Intensity-based modal analysis of partially coherent beams with Hermite–Gaussian modes

F. Gori and M. Santarsiero

Dipartimento di Fisica, Università degli Studi Roma Tre, and Istituto Nazionale per la Fisica della Materia,
Via della Vasca Navale 84, I-00146 Rome, Italy

R. Borghi and G. Guattari

Dipartimento di Ingegneria Elettronica, Università degli Studi Roma Tre, and Istituto Nazionale per la Fisica della Materia,
Via della Vasca Navale 84, I-00146 Rome, Italy

Received March 19, 1998

Many partially coherent beams are made up of a superposition of mutually uncorrelated Hermite–Gaussian modes. We prove that knowledge of the transverse intensity profile of such a beam is sufficient for evaluating the weights of the modes in an exact way. Simulations indicate that the proposed method resists noise well. © 1998 Optical Society of America

OCIS codes: 030.4070, 030.1670.

The output beam of a stable-cavity laser oscillating in many transverse modes can be written as an incoherent superposition of Hermite–Gaussian (HG) beams.1,2 As is well known,3 this superposition gives rise to partially coherent beams. The same type of representation holds true, according to Wolf’s modal theory of coherence,4 for several partially coherent beams of interest.3,5–8 For all cases, the problem consists of determining the weights of the underlying HG modes.1,2 As is well known,3 this superposition gives incoherent superposition of Hermite–Gaussian (HG) beams.1,2 Since the modes are independent of one another, their time-averaged intensities are additive, and the transverse intensity profile of the beam at its waist plane is

\[ I(x) = \sum_{n=0}^{\infty} c_n G_n^2(x; v_0), \]

where \( c_n \) is a positive coefficient representing the power content of the \( n \)th mode. The problem of interest to us is to evaluate the \( c_n \) coefficients when \( I(x) \) is known. In doing so, we assume that the spot size of the modes is known.

The difficulty with this evaluation stems from the fact that, although the \( G_n \) constitute an orthogonal set, their squares obviously do not. Consequently, the usual scalar-product rule that we would adopt for evaluating the coefficients of a series expansion into orthogonal functions cannot be applied. We can even wonder whether a unique solution exists for the values of \( c_n \). Surprisingly, we have found that when we pass to the Fourier-transform domain the coefficients can be evaluated by the scalar product rule; this may sound contradictory because the scalar product is conserved under Fourier transformation. As a matter of fact, the property that we exploit here is slightly subtler. Briefly, it turns out that the Fourier transforms of the \( G_n^2 \), although they are not orthogonal on the whole \( p \) axis \((p \) is a spatial-frequency variable), are indeed orthogonal on the half-axis \( p \geq 0 \) with respect to the variable \( p^2 \).

Our derivation starts from the following Fourier-transform relation:

\[ \mathcal{F}\{G_n^2(x; v_0)\}(p) = \Psi_n(\pi^2 v_0^2 p^2), \]

where \( \mathcal{F} \) denotes the Fourier-transform operator with respect to \( x \) and the functions

\[ \Psi_n(t) = L_n(t) \exp\left(-\frac{t^2}{2}\right) \]

are introduced, where \( L_n \) is the \( n \)th-order Laguerre polynomial.15 Equation (4) can be easily derived from formula (22.13.20) of Ref. 15.
It is well known that the $\Psi_n$ functions are orthogonal on the half-axis $t \geq 0$, i.e.,

$$\int_0^\infty \Psi_n(t)\Psi_m(t)dt = \delta_{n,m}, \quad (6)$$

where $\delta_{n,m}$ is the Kronecker symbol.

On Fourier transforming both sides of Eq. (3) and using Eq. (4), we obtain

$$\tilde{I}(p) = \sum_{n=0}^\infty c_n \Psi_n(\pi^2 v_0^2 p^2), \quad (7)$$

where the tilde stands for the Fourier transform. Then, by exploiting the orthogonality of the $\Psi_n$ functions [see Eq. (6)], we find the following expression for the expansion coefficient $c_n$:

$$c_n = 2\pi^2 v_0^2 \int_0^\infty \tilde{I}(p)\Psi_n(\pi^2 v_0^2 p^2)pdp. \quad (8)$$

This amazingly simple equation completely solves the problem of finding the powers that are contributed to the beam by the various modes. Furthermore, thanks to the completeness of the set of $\Psi_n$ functions, Eq. (8) proves that $c_n$ values are indeed unique.

We have referred to the case in which the intensity is known across the waist plane, which is easily generalized. Indeed, any beam constituted by an incoherent superposition of HG modes is endowed with well-known shape-invariance properties.6 To a constant factor accounting for energy conservation, the intensity distribution at any plane $z \neq 0$ is a magnified version of the one specified by Eq. (3), and the same magnification occurs for the spot sizes of the modes. Accordingly, we could rephrase our analysis for a plane $z \neq 0$, substituting the value of the spot size of the propagated modes, $v_z$, for $v_0$. The spot size of the modes as well as the waist position can be determined in various ways. For example, if the beam is generated by a laser, knowledge of the cavity parameters allows us to specify the mode characteristics. On the other hand, if the laser cavity is unknown or if the beam is generated by a synthetic source, the mode parameters can be evaluated starting from measurements of the intensity profile at distinct cross sections.

Let us now discuss how Eq. (8) can be exploited in practice. Suppose that the intensity profile $I(x)$ has been measured. Using standard sampling criteria, we can evaluate the Fourier transform $\tilde{I}(p)$ by use of fast Fourier-transform and interpolation techniques. We can then use a quadrature rule to compute the values of $c_n$ through Eq. (8). An important question is how resistant such a procedure is to noise. Without going into detailed analysis, we present here the results of a few simulations from which some hints about the performance of the method can be gained.

We simulated a flat-top intensity distribution by means of a flattened Gaussian profile10 of order $N = 20$. Noise was added to the intensity value of each sample as a random number with a uniform distribution in the interval $(-\varepsilon I_0, \varepsilon I_0)$, where $I_0$ is the maximum value of the intensity profile and $\varepsilon$ is a positive quantity. Possible negative values of the corrupted intensity were set to zero.

In Fig. 1(a) we show intensity profiles for $\varepsilon = 0, 0.05, 0.1$. In Fig. 1(b) the $c_n$ values obtained from Eq. (8) for these three cases are shown. For $\varepsilon = 0$ they are joined by a dashed curve and coincide with the theoretical values that were derived in the study reported in Ref. 14. For $\varepsilon \neq 0$ it should be noted that noise breaks down the even character of the intensity profile, which leads to possibly complex values for the coefficients, whereas the true values are real and positive. For this reason we show in Fig. 1(b) the real part of the values of $c_n$; the imaginary part can be ascribed to only noise. Some negative, and hence meaningless, values are exhibited, but this occurs only for those values of $n$ for which the true $c_n$ is near zero. Accordingly, the performance of the method with noisy data seems satisfactory.

In summary, a simple method of evaluating the modal content of a partially coherent beam obtained as an incoherent superposition of HG beams has been presented. The method requires the knowledge of the propagation characteristics of the modes and uses the intensity distribution of the beam at an arbitrarily chosen transverse plane to yield the set of the mode weights. In turn, this method permits the evaluation of the cross-spectral density function of the field and then leads to a complete characterization of the beam.3 Although transverse modes emitted from most stable-cavity resonators are well described by HG

![Fig. 1](image-url)
beams, sometimes the beams of interest can be of the Laguerre–Gauss family. The possibility of extending the present method to Laguerre–Gauss modes remains to be seen.

References