1}, \tag{12}$$

condition (11) becomes

$$G_N(u_F) = \tfrac{1}{2}(F-u_F). \tag{13}$$

Eq. (13) can be solved numerically for any value of N, after inserting the expression of $I_N^{(\infty)}$, given in Eq. (4), into Eq. (12). The corresponding values of z_M are obtained by means of Eqs. (2) and (7). Among the various solutions, the one pertaining to the global maximum is found to be characterized for being the closest to the coordinate $u=0$.

It is useful to explicit Eq. (12) for $N=0$ and $N\to\infty$. In such cases, indeed, from Eqs. (5) and (6) we have

$$G_0(u_F) = -\left(\frac{u_F}{2} + \frac{2}{u_F}\right), \tag{14}$$

and

$$G_\infty(u_F) = \frac{2}{1/\tan(u_F/4) - 4/u_F}, \tag{15}$$

which, together with Eq. (13), lead to the already known expressions for the focal shift of Gaussian beams and spherical waves diffracted by a circular aperture, respectively (see, e.g., Ref. [5]).

The solution of Eq. (13) in the general case gives results shown in Fig. 5, where the relative focal shift $\Delta f/f = (z_M - f)/f$ is reported versus F for some values of the order N of the FGB. Several considerations can be made. First, the relative shift is always negative and tends to vanish for high values of F, as was expected. Moreover, when F approaches zero, it tends to -1, corresponding to the fact that in this limit the axial intensity is maximum at the lens plane ($z_M = 0$). As regards the dependence on the steepness of the flattened Gaussian profile, we note that for any fixed value of the Fresnel number the relative focal shift increases (in modulus) with increasing N. Finally, the relative intensity excess, defined as $\Delta I/I_0 = (I_M - I_0)/I_0$, is shown in Fig. 6.

The cases $N=0$ and $N\to\infty$ can be directly compared to those of Figs. 4 and 5 of Ref. [9], where focal shifts and intensity excesses of focused truncated Gaussian beams in the weak and strong truncation limit, respectively, are reported.

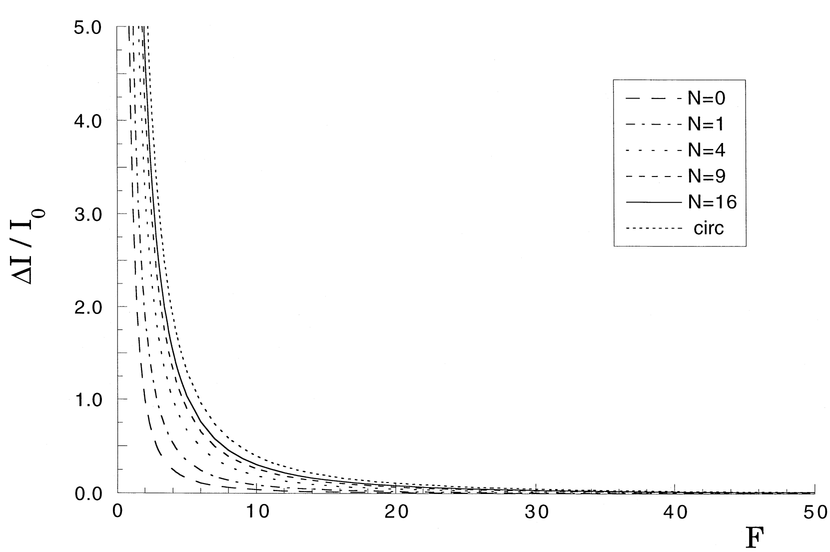

Fig. 6. Relative intensity excess versus F for some values of N.

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