

Plane-wave scattering by a dielectric circular cylinder parallel to a general reflecting flat surface

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We present a generalization of a method developed for treating the plane-wave scattering by a perfectly conducting circular cylinder in front of a plane surface to the case of a generic dielectric circular cylinder. Thanks to this formulation, the problem can be treated in a very efficient way for both the near and the far field, and an accurate determination of the field inside the cylinder is possible. Numerical results and comparisons with other methods are presented. © 1997 Optical Society of America [S0740-3232(97)02806-8]

1. INTRODUCTION

Recently,¹ we studied the scattering problem of a plane wave by a perfectly conducting circular infinite cylinder placed in front of a plane discontinuity for the electromagnetic constants. The proposed method, starting from the customary expansion of the scattered field, exploits the plane-wave representation of cylindrical waves² and provides a rigorous solution for this problem, at least in its theoretical basis. In a practical implementation of the algorithm, even if some numerical approximations must be made because of the necessity of truncating the involved series, a very rapid convergence and a remarkable stability with respect to the input data are obtained. These characteristics make the technique attractive for a wide range of applications. In this paper we intend to generalize the aforementioned approach to the case of a dielectric cylinder, characterized by a refractive index n_c , which can also assume complex values, corresponding to lossy materials.

The study of the scattering of a plane wave by a dielectric cylinder, with or without a reflecting surface, has a great number of applications, such as optical fiber characterization, microwave heating, defect detection in semiconductors, and near-field optics.³⁻¹⁰ The scattering problem for an isolated cylinder was first studied by Lord Rayleigh¹¹ and then generalized by Wait¹² to the case of oblique incidence. Richmond¹³ studied a cylinder with arbitrary cross section and solved the problem numerically, making use of the concept of equivalent currents.

When a plane interface is introduced, even in the case of a circular cylinder, the problem turns out to be much more difficult to deal with and only numerical approaches have been used to solve it. In particular, the far-zone scattered field has been determined by using coupled-dipole or integral-equation methods.^{14,15}

The present generalization of the method proposed in

Ref. 1 affords a solution for a wide class of scattering problems. More precisely, since the presence of the interface is taken into account only through the reflection coefficient, very different types of surfaces can be considered. Moreover, both the near- and far-zone diffracted fields can be determined for both polarization states (TE and TM, with respect to the cylinder axis).

In Section 2 the theoretical approach is described, while in Section 3 the convergence properties are shown together with numerical results for a test case, both in the near and the far region. Comparisons with the results given in Ref. 14 are also reported.

2. THEORETICAL ANALYSIS

Figure 1 shows the geometrical layout of our problem: A monochromatic plane wave with wavelength λ impinges on a dielectric circular cylinder of radius a and refractive index n_c . The cylinder axis is placed at a distance h from a general reflecting flat surface. This diffractive structure is assumed to be infinite along the y direction, which is parallel to the cylinder axis, so that the problem is reduced to a two-dimensional form. Moreover, we utilize the following dimensionless variables:

$$\xi = k_0 x, \quad \zeta = k_0 z, \quad \chi = k_0 h, \quad \rho = k_0 r, \quad (1)$$

where $k_0 = 2\pi/\lambda$ is the wave number in vacuum. In the following, $\mathbf{n} = \mathbf{k}/k_0$ denotes the unit vector associated with a typical plane wave whose wave vector is \mathbf{k} , while n_{\parallel} and n_{\perp} are the components of \mathbf{n} parallel and perpendicular to the reflecting surface, respectively. The latter is characterized by a complex reflection coefficient Γ , which is a function of n_{\parallel} . Moreover, \mathbf{k}^i is the wave vector of the incident field, and ϕ denotes the incidence angle with respect to the x axis. The polarization of the fields involved is assumed to be either TM or TE (electric or magnetic field directed along the axis of the cylinder). In

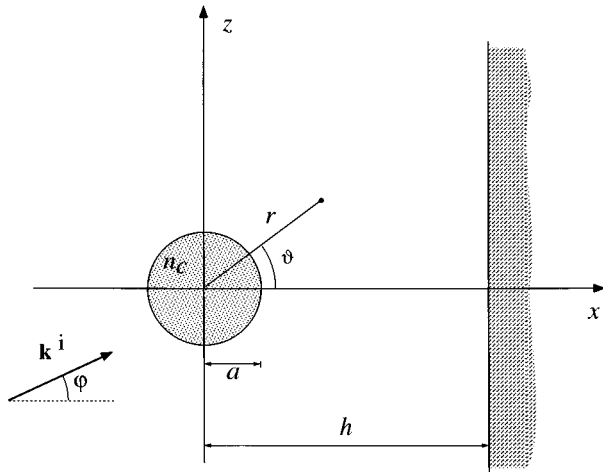


Fig. 1. Geometrical layout of the scattering problem.

both cases the amplitude of the field parallel to the cylinder axis is represented by the scalar function $V(\xi, \zeta)$.

To solve the scattering problem, we consider the field $V(\xi, \zeta)$, which is due to the interaction between the incident plane wave and the diffractive structure, as the sum of five contributions, say,

- V_i : field of the incident plane wave;
- V_r : field that is due to the reflection of V_i by the plane surface;
- V_c : field present inside the cylinder;
- V_d : field diffracted by the cylinder;
- V_{dr} : field that is due to the reflection of V_d by the plane surface.

This approach parallels the one used in Ref. 1 for the case of a perfectly conducting cylinder. Here, however, we have introduced the term $V_c(\xi, \zeta)$, which accounts for the field distribution inside the cylinder.

The boundary conditions on the cylinder surface can be easily imposed by expressing each contribution to the total field in a reference frame having cylindrical symmetry. The expressions of V_i , V_r , V_d , and V_{dr} are discussed in Ref. 1 and have the form

$$V_i(\xi, \zeta) = V_0 \sum_{m=-\infty}^{+\infty} i^m \exp(-im\phi) J_m(\rho) \exp(im\theta), \quad (2a)$$

$$V_r(\xi, \zeta) = V_0 \Gamma(n_{\parallel}^i) \exp(in_{\perp}^i 2\chi) \sum_{m=-\infty}^{+\infty} i^m J_m(\rho) \times \exp[im(\theta + \phi - \pi)], \quad (2b)$$

$$V_d(\xi, \zeta) = V_0 \sum_{m=-\infty}^{+\infty} i^m \exp(-im\phi) c_m \text{CW}_m(\xi, \zeta), \quad (2c)$$

$$V_{dr}(\xi, \zeta) = V_0 \sum_{m=-\infty}^{+\infty} i^m \exp(-im\phi) c_m \text{RW}_m(2\chi - \xi, \zeta) \\ = V_0 \sum_{m=-\infty}^{+\infty} i^m \exp(-im\phi) c_m \sum_{l=-\infty}^{+\infty} i^l J_l(\rho) \\ \times \exp(il\theta) \text{RW}_{l+m}(2\chi, 0). \quad (2d)$$

In these equations V_0 is the amplitude of the incident field, (ρ, θ) are coordinates of the polar reference frame centered on the cylinder axis, J_m is the Bessel function of the first kind of m th order, and the coefficients c_m represent the unknown quantities of our problem. Moreover, the function CW and RW are defined as¹

$$\text{CW}_m(\xi, \zeta) = H_m(\rho) \exp(im\theta), \quad (3a)$$

$$\text{RW}_m(\xi, \zeta) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \Gamma(n_{\parallel}) F_m(\xi, n_{\parallel}) \exp(in_{\parallel} \zeta) dn_{\parallel}, \quad (3b)$$

where $H_m(x)$ is the Hankel function of the first kind of m th order, while the function $F_m(\xi, n_{\parallel})$, representing the angular spectrum of cylindrical functions $\text{CW}_m(\xi, \zeta)$, is^{1,2}

$$F_m(\xi, n_{\parallel}) = \frac{2 \exp(in_{\perp} \xi)}{n_{\perp}} \exp(-im \arccos n_{\parallel}). \quad (4)$$

The field transmitted into the cylinder can be written as¹⁶

$$V_c(\xi, \zeta) = V_0 \sum_{m=-\infty}^{+\infty} i^m \exp(-im\phi) d_m J_m(n_c \rho) \exp(im\theta), \quad (5)$$

with unknown coefficients d_m .

Now, from Eqs. (2a)–(2d) and (5), it is straightforward to impose the appropriate boundary conditions on the surface of the cylinder in order to determine the values of the c_m and d_m coefficients. Let us analyze this procedure in some detail for both polarization states. If we denote with $\tilde{\nabla}$ the gradient operator with respect to the dimensionless coordinates (ξ, ζ) , Maxwell's equations read as¹

$$\mathbf{E}(\xi, \zeta) = (iZ_0/\nu^2) \tilde{\nabla} \times \mathbf{H}(\xi, \zeta), \quad (6)$$

$$\mathbf{H}(\xi, \zeta) = -iY_0 \tilde{\nabla} \times \mathbf{E}(\xi, \zeta), \quad (7)$$

Z_0 and Y_0 being the characteristic impedance and admittance, respectively, of the medium outside the cylinder, while ν is defined as

$$\nu = \begin{cases} 1 & \text{outside the cylinder} \\ n_c & \text{inside the cylinder} \end{cases}. \quad (8)$$

The expression of the curl operator in cylindrical coordinates is

$$\tilde{\nabla} \times [V(\xi, \zeta) \hat{\mathbf{y}}_0] = (1/\rho) \partial_{\theta} V(\xi, \zeta) \hat{\mathbf{r}}_0 - \partial_{\rho} V(\xi, \zeta) \hat{\theta}_0, \quad (9)$$

where $\hat{\mathbf{y}}_0$, $\hat{\mathbf{r}}_0$, and $\hat{\theta}_0$ are the unit vectors associated with the cylindrical reference frame. In the case of TM polarization, we have $\mathbf{E}(\xi, \zeta) = V(\xi, \zeta) \hat{\mathbf{y}}_0$. The boundary conditions, arising from the continuity of the tangential components of electric and magnetic fields, have to be written as

$$(V_i + V_r + V_d + V_{dr})_{\rho=ka} = (V_c)_{\rho=ka}, \\ (\partial_{\rho} V_i + \partial_{\rho} V_r + \partial_{\rho} V_d + \partial_{\rho} V_{dr})_{\rho=ka} = (\partial_{\rho} V_c)_{\rho=ka}. \quad (10)$$

For TE polarization the roles of $V(\xi, \zeta)$ and $\partial_{\rho} V(\xi, \zeta)$ are interchanged, so that the boundary conditions become

$$(V_i + V_r + V_d + V_{dr})_{\rho=ka} = (V_c)_{\rho=ka},$$

$$(\partial_\rho V_i + \partial_\rho V_r + \partial_\rho V_d + \partial_\rho V_{dr})_{\rho=ka} = (1/n_c^2)(\partial_\rho V_c)_{\rho=ka}. \quad (11)$$

By using Eqs. (2a)–(2d), (5), (10), and (11), after some algebra we obtain the following linear system for the unknown coefficients c_m and d_m :

$$\sum_{l=-\infty}^{+\infty} A_{ml}^{(1)} c_l - G_m^{(1)} d_m = b_m^{(1)}, \quad (12a)$$

$$\sum_{l=-\infty}^{+\infty} A_{ml}^{(2)} c_l - G_m^{(2)} d_m = b_m^{(2)}, \quad (12b)$$

where the superscripts (1) and (2) refer to the boundary conditions on the field and on its normal derivative, respectively. Here

$$G_m^{(1)} = \exp(-im\phi) \frac{J_m(n_c ka)}{H_m(ka)}, \quad (13a)$$

$$G_m^{(2)} = p \exp(-im\phi) \frac{J'_m(n_c ka)}{H'_m(ka)}, \quad (13b)$$

where the prime denotes differentiation and p is defined as

$$p = \begin{cases} n_c & \text{for TM polarization} \\ n_c^{-1} & \text{for TE polarization} \end{cases}. \quad (14)$$

Furthermore, the coefficients of systems (12) are

$$A_{ml}^{(1,2)} = \exp(-il\phi) [\delta_{ml} + i^{l-m} T_m^{(1,2)}(ka) \times \text{RW}_{l+m}(2\chi, 0)], \quad (15a)$$

$$b_m^{(1,2)} = -T_m^{(1,2)}(ka) \{ \exp(-im\phi) + \Gamma(n_\parallel^i) \times \exp(in_\perp^i 2\chi) \exp[-im(\pi - \phi)] \}, \quad (15b)$$

where the symbol δ_{ml} denotes the Kronecker delta, while the function $T_m^{(j)}$, which contains the information regarding the boundary conditions, is defined as

$$T_m^{(j)}(x) = \begin{cases} J_m(x)/H_m(x) & \text{for } j = 1 \\ J'_m(x)/H'_m(x) & \text{for } j = 2 \end{cases}. \quad (16)$$

A way to solve systems (12) is to eliminate the coefficients d_m , thus obtaining the following linear system for the sole c_l coefficients:

$$\sum_{l=-\infty}^{+\infty} D_{ml} c_l = L_m, \quad (17)$$

where

$$D_{ml} = G_m^{(2)} A_{ml}^{(1)} - G_m^{(1)} A_{ml}^{(2)}, \quad (18a)$$

$$L_m = G_m^{(2)} b_m^{(1)} - G_m^{(1)} b_m^{(2)}. \quad (18b)$$

In particular, Eq. (17) shows that the computational effort is the same as that for the perfectly conducting cylinder case. This will be confirmed by the numerical test presented in the next section.

Once the c_l coefficients are known, it is possible to evaluate the internal field V_c by means of d_m coefficients in a straightforward way. Indeed, by subtracting term by term Eqs. (12), we obtain

$$d_m = -\frac{1}{G_m^{(1)} - G_m^{(2)}} \times \left\{ b_m^{(1)} - b_m^{(2)} - \sum_{l=-\infty}^{+\infty} [A_{ml}^{(1)} - A_{ml}^{(2)}] c_l \right\}, \quad (19)$$

and, by recalling Eqs. (14)–(16), after some algebra we get the equality

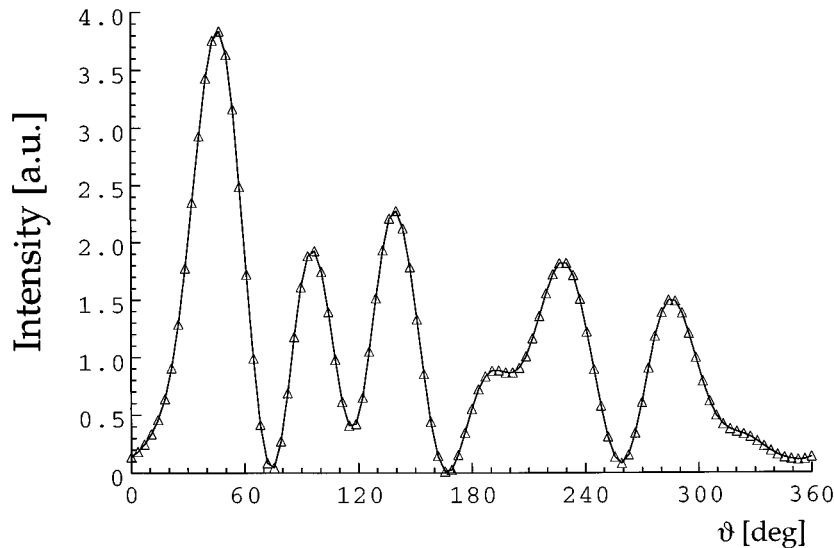


Fig. 2. Comparison between the intensity values, in arbitrary units, at the cylinder surface, evaluated from the internal (solid curve) and external (triangles) expansions, where $\lambda = 632.8$ nm, $\phi = 30^\circ$, $n_c = 1.46$, $ka = 3.95$, and $N = 12$. The plane surface is the interface between vacuum and a homogeneous dielectric medium (silicon, $n_s = 3.8$).

$$d_m = \frac{J_m(ka)H'_m(ka) - J'_m(ka)H_m(ka)}{J_m(n_c ka)H'_m(ka) - pJ'_m(n_c ka)H_m(ka)} \times \left\{ 1 + \Gamma(n_{\parallel}^i) \exp(in_{\perp}^i 2\chi) \exp[i(m-2)\phi - \pi] + \sum_{l=-\infty}^{+\infty} \text{RW}_{l+m}(2\chi, 0) i^{l-m} \exp[i(m-l)\phi] c_l \right\},$$

$$m = 0, \pm 1, \pm 2, \dots \quad (20)$$

From Eq. (20) it is easily seen how the effect of the plane surface on the internal field is contained in the last two terms in curly braces, which take into account the reflected fields V_r and V_{dr} . It should be noted, in fact, that, in the absence of the surface ($\Gamma \equiv 0$), these terms vanish, and the expression of d_m tends to be the same as that for the isolated cylinder.¹⁶

By knowledge of the c_m and d_m coefficients ($m = 0, \pm 1, \pm 2, \dots$), the scattering problem is exactly solved. Of course, since the outlined procedure involves infinite series, some truncation criterion is to be adopted. As we shall see in the next section, one can use the same criteria that proved to be adequate for other problems involving scattering from cylinders.^{1,17-19}

In Section 3 we will present some numerical results, and we will show that for obtaining an accurate description of the electromagnetic field, the required value of the index of truncation of the series can be reasonably small. In particular, we will use as a test case one of the problems studied by Taubenblatt by means of the so-called coupled-dipole method.¹⁴ As we shall see, the agreement between our results and Taubenblatt's is excellent. Furthermore, a complete and accurate two-dimensional map of the electromagnetic intensity inside the cylinder is also presented. The importance of such information was stressed by Owen *et al.* for the case of an isolated dielectric cylinder,⁴ and it arises in a wide range of practical applications.⁸⁻¹⁰

3. NUMERICAL RESULTS

The case under test is one of those studied in Ref. 14, i.e., the scattering problem of a monochromatic plane wave of wavelength $\lambda = 632.8$ nm by a dielectric circular cylinder of SiO_2 ($n_c = 1.46$), whose radius is $a = 0.35$ μm , placed onto a flat substrate of silicon (refractive index $n_s = 3.8$). The incidence angle of the impinging wave is $\phi = 30^\circ$, and the reflection coefficient of the surface has been evaluated by means of the well-known Fresnel formulas.¹⁶ The polarization is assumed to be TM.

It is well known that, in the truncation of series relating to scattering from circular cylinders, the truncation index, say N , can be related to the cylinder radius a by the rule of thumb $N \approx 3ka$.^{1,17-19} We verified that this choice leads to good results also in the present problem by controlling the convergence of the expansion coefficients for increasing values of N .

To show the good matching between the internal (V_c) and the external ($V_i + V_r + V_d + V_{dr}$) field on the cylinder surface, the squared modulus of the electric field (here and in the following loosely referred to as the inten-

sity) is reported in Fig. 2 for $\rho \rightarrow ka^-$ (internal expansion) as a solid curve and for $\rho \rightarrow ka^+$ (external expansion) as triangles for values of θ ranging from 0° to 360° .

Figure 3 shows the far-zone diffracted intensity I as a function of the scattering angle θ (see Fig. 1) for the case under test, where for the sake of comparison we have used a different system of visualization. Our result is in perfect agreement with Fig. 2 of Ref. 14, which refers to the same geometry. It should be stressed that, even though the coupled-dipole method gives the far-zone field produced by the scattering from arbitrarily shaped scatterers, it requires heavier and heavier computational effort with increasing cross-section dimensions.

Finally, Fig. 4 shows a two-dimensional plot of the intensity distribution inside the dielectric cylinder.³⁻¹⁰

4. CONCLUSIONS

A very efficient technique has been described to treat the general scattering problem by a circular dielectric cylin-

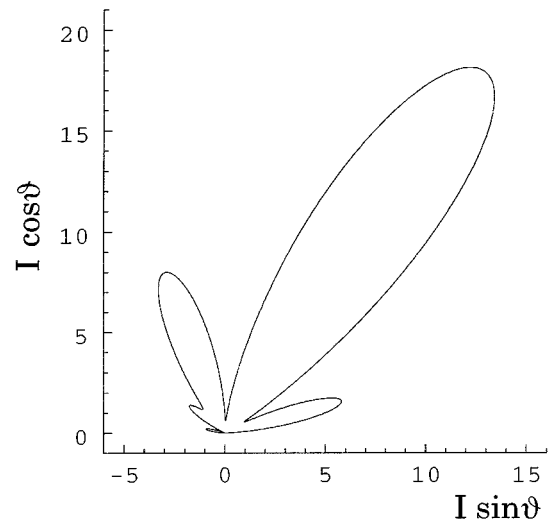


Fig. 3. Far-field diffracted intensity I as a function of the scattering angle θ for the case of Fig. 2. The same presentation of the values has been chosen as that of Ref. 14.

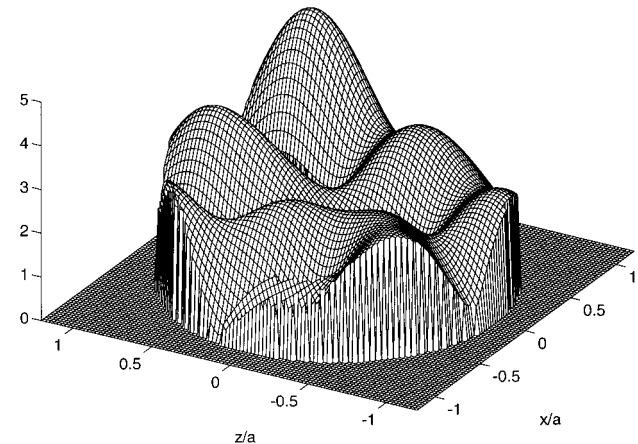


Fig. 4. Intensity distribution inside the dielectric cylinder for the case of Figs. 2 and 3. For clarity, the field outside the cylinder has been set to zero.

der near an arbitrary plane interface. Both polarizations may be studied, and both the near and the far field may be obtained. A comparison with results in the literature was presented, showing very good agreement. The case of a lossy cylinder could be treated by means of the same procedures by suitably choosing the complex refractive index of the cylinder.

Since an arbitrary scatterer may be simulated with a suitable array of circular cylinders,^{14,18} work is in progress to generalize our method to deal with several dielectric cylinders arbitrarily placed in the presence of a flat surface, as was done for the case of perfectly conducting cylinders.²⁰ Furthermore, by making use of a plane-wave spectrum expansion, the method could be extended to cases in which the incident field differs from a single plane wave.

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