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Imaging of generalized Bessel-Gauss beams

C. PALMA

Physics Department, Third University of Rome, Via della Vasca Navale 84, 00146 Rome, Italy

G. CINCOTTI, G. GUATTARI

Electronics Department, Third University of Rome, Via della Vasca Navale 84, 00146 Rome, Italy

and M. SANTARSIERO

Physics Department, University of Rome 'La Sapienza', Piazzale Aldo Moro 2, 00185 Rome, Italy

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Abstract. Different kinds of Bessel–Gauss beam have been recently introduced. By considering the effect of a lens on the field, we analyse how these different sets of fields transform into one another and illustrate how a superposition model of Gaussian beams allows this transformation to be clearly interpreted.

1. Introduction

Light beams, or more generally wave beams, are physical entities often met in practical experience and represented by physicomathematical models. Nevertheless, only a few of them have analytically simple behaviour in propagation and have been the subject of theoretical analysis: the Gaussian beams [1, 2] and relatively few other beams, both coherent [3–6] and partially coherent [7–9]. We recall, among them, the non-diffracting beams [10–13], which keep the same intensity distribution while propagating. Unfortunately, these fields are not physically realizable, because they carry an infinite amount of power through any section normal to the propagation direction. Moreover, any experimental realization necessarily requires finite aperture elements limiting the propagation length over which the beam is non-diffracting [14–17]. All these reasons prompted the introduction of related finite-energy beams, such as Bessel–Gauss beams (BGBs) [18].

In recent work [19], some extensions of the fundamental BGB, that may find practical applications, have been introduced and studied in free propagation. We briefly describe their essential features in the next section. We stress that unconventional coherent beams, besides the interest occurring in their specific name and usefulness, may be important as eigenmodes of cross-spectral density functions [20]. When this happens, they assume the quality of coherent members of an entire class of beams described by that cross-spectral density and may enlighten us concerning the overall behaviour of the class. This is, for instance, the case with the celebrated Gaussian beams, which are the coherent members of the class of Gauss–Schell model beams [21, 22]. In particular, this is also the case with the ordinary BGBs (whose definition is recalled in the next section) which are the coherent members or modes of the class of beams originating from J_0-correlated sources [23]. This feature adds further relevance to these coherent beams.

The aim of this work is to show that the generalizations recently introduced have a close interconnection that clearly appears when studying the classical effect on the field of a lens met on the propagation axis.

2. Extensions of Bessel-Gauss beams

We briefly quote and describe the expressions of the fundamental BGB and of the above-mentioned extensions [19] and we add some final unifying considerations.

(1) A BGB of order n is represented by the following distribution in cylindrical coordinates ρ, θ and z on the transverse plane $z = 0$:

$$V_n(\rho, \theta, 0) = A\mathrm{i}^n \exp\left(-\frac{\rho^2}{w_0^2}\right) J_n(\beta\rho) \exp(\mathrm{i}n\theta), \tag{1}$$

where A is a (possibly) complex amplitude factor, w_0 and β are real quantities and J_n is the Bessel function of the first kind of integer order n. From now on, we label these beams as ordinary BGBs. We recall that for the case $n = 0$ the ordinary BGB has been interpreted as a superposition of equiamplitude tilted Gaussian beams [18, 19], whose axes lie on the surface of a cone around the propagation axis having a (small) semiaperture or tilting angle

$$\alpha = \sin^{-1}\left(\frac{\beta}{k}\right), \tag{2}$$

where $k = 2\pi/\lambda$ is the wavenumber. Higher-order beams can be obtained by suitably dephasing the component beams.

It should be noted that tilted Gaussian beams may be related to the recently introduced decentred Gaussian beams [24].

(2) A modified BGB (see also [25]) has the following field distribution on the plane $z = 0$:

$$V_n(\rho, \theta, 0) = A \exp\left(-\frac{\rho^2 + a^2}{w_0^2}\right) I_n\left(\frac{2a\rho}{w_0^2}\right) \exp(\mathrm{i}n\theta), \tag{3}$$

where a is a positive constant and I_n is the modified Bessel function of the first kind of order n. These beams may be obtained, proceeding as before, by superimposing Gaussian beams whose axes lie on the surface of a cylinder of radius a [19].

(3) Finally, a generalized BGB has the following field distribution on the plane $z = 0$:

$$V_n(\rho, \theta, 0) = A \exp\left(-\frac{\rho^2 + a^2}{w_0^2}\right) I_n\left[\left(\frac{2a}{w_0^2} + \mathrm{i}\beta\right)\rho\right] \exp(\mathrm{i}n\theta). \tag{4}$$

The generalized BGB may be considered as the superposition of Gaussian beams whose axes, starting from the waists, generate the surface of a frustum of a cone. The parameter a is now the radius of the base of the frustum and β is related to the semiaperture of the cone as in equation (2)

[19]. Note that, since the argument of I_n in equation (4) is complex, it is immaterial whether we use ordinary or modified Bessel functions, because [26]

$$I_n(u) = (-\mathrm{i})^n J_n(\mathrm{i}u), \qquad J_n(-u) = (-1)^n J_n(u), \tag{5}$$

and similar relations hold, inverting the role of the functions J_n and I_n. Moreover, for the same reason, the field in equation (4) has a phase distribution depending on ρ; therefore the plane $z = 0$ is not necessarily a waist plane, the wave front there being not planar.

A first glimpse into the intimate relationship among all these beams comes from the following remark. An ordinary BGB, as observed before, is given by the superposition of Gaussian beams lying on a cone whose apex is in their waist plane. Thus, after propagation to a plane z = constant, the same component beams lie on the frustum of a cone having its base on that plane, where (different from the generalized BGB) they are each endowed with a parabolic curvature. In fact, the ordinary BGB propagated at a distance z is, in the paraxial approximation [18, 19],

$$\begin{aligned} V_n(\rho, \theta, z) &= \frac{Aq(0)}{q(z)}\,\mathrm{i}^n \exp\left[\mathrm{i}\left(k - \frac{\beta^2}{2k}\right)z\right] \exp\left(\frac{\mathrm{i}k(\rho^2 + \beta^2 z^2/k^2)}{2q(z)}\right) \\ &\quad \times J_n\left(\frac{-\mathrm{i}\beta L}{q(z)}\,\rho\right) \exp(\mathrm{i}n\theta) \\ &= \frac{Aq(0)}{q(z)} \exp\left[\mathrm{i}\left(k - \frac{\beta^2}{2k}\right)z\right] \exp\left(\frac{\mathrm{i}k(\rho^2 + \beta^2 z^2/k^2)}{2q(z)}\right) I_n\left(\frac{\beta L}{q(z)}\,\rho\right) \exp(\mathrm{i}n\theta), \end{aligned} \tag{6}$$

where the parameters L and $q(z)$, well known from Gaussian beam theory [1], have the following expressions:

$$L = \frac{kw_0^2}{2}, \qquad q(z) = z - \mathrm{i}L. \tag{7}$$

Equation (6) has a meaningful resemblance with equation (4), that represents a generalized BGB on the plane $z = 0$; the argument of the Bessel function in both equations is complex, with non-vanishing real and imaginary parts; the exponentials in ρ^2 have the same real part but in equation (6) there is also an additional imaginary exponent accounting for a different phase distribution. In fact, the generalized BGB can be obtained from the ordinary BGB by shifting the waist plane of the component Gaussian beams along their propagation directions, from the plane $z = 0$, where their waists overlap, to a suitable plane $z \neq 0$, where their waists are centred on a circumference.

The essential difference among the mathematical description of the three kinds of BGB is in the argument of the Bessel function, which is real, imaginary or complex respectively. A compact notation for BGB on the plane $z = 0$, formally identical with the ordinary beam of equation (1) but comprising all kinds of beams, is

$$V_n(\rho, \theta, 0) = B\mathrm{i}^n \exp\left(-\frac{\rho^2}{w_0^2}\right) J_n(\chi\rho) \exp(\mathrm{i}n\theta), \tag{8}$$

where

$$B = A \exp\left(-\frac{a^2}{w_0^2}\right), \qquad \chi = \beta - \mathrm{i}\frac{2a}{w_0^2}. \tag{9}$$

Using this unifying notation, it becomes more evident that the ordinary, modified and generalized BGBs constitute a family. The quality of its members depend on whether the parameter χ is real (ordinary BGB), imaginary (modified BGB) or fully complex (generalized BGB). Clearly, in the last case, the real part of χ accounts for the inclination of the constituting Gaussian beams and the imaginary part for the 'separation' of their waists (more precisely, for the radius of the circumference on which they lie).

3. Imaging of generalized Bessel-Gauss beams

To set up our analysis, we refer to the geometry in the figure. The plane $z = 0$ is the first plane of the figure. A thin lens of focal length f lies on the second plane set at a distance z_1. Finally, the third plane, at a distance z_2 from the lens, is the exit plane where the propagated field will be calculated.

We denote by the subscripts 0, 1, 2 the fields $U_0(\rho, \theta)$, $U_1(\sigma, \psi)$ and $U_2(r, \varphi)$ respectively on the three planes in the figure, successively reached in propagation from the left.

Under the paraxial approximation, the field $U_1(\sigma, \psi)$ impinging on the lens is given by a Fresnel diffraction integral acting on the field $U_0(\rho, \theta)$, on the waist plane. As the dependence on the anomaly θ is only in the linear phase term $\exp(\mathrm{i}n\theta)$ the diffraction integral is simplified [27]:

$$U_1(\sigma, \psi) = (-\mathrm{i})^{n+1} \exp(\mathrm{i}n\psi) \frac{k \exp(\mathrm{i}kz_1)}{z_1} \exp\left(\mathrm{i}\frac{k\sigma^2}{2z_1}\right) \int_0^\infty U_0(\rho, \theta) \exp(-\mathrm{i}n\theta) \times \exp\left(\mathrm{i}\frac{k\rho^2}{2z_1}\right) J_n\left(\frac{k\sigma\rho}{z_1}\right) \rho \, \mathrm{d}\rho. \tag{10}$$

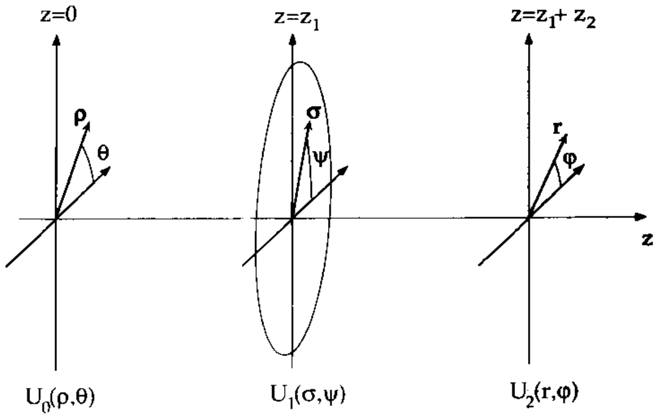

Figure Geometry and notations for the propagation problem.

The field $U_1(\sigma, \psi)$ impinging on the lens, multiplied by the lens transmission function [28], gives the field $U'_1(\sigma, \psi)$ leaving the lens:

$$U'_1(\sigma, \psi) = U_1(\sigma, \psi) \exp\left(-\mathrm{i}\frac{k\sigma^2}{2f}\right). \tag{11}$$

Finally, the field on the exit plane is given by another Fresnel diffraction integral

$$U_2(r, \varphi) = (-\mathrm{i})^{n+1} \exp(\mathrm{i}n\varphi) \frac{k \exp(\mathrm{i}kz_2)}{z_2} \exp\left(\mathrm{i}\frac{kr^2}{2z_2}\right) \int_0^\infty U'_1(\sigma, \psi) \exp(-\mathrm{i}n\psi) \times \exp\left(\mathrm{i}\frac{k\sigma^2}{2z_2}\right) J_n\left(\frac{kr\sigma}{z_2}\right) \sigma\,\mathrm{d}\sigma. \tag{12}$$

We shall use the equation (8) for the field $U_0(\rho, \theta)$. Performing the integrals in equations (10) and (12) we obtain the field at a distance z_2 beyond the lens originated by a BGB with the waist at a distance z_1 before the lens. The corresponding computation, which is quite long, is reported in the appendix. The result is

$$U_2(r, \varphi) = -B(-\mathrm{i})^n \exp(\mathrm{i}n\varphi) \frac{\chi'}{\chi} \exp[\mathrm{i}k(z_1 + z_2)] \times \exp\left[\frac{\mathrm{i}L}{w_0^2 z_2}\left(1 + \mathrm{i}\frac{\chi'}{\chi}\frac{(z_1 - \mathrm{i}L)}{L}\right) r^2\right] \exp\left(\mathrm{i}w_0^2 \chi\chi' \frac{z_2(1 + \epsilon z_1)}{4L}\right) J_n[\chi' r], \tag{13}$$

where

$$\chi' = \frac{\mathrm{i}\chi L}{z_z[1 + \epsilon(z_1 - \mathrm{i}L)]}, \qquad \epsilon = \frac{1}{z_2} - \frac{1}{f}. \tag{14}$$

Although the general result looks cumbersome to dwell with, it may be seen that, at a generic plane $z = \text{constant}$, the field has the mathematical structure of a generalized BGB whose beam parameters depend on z_1, z_2 and f. The field structure becomes simpler in the following illustrative cases.

(*a*) *Propagation from the first focal plane to the second plane.* In this case, it is

$$z_1 = z_2 = f, \qquad \epsilon = 0, \qquad \chi' = \frac{\mathrm{i}\chi L}{f}, \tag{15}$$

and a Fourier transform relation links the fields on the two planes [28]. Equation (13) becomes

$$V_2(r, \varphi) = B(-\mathrm{i})^{n+1} \exp(\mathrm{i}n\varphi) \frac{L}{f} \exp(2\mathrm{i}kf) \exp\left(-\frac{L^2}{w_0^2 f^2} r^2\right) \times \exp\left(-\frac{w_0^2\chi^2}{4}\right) J_n\left(\frac{\mathrm{i}\chi L}{f} r\right). \tag{16}$$

This is clearly a generalized BGB in which the roles of the real and imaginary parts of χ are inverted. It means, for instance, that an ordinary BGB distribution at the waist on the first focal plane becomes a modified BGB at the waist in the second focal plane and vice versa. In this particular case, it is $\chi = \beta$ and the parameters $\bar{w}_0$ and $\bar{a}$ of the modified BGB (see equation (3)), are easily seen to be

$$\bar{w}_0 = w_0 \frac{f}{L}, \qquad \bar{a} = w_0^2 \frac{\beta f}{2L} = \bar{w}_0^2 \frac{\beta L}{2f}. \tag{17}$$

In other words, the transformation may be interpreted affirming that all the component Gaussian beams, lying on a cone before the lens, lie on a cylinder of radius $\bar{a}$ after the lens and have spot size $\bar{w}_0$ on their waists. Clearly, owing to the symmetry of the Fourier transform (or to the invertibility of the path direction), the reverse is also true; a modified BGB with the waist lying on the first focal plane is transformed by the lens in an ordinary BGB with the waist on the second focal plane. We could also have obtained this result using the properties of the Fourier transform [29], for the transform of a product of two functions (a Gaussian function multiplied by a Bessel function) is the convolution of the two transforms (the convolution of a Gaussian function with an annulus).

(*b*) *Case* $z_1 = 0$. Performing the limit of the function in equation (13) for $z_1 \to 0$, we obtain

$$U_2(r, \varphi) = B(-\mathrm{i})^{n+1} \exp(\mathrm{i}n\varphi) \frac{L \exp(\mathrm{i}kz_2) \exp\left[\dfrac{\mathrm{i}L}{w_0^2 z_2}\left(1 + \dfrac{\mathrm{i}L}{z_2(1-\mathrm{i}L\epsilon)}\right) r^2\right]}{z_2(1-\mathrm{i}L\epsilon)}$$
$$\times \exp\left(-\frac{w_0^2\chi^2}{4(1-\mathrm{i}L\epsilon)}\right) J_n\left(\frac{\mathrm{i}\chi L r}{z_2(1-\mathrm{i}L\epsilon)}\right). \tag{18}$$

In particular, the field on the second focal plane ($z_2 = f$ and $\epsilon = 0$) is

$$U_2(r, \varphi) = B(-\mathrm{i})^{n+1} \exp(\mathrm{i}n\varphi) \frac{L \exp(\mathrm{i}kf) \exp\left[\dfrac{\mathrm{i}L}{w_0^2 f}\left(1 + \dfrac{\mathrm{i}L}{f}\right) r^2\right]}{f}$$
$$\times \exp\left(-\frac{w_0^2\chi^2}{4}\right) J_n\left(\frac{\mathrm{i}\chi L r}{f}\right). \tag{19}$$

This field is the same as in equation (16), except for a quadratic phase factor, that can be exactly cancelled putting a lens, identical with the first lens, on the exit plane. Here too, we may invert the path direction and the role of the two fields.

(*c*) *Perfect imaging.* When

$$\frac{1}{z_1} + \frac{1}{z_2} - \frac{1}{f} = 0, \qquad 1 + \epsilon z_1 = 0, \tag{20}$$

equation (13) becomes

$$U_2(r, \varphi) = -B(-\mathrm{i})^n \exp(\mathrm{i}n\varphi) \frac{z_1}{z_2} \exp[\mathrm{i}k(z_1 + z_2)]$$
$$\times \exp\left[\frac{\mathrm{i}L}{w_0^2 z_2}\left(1 + \mathrm{i}\frac{z_1}{z_2 L}(z_1 - \mathrm{i}L)\right) r^2\right] J_n\left(\frac{z_1 \chi r}{z_2}\right). \tag{21}$$

If we introduce, as usual in geometric optics, the magnification

$$M = -\frac{z_2}{z_1}, \tag{22}$$

equation (21) becomes

$$U_2(r,\ \varphi) = \frac{1}{M} \exp\left[\mathrm{i}k(z_1 + z_2)\right] \exp\left[\frac{\mathrm{i}k}{2z_2}\left(1 - \frac{1}{M}\right)r^2\right] U_0\left[\frac{r}{M},\ \varphi\right]. \tag{23}$$

In this case there is perfect imaging; the beam has a radial and angular distribution that is an exact replica of the distribution at the entrance, magnified by a factor M [28]. However, because of the quadratic phase factor, this distribution is on a spherical surface.

4. Conclusions

We have analysed the effect of a lens on the propagation of generalized BGB. This simple effect clearly shows that all kinds of BGB are interconnected to one another; starting from a BGB of a determined kind, a proper use of propagation and passage through a lens allows us to obtain BGBs of any other kind. Therefore these beams are not physically shape invariant and yet have the weaker property of transforming their physical shapes into one another. In fact these beams belong to a class that has been shown, from a very general point of view, to maintain the same abstract mathematical structure [30, 31] under paraxial transformations. This is confirmed in the practical expressions that we have obtained. Furthermore, as already mentioned in the introduction, the BGBs are the coherent members of the class of beams originated by J_0-correlated sources; hence, this feature helps also to clarify why partially coherent beams from those sources do not keep the same correlation property during propagation [23], which is different from the situation with Gauss–Schell model beams [7–9].

Appendix

Substituting equation (8) into equation (10) we obtain

$$U_1(\sigma,\ \psi) = -\mathrm{i}B \exp(\mathrm{i}n\psi)\, \frac{k \exp(\mathrm{i}kz_1)}{z_1} \exp\left(\mathrm{i}\,\frac{k\sigma^2}{2z_1}\right) \int_0^\infty J_n(\chi\rho) J_n\left(\frac{k\sigma\rho}{z_2}\right)$$
$$\times \exp\left[-\left(\frac{1}{w_0^2} - \frac{\mathrm{i}k}{2z_1}\right)\rho^2\right] \rho\, \mathrm{d}\rho. \tag{A 1}$$

This integral can be easily solved by using the integral formula [32]

$$\int_0^\infty J_n(2\gamma x^{1/2}) J_n(2\delta x^{1/2}) \exp(-\alpha x)\, \mathrm{d}x = \frac{1}{\alpha}\, I_n\left(\frac{2\gamma\delta}{\alpha}\right) \exp\left(-\frac{\gamma^2 + \delta^2}{\alpha}\right), \tag{A 2}$$

which holds for any choice of the parameters α, γ and δ. This yields

$$U_1(\sigma,\ \psi) = -\mathrm{i}B \exp(\mathrm{i}n\psi)\, \frac{kw_0^2}{2z_1(1 - \mathrm{i}L/z_1)} \exp(\mathrm{i}kz_1) \exp\left(-\frac{w_0^2\chi^2}{1 - \mathrm{i}L/z_1}\right)$$
$$\times I_n\left(\frac{\chi L\sigma}{z_1(1 - \mathrm{i}L/z_1)}\right) \exp\left[-\left(\frac{L^2}{z_1^2 w_0^2(1 - \mathrm{i}L/z_1)} - \mathrm{i}\,\frac{k}{2z_1}\right)\sigma^2\right]. \tag{A 3}$$

Inserting equation (A 3) into equation (12) and taking into account equation (11), we get

$$U_2(r, \varphi) = (-1)^{n+1} B \exp(\mathrm{i}n\varphi) \frac{k^2 w_0^2 \exp[\mathrm{i}k(z_1 + z_2)]}{2 z_1 z_2 (1 - \mathrm{i}L/z_1)} \exp\left(- \frac{w_0^2 \chi^2}{1 - \mathrm{i}L/z_1} \right)$$
$$\times \exp\left(\mathrm{i} \frac{kr^2}{2z_2} \right) \int_0^\infty J_n\left(\frac{\mathrm{i}\chi L \sigma}{z_1(1 - \mathrm{i}L/z_1)} \right)$$
$$\times \exp\left\{ - \left[\frac{L^2}{z_1^2 w_0^2 (1 - \mathrm{i}L/z_1)} - \mathrm{i} \frac{k}{2} \left(\frac{1}{z_1} + \frac{1}{z_2} - \frac{1}{f} \right) \right] \sigma^2 \right\} J_n\left(\frac{kr\sigma}{z_2} \right) \sigma \, \mathrm{d}\sigma, \tag{A 4}$$

and then, using equation (A 2) again, we obtain after simple algebra

$$U_2(r, \varphi) = -\mathrm{i}B \exp(\mathrm{i}n\varphi) \frac{L \exp[\mathrm{i}k(z_1 + z_2)]}{z_2[1 + \epsilon(z_1 - \mathrm{i}L)]} \exp\left(- \frac{w_0^2 \chi^2 (1 + \epsilon z_1)}{4[1 + \epsilon(z_1 - \mathrm{i}L)]} \right)$$
$$\times \exp\left[\mathrm{i} \frac{kr^2}{2z_2^2} \left(1 - \frac{z_1 - \mathrm{i}L}{z_2[1 + \epsilon(z_1 - \mathrm{i}L)]} \right) \right] I_n\left(\frac{\chi L r}{z_2[1 + \epsilon(z_1 - \mathrm{i}L)]} \right) \tag{A 5}$$

This expression is equivalent to equation (13) of section 2.

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References

[1] SIEGMAN, A. E., 1986, *Lasers* (Mill Valley, California: University Science Books).
[2] YARIV, A., 1971, *Introduction to Optical Electronics* (New York: Holt, Rinehart and Winston).
[3] GORI, F., 1994, *Optics Commun.*, **107,** 335.
[4] DE SILVESTRI, S., LAPORTA, P., MAGNI, V., and SVELTO, O., 1988, *IEEE J quant. Electron.*, **24,** 1172.
[5] BAGINI, V., GORI, F., SANTARSIERO, M., FREZZA, F., SCHETTINI, G., RICHETTA, M., and SCHIRRIPA SPAGNOLO, G., 1993, *Proceedings of the Workshop on Laser Beam Characterization*, (Madrid: Sociedad Española de Optica) edited by P. M. Mejìas, H. Weber, R. Martìnez-Herrero and R. Gonzàlez-Ureña Eds.) p. 31.
[6] SHEPPARD, C. J. R., and WILSON, T., 1978, *Microwaves, Optics Acoustics.*, **2,** 105.
[7] COLLETT, E., and WOLF, E., 1978, *Optics Lett.*, **2,** 27.
[8] TURUNEN, J., VASARA, A., and FRIBERG, A. T., 1991, *J. Opt. Soc. Am.* A, **8,** 282.
[9] SIMON, R., and MUKUNDA, N., 1993, *J. opt. Soc. Am.* A, **10,** 95.
[10] DURNIN, J., 1987, *J. opt. Soc. Am.* A, **4,** 651.
[11] DURNIN, J., MICELI, J. J., JR., and EBERLY, J. H., 1987, *Phys. Rev. Lett.*, **58,** 1499.
[12] MISHRA, S. R., 1991, *Optics Commun.*, **85,** 159.
[13] BLOISI, F., and VICARI, L., 1990, *Optics Commun.*, **75,** 353.
[14] INDEBETOUW, G., 1989, *J. opt. Soc. Am.* A, **6,** 150.
[15] VASARA, A., TURUNEN, J., and FRIBERG, A. T., 1989, *J. opt. Soc. Am.* A, **6,** 1748.
[16] VICARI, L., 1989, *Optics Commun.,* **70,** 273.
[17] HERMAN, R. M., and WIGGINS, T. A., 1991, *J. opt. Soc. Am.* A, **8,** 932.
[18] GORI, F., GUATTARI, G., and PADOVANI, C., 1987, *Optics Commun.*, **64,** 491.
[19] BAGINI, V., FREZZA, F., SANTARSIERO, M., SCHETTINI, G., and SCHIRRIPA SPAGNOLO, G., 1996, *J. mod. Optics*, **43,** 1155.
[20] WOLF, E., 1982, *J. opt. Soc. Am.*, **72,** 343.

[21] GORI, F., 1980, *Optics Commun.*, **34,** 301.
[22] STARIKOV, A., and WOLF, E., 1982, *J. opt. Soc. Am.*, **72,** 923.
[23] PALMA, C., BORGHI, R., and CINCOTTI, G., 1996, *Optics Commun.* (to be published).
[24] AL-RASHED, A.-A. R., and SALEH, B. E. A., 1995, *Appl. Optics,* **34,** 6819.
[25] RUSCHIN, S., 1994, *J. opt. Soc. Am.*, **11,** 3224.
[26] WATSON, G. N., 1958, *A Treatise on the Theory of the Bessel Functions*, second edition (Cambridge University Press).
[27] GORI, F., SANTARSIERO, M., and GUATTARI, G., 1993, *J. Opt. Soc. Am.* A, **10,** 673.
[28] GOODMAN, J. W., 1968, *Introduction to Fourier Optics* (New York: McGraw-Hill).
[29] BRACEWELL, R. N., 1986, *The Fourier Transform and its Applications*, second edition (New York: McGraw-Hill).
[30] WOLF, K. B., 1988, *Phys. Rev. Lett.*, **60,** 757.
[31] FRIBERG, A. T., VASARA, A., and TURUNEN, J., 1991, *Phys. Rev.* A, **43,** 7079.
[32] GRADSHTEYN, I. S., and RYZHIK, I. M., 1980, *Table of Integrals, Series and Products* (New York: Academic Press).