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Propagation of generalized Bessel–Gauss beams through *ABCD* optical systems

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Abstract

We study the propagation of generalized Bessel–Gauss beams through *ABCD* optical systems, starting from a representation in terms of tilted and shifted Gaussian beams. We show that generalized Bessel–Gauss beams constitute a class of fields whose mathematical expression remains the same after any paraxial transformation, and give the explicit form of the propagated field.

1. Introduction

Bessel beams (also called non-diffracting, or propagation-invariant beams) [1–4] are particular solutions of the Helmholtz equation that maintain the same intensity profile at any plane orthogonal to the propagation direction. Bessel beams owe their name to the fact that, once a cylindrical reference frame (r, ϑ, z) is fixed with the z axis coincident with the propagation direction, their amplitude turns out to be

$$U_z^{(n)}(r, \vartheta) = FJ_n(\beta r)\exp(\mathrm{i}n\vartheta)\exp\left(\mathrm{i}\sqrt{k^2-\beta^2}\,z\right), \tag{1}$$

where J_n is the Bessel function of the first type of order n [5], F is an amplitude factor, k is the wave number, and β is a positive parameter smaller than k. In particular, the transverse pattern of the Bessel beam of zero order presents a central bright spot, having linear dimension of the order of β^{-1}. The possibility of producing a virtually diffraction-free pencil of light made possible the devising of practical applications, hardly conceivable with conventional beams [6–10].

An undesirable feature of Bessel beams is that they should carry an infinite amount of energy, because the functions in Eq. (1) are not square integrable. In fact, every attempt to produce such beams experimentally will always lead to beams that maintain (at least approximately) the same transverse intensity profile only within a finite range of values of z. This is essentially due to finite aperture optical elements unavoidably present in any practical realization [4,11–14].

For this reason several papers have been devoted to the study of apertured Bessel beams, i.e. beams that, in a given transverse plane, show the field distribution (1) multiplied by a windowing profile [15–20]. Among all the possible choices for this profile, the Gaussian function leads to the so-called

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Bessel–Gauss beams (BGBs from now on) [21], having, on the plane $z = 0$, the field distribution

$$U_0^{(n)}(r, \vartheta) = F J_n(\beta r) \exp\left[-(r/w_0)^2\right] \exp(\mathrm{i} n\vartheta), \tag{2}$$

where w_0 is a positive parameter. In the following, these beams will be referred to as ordinary BGBs. One of the qualities of BGBs is that their expression in paraxial propagation can be given a closed form. Moreover, it has been shown [21] that they maintain approximately the same transverse profile as far as z remains smaller than a certain value, say z_M, depending on β and w_0. When $z \gg z_M$ the transverse profile is very different from the one at $z = 0$ and may also present an annular shape. Incidentally, we recall that partially coherent BGBs have been studied, too [22].

In a recent work [23], a generalization of BGBs, the so called generalized BGBs, has been introduced. They comprise ordinary BGBs as particular cases, but can also exhibit annular or flattened profiles, depending on the values of their parameters. In the present paper we show that generalized BGBs are described by one and the same mathematical expression after propagating through any paraxial optical system, described by a 2×2 $ABCD$ matrix. Our approach makes use of a representation of BGBs in terms of a continuous superposition of suitably tilted and shifted Gaussian beams. As we shall see, in many cases this makes the paraxial propagation process more intuitive and allows the form of the output field to be predicted by means of simple arguments, based on the propagation features of geometrical rays. At a more abstract level our results could be derived following an approach based on very general considerations of group theory, as was done by Wolf [24].

Although a more complete analysis of the subject would require a vectorial treatment [25,26], in this work we use a scalar representation of the involved fields. As shown in Ref. [25], this approach leads to correct results when linearly polarized beams are studied and the conditions ensuring the validity of the paraxial approximation are fulfilled.

In Section 2 we recall the definition of generalized BGBs, and discuss their structure for some particular choices of their parameters. In Section 3 we work out the expression of a tilted and shifted Gaussian beam propagating in an $ABCD$ optical system. Finally, in Section 4 we derive the expression of the propagated generalized BGB and discuss our results for some particular cases.

2. Generalized Bessel–Gauss beams

Generalized Bessel–Gauss beams are characterized by the following field distribution across the plane $z = 0$ [23]:

$$U_0^{(n)}(r, \vartheta) = F J_n\left[\left(\beta - \mathrm{i}\frac{2a}{w_0^2}\right) r\right] \exp\left(-\frac{r^2 + a^2}{w_0^2}\right) \times \exp(\mathrm{i} n\vartheta). \tag{3}$$

Differently from ordinary BGBs (see Eq. (2)), in expression (3) the positive parameter a is present, so the argument of the Bessel function is now complex. In particular, when $a = 0$ we obtain ordinary BGBs, whereas when $a \neq 0$ and $\beta = 0$, Eq. (3) gives

$$U_0^{(n)}(r, \vartheta) = F \mathrm{i}^{-n} I_n\left(\frac{2a}{w_0^2} r\right) \exp\left(-\frac{r^2 + a^2}{w_0^2}\right) \times \exp(\mathrm{i} n\vartheta). \tag{4}$$

Here, the modified Bessel function of order n has been introduced [5] and the following property,

$$I_n(x) = \mathrm{i}^n J_n(-\mathrm{i} x), \tag{5}$$

has been used. Field distributions of the type of Eq. (4) give rise to the so-called modified BGBs [23,27].

The expression of a generalized BGB in free paraxial propagation was given in Ref. [23], where it was shown that, for suitable choices of the parameters and of the propagation distance, the transverse profile can also take annular or flattened shapes.

A useful approach to solve propagation problems for BGBs consists in an expansion in terms of Gaussian beams. Indeed, a field of the form of Eq. (3) can be thought of as the superposition of a continuous set of Gaussian beams, whose propagation axes are evenly distributed on the surface of a cone [23]. The axis of the latter coincides with the z axis (see Fig. 1) and its aperture angle ϑ is related to β by the relation

$$\beta = k \sin \vartheta. \tag{6}$$

All Gaussian beams are supposed to be linearly polarized along the same direction, and the angle ϑ

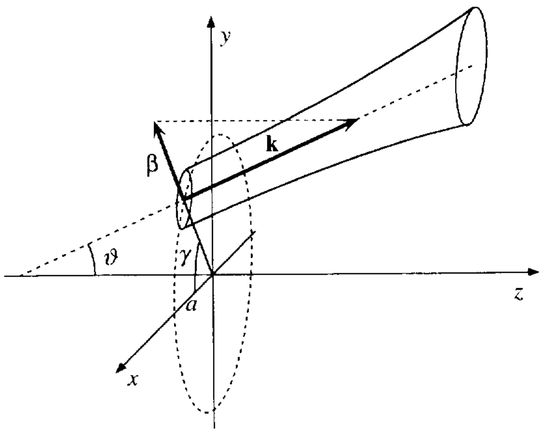

Fig. 1. One of the Gaussian beams constituting a BGB.

is required to be small enough to permit scalar and paraxial treatment for the propagation of the total field. The centers of the transverse distributions of the Gaussian beams at their waist planes are placed on a circumference with radius a, lying on the plane $z=0$, which is placed at a distance $a/\tan\vartheta$ from the cone apex.

Let us translate these ideas into formulas. If we denote by $\boldsymbol{r}$ the typical position vector on the plane $z=0$, each Gaussian field across such plane can be written as

$$V_0(\boldsymbol{r}) = G\exp\left[-\left(\frac{\boldsymbol{r}-\boldsymbol{a}}{w_0}\right)^2\right]\exp(\mathrm{i}\boldsymbol{\beta}\cdot\boldsymbol{r}), \qquad (7)$$

where $\boldsymbol{a}$ is the center of symmetry of its transverse profile, $\boldsymbol{\beta}$ is the projection of the mean wave vector on the plane $z=0$, w_0 is the waist size, and G is an amplitude factor. We note that the ellipticity of the beam section has been neglected, since we assume that β is small enough with respect to k. Fields of the type of Eq. (7) may be related to the recently introduced decentered Gaussian beams [28].

To obtain a generalized BGB of order n we have to sum fields of the type of Eq. (7), with suitable weights, for all the values of $\boldsymbol{a}$ with modulus a, and with $\boldsymbol{\beta}$ parallel to $\boldsymbol{a}$. In particular, denoting by (a, γ) the polar coordinates of $\boldsymbol{a}$, the amplitude factors must be chosen as

$$G = \frac{F\exp(\mathrm{i}n\gamma)}{2\pi\mathrm{i}^n}. \qquad (8)$$

In conclusion, it is easy to prove [23] that the field (3) can be written as

$$U_0^{(n)}(r,\vartheta) = \frac{F}{2\pi\mathrm{i}^n}\int_0^{2\pi}\exp\left[-\left(\frac{\boldsymbol{r}-\boldsymbol{a}}{w_0}\right)^2\right] \times\exp(\mathrm{i}\boldsymbol{\beta}\cdot\boldsymbol{r})\exp(\mathrm{i}n\gamma)\,\mathrm{d}\gamma. \qquad (9)$$

As it is evident, the problem of the propagation of a generalized BGB in an optical system can be easily solved if one knows the solution of the analogous problem for a single tilted and shifted Gaussian beam. This will be dealt with in the next section.

3. Gaussian beams and *ABCD* systems

First of all we introduce the dimensionless parameter

$$\boldsymbol{\epsilon} = \frac{\boldsymbol{\beta}}{k}. \qquad (10)$$

We stress that the pair (a, ϵ), characterizing the propagation axes of the component Gaussian beams, is equivalent to the vector representing, in the ray-matrix formalism [29], a meridian ray propagating in a symmetrical optical system. Let us further recall that the complex radius of curvature q of a Gaussian beam [30] is defined by the relation

$$\frac{1}{q} = \frac{1}{R} + \mathrm{i}\frac{2}{kw^2}, \qquad (11)$$

where R and w are the real radius of curvature and the spot size of the beam, respectively, at a given transverse plane.

It can be noted that the right-hand side of Eq. (7) can be factorized into the product of two functions, each depending on one transverse coordinate; hence, we shall first treat the one-dimensional case, and then extend our results to the two-dimensional one. If we denote by suffixes 0 and 1 the quantities pertinent to the input and the output planes, respectively, our aim is to solve the propagation problem for the field

$$V_{x0}(x) = G_{x0}\exp\left[\mathrm{i}\frac{k}{2q_0}(x-a_{x0})^2\right]\exp(\mathrm{i}k\epsilon_{x0}x). \qquad (12)$$

Here, a_{x0} and ϵ_{x0} are the x-component of $\boldsymbol{a}$ and $\boldsymbol{\epsilon}$ respectively, whereas the symbols V_{x0} and G_{x0} are used only to remind that the actual field V_0 is presently replaced by a one-dimensional version.

We start from the Collins integral [31] that, within the paraxial approximation, allows us to write

$$V_{x1}(x) = \exp(\mathrm{i}k\ell)\sqrt{\frac{-\mathrm{i}}{\lambda B}}\int_{-\infty}^{\infty} V_{x0}(\xi) \times \exp\left[\mathrm{i}\frac{k}{2B}\left(A\xi^2 - 2x\xi + Dx^2\right)\right] \mathrm{d}\xi, \tag{13}$$

where ℓ is the optical path length from the input to the output plane, measured along the optical axis and, of course, A, B and D are elements of the pertinent $ABCD$ matrix. By substituting from Eq. (12) into Eq. (13), and using the relation [5]

$$\int_{-\infty}^{\infty} \exp(-rt^2)\exp(-2st)\,\mathrm{d}t = \sqrt{\frac{\pi}{r}}\exp\left(\frac{s^2}{r}\right), \tag{14}$$

after some calculations, the output field can be written in the form

$$V_{x1}(x) = \exp(\mathrm{i}k\ell)G_{x1}\exp\left[\mathrm{i}\frac{k}{2q_1}(x - a_{x1})^2\right] \times \exp(\mathrm{i}k\epsilon_{x1}x), \tag{15}$$

where

$$G_{x1} = G_{x0}\sqrt{\frac{1}{A + B/q_0}} \times \exp\left[-\mathrm{i}\frac{k}{2}(Ca_{x0}a_{x1} + B\epsilon_{x0}\epsilon_{x1})\right], \tag{16}$$

$$\begin{pmatrix} a_{x1} \\ \epsilon_{x1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} a_{x0} \\ \epsilon_{x0} \end{pmatrix}, \tag{17}$$

and

$$q_1 = \frac{Aq_0 + B}{Cq_0 + D}. \tag{18}$$

The extension to the two-dimensional case is straightforward, since it is enough to multiply Eq. (15) by a similar expression depending on y. In particular, if we consider only meridional directions (for which $\boldsymbol{\epsilon}$ is parallel to $\boldsymbol{a}$), we obtain

$$V_1(\boldsymbol{r}) = G_1\exp\left[\mathrm{i}\frac{k}{2q_1}(\boldsymbol{r} - \boldsymbol{a}_1)^2\right]\exp(\mathrm{i}k\boldsymbol{\epsilon}_1\cdot\boldsymbol{r}), \tag{19}$$

where q_1 is given by Eq. (18) and

$$G_1 = \frac{G_0}{A + B/q_0}\exp\left[-\mathrm{i}\frac{k}{2}(Ca_0a_1 + B\epsilon_0\epsilon_1)\right]. \tag{20}$$

$$\begin{pmatrix} a_1 \\ \epsilon_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} a_0 \\ \epsilon_0 \end{pmatrix}, \tag{21}$$

the angle γ being unchanged.

The above relations can be interpreted as follows: the propagation axis of the Gaussian beam exactly follows the geometrical path, as expected from the ray-matrix approach (Eq. (21)); spot size and curvature radius are the same as those of a centered Gaussian beam propagating through the optical system (Eq. (18)). Moreover, the field is to be multiplied by a complex factor, accounting for the conservation of the total energy and for the phase shift introduced by the system.

4. Propagation of generalized BGBs in *ABCD* systems

The expression of a generalized BGB propagated through an *ABCD* optical system can be simply found by summing all the Gaussian fields of the expansion (9), using the new parameters evaluated by means of Eqs. (18), (20) and (21). This gives

$$U_1^{(n)}(n, \vartheta) = \frac{F}{A + B/q_0}\exp[\mathrm{i}k(\ell - s)] \times J_n\left[\left(\epsilon_1 - \frac{a_1}{q_1}\right)kr\right] \times \exp\left[\mathrm{i}\frac{k}{2q_1}\left(r^2 + a_1^2\right)\right]\exp(\mathrm{i}n\vartheta), \tag{22}$$

where

$$s = \frac{Ca_0a_1 + B\epsilon_0\epsilon_1}{2}. \tag{23}$$

As it is evident, the output field has the same structure as the input field, (3), apart from an amplitude and phase term, provided that the complex radius of curvature, (11), is introduced. Hence, we can affirm that generalized BGBs constitute a class of fields whose form is invariant after propagation in any paraxial optical system.

Our assertion deserves further specifications. As is well known, certain beams, such as Hermite–Gauss or Laguerre–Gauss beams [30], maintain the same transverse intensity distribution upon propagation, except for a magnification (or demagnification) factor. Fields of this type are said to be shape-invariant. Eq. (22) has an invariant form. Yet the corresponding field is not shape-invariant. In order to reconcile these two seemingly contradictory statements, we note that the function J_n in Eq. (22) depends on a complex variable. On varying r the argument of J_n spans a (half-) line in the complex plane. Now, the complex coefficients of r pertaining to the input and output planes are not, generally speaking, simply proportional to each other through a real factor. As a consequence, two distinct lines of the complex plane are spanned upon variation of r and this gives rise to different profiles.

We can further visualize the situation by considering the surface that represents the (three-dimensional) graph of $J_n(\zeta)$ with respect to the complex variable ζ. Then the input and output profiles are determined by the sections of such a surface with two different half-planes originating from the line orthogonal to the complex plane at the point $\zeta = 0$. In a sense, this is the basic trick by which a single function can account for infinitely many profiles. For the sake of completeness we note that a further reason why shape-invariance is not exhibited is the fact that the coefficients of r within the Bessel function and the quadratic exponential change on propagation according to different laws.

In the following we shall see how the expression of the output field can be evaluated for the propagation through some simple optical systems. For simplicity, the plane where the centers of symmetry of the component Gaussian beams lie will be chosen as the input plane. Consequently, the radius of curvature q_0 is purely imaginary, i.e.

$$q_0 = -\mathrm{i}L, \tag{24}$$

where

$$L = \frac{kw_0^2}{2} \tag{25}$$

is the Rayleigh distance.

As a first example, we consider free propagation along a distance z, which is represented by the matrix [29]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix}. \tag{26}$$

Taking into account that $\ell = z$ and using Eqs. (18), (21) and (23) we obtain

$$q_1 = q_0 + z, \quad a_1 = a_0 + \epsilon_0 z, \quad \epsilon_1 = \epsilon_0, \quad s = \frac{\epsilon_0^2}{2} z. \tag{27}$$

According to Eq. (22), the total propagated field is then given by

$$\begin{aligned} U_1^{(n)}(r, \vartheta) &= \frac{F}{1 + \mathrm{i}z/L} \exp\left[\mathrm{i}kz\left(1 - \epsilon_0^2/2\right)\right] \exp(\mathrm{i}n\vartheta) \\ &\times J_n\left[\left(\epsilon_0 - \frac{a_0 + \epsilon_0 z}{z - \mathrm{i}L}\right) kr\right] \\ &\times \exp\left(\mathrm{i}\frac{k}{2(z - \mathrm{i}L)}\left[r^2 + (a_0 + \epsilon_0 z)^2\right]\right). \end{aligned} \tag{28}$$

This equation agrees with Eq. (20) of Ref. [23], where a superposition of generalized BGBs with different indexes was studied in free propagation. In that paper the quantity $F(z)$ was used, which turns out to be, with the present notations,

$$F(z) = \frac{1}{w^2(z)} - \mathrm{i}\frac{k}{2R(z)} = -\mathrm{i}\frac{k}{2(z - \mathrm{i}L)}. \tag{29}$$

In the particular case of $a_0 = 0$ and $n = 0$, Eq. (28) coincides with Eq. (2.7) of Ref. [21], concerning the free paraxial propagation of an ordinary BGB of zero order.

Using the model adopted in this paper it appears evident that the field produced by propagating an ordinary BGB through a distance z can take an annular shape for large values of z, since the geometrical rays associated with the propagation axes of the component Gaussian beams move away from the z-axis with increasing z.

The second example refers to a Fourier transformer, realized through propagation from the first to the second focal plane of a converging lens. The corresponding matrix is [29]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 0 & f \\ -1/f & 0 \end{pmatrix}, \tag{30}$$

where f is the focal length of the lens. Letting $\ell = 2f$ and proceeding as above we obtain

$$q_1 = -\frac{f^2}{q_0}, \quad a_1 = f\epsilon_0, \quad \epsilon_1 = -\frac{a_0}{f}, \quad s = -a_0\epsilon_0. \tag{31}$$

Hence Eq. (22) gives

$$U_1^{(n)}(r, \vartheta) = \frac{F}{\mathrm{i}f/L}\exp[\mathrm{i}k(2f + a_0\epsilon_0)] \exp(\mathrm{i}n\vartheta) \times J_n\left[-\left(\frac{2a_0}{w_0^2} + \mathrm{i}k\epsilon_0\right)\frac{Lr}{f}\right] \times \exp\left[\left(-\frac{kL}{2f^2}\left[r^2 + (f\epsilon_0)^2\right]\right]\right). \tag{32}$$

By comparing Eq. (32) with Eq. (3), it can be noted that the roles of $\boldsymbol{a}$ and $\boldsymbol{\epsilon}$ are somehow interchanged. This means that, if an ordinary BGB is the input beam, a modified BGB will be obtained at the output, and vice versa. This can be easily understood if one considers that, in the case of an ordinary BGB, the rays associated to the propagation directions of the component Gaussian beams lie on the surface of a cone, whose apex is in the focus of the lens. Therefore, beyond the lens they will be lying horizontally on the surface of a cylinder. At a distance f from the lens, where all Gaussian beams have their new waist planes, the typical structure of a modified BGB is then found. Of course, the same scheme can be used to understand the transformation from modified to ordinary BGBs.

Finally, we consider the propagation of a BGB in a gradient refractive-index (GRIN) fiber [32]. Such elements are characterized by a radially dependent refractive index of the type

$$n(r) = n_0\left(1 - \tfrac{1}{2}\eta^2 r^2\right), \tag{33}$$

with n_0 and η real parameters. The matrix describing the paraxial propagation through a distance z in such a fiber is [32]

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \chi & \dfrac{\sigma}{\alpha} \\ -\alpha\sigma & \chi \end{pmatrix}, \tag{34}$$

where

$$\chi = \cos(\eta z), \quad \sigma = \sin(\eta z), \quad \alpha = n_0\eta. \tag{35}$$

As a consequence, the parameters of the output field are

$$\ell = n_0 z, \quad q_1 = \frac{1}{\alpha}\frac{\sigma - \mathrm{i}\alpha L\chi}{\chi + \mathrm{i}\alpha L\sigma},$$
$$a_1 = \chi a_0 + \frac{\sigma}{\alpha}\epsilon_0, \quad \epsilon_1 = \chi\epsilon_0 - \alpha\sigma a_0 \tag{36}$$

and

$$s = \frac{\chi\sigma}{2}\left(\frac{\varepsilon_0^2}{\alpha} - \alpha a_0^2\right) - \sigma^2 a_0\epsilon_0. \tag{37}$$

Eq. (36), in particular, states that the geometrical ray associated to the pair (a_1, ε_1) follows a sinuisodal path with period $2\pi/\eta$. More precisely,

$$a_1 = a_M \sin(\eta z + \psi), \quad \epsilon_1 = \alpha a_M \cos(\eta z + \psi), \tag{38}$$

with

$$a_M = \sqrt{a_0^2 + \left(\frac{\epsilon_0}{\alpha}\right)^2}, \quad \psi = \operatorname{atan}\left(\frac{\alpha a_0}{\epsilon_0}\right). \tag{39}$$

When the condition

$$L = \frac{1}{\alpha} \tag{40}$$

is satisfied, the expression of the propagated field takes a particularly simple form. In this case we see from Eq. (36) that $q_1 = q_0$ for each value of z and, disregarding overall constant factors, the final field can be written as

$$U_1^{(n)}(r, \vartheta) \propto \exp(\mathrm{i}n\vartheta) J_n\left(\frac{a_M kr}{L}\exp[-\mathrm{i}(\eta z + \psi)]\right) \times \exp\left(-\frac{k}{2L}\left[r^2 + a_M^2 \sin^2(\eta z + \psi)\right]\right). \tag{41}$$

In particular, the output field will show the typical profile of an ordinary BGB when

$$\eta z+\psi=m\pi, \qquad (m=0,\ \pm 1,\ \pm 2,\cdots) \tag{42}$$

and that of a modified BGB when

$$\eta z+\psi=\left(m+\tfrac{1}{2}\right)\pi, \qquad (m=0,\ \pm 1,\ \pm 2,\cdots) \tag{43}$$

These conditions correspond to values of z where the rays associated to the propagation axes of the component Gaussian beams intercept, and where they are as far as possible from the optical axis, respectively. Moreover, the requirement (40) assures that the radius of curvature of the Gaussian beams is invariant through propagation.

From the previous examples it appears that the form of an FGB propagating through an optical system can be easily predicted if one knows how the same system acts on geometrical rays and on centered Gaussian beams.

5. Conclusions

We have given a simple procedure for evaluating the expression of a generalized Bessel–Gauss beam propagating in a paraxial optical system, characterized by an *ABCD* matrix. We showed that the mathematical form of such beams remains unchanged upon propagation, and that its characterizing parameters can be related to those of a geometrical meridian ray and of a centered Gaussian beam, propagating in the same system. In this way, the evaluation of the output field turns out to be particularly simple and can acquire an intuitive meaning, as it has been shown in some examples.

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