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Experimental determination of the size of a source from spectral measurements

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Abstract

We show how spectral changes measurements in a Young interferometer can be used to determine the dimension of an incoherent source. The experimental results confirm the theoretical predictions.

1. Introduction

The numerous studies of the spectral changes that take place in partially coherent wave fields on propagation because of the properties of spatial coherence on the source plane [1] demonstrate that many implications are connected with this effect. For example, the theoretical [2,3] and the experimental [4,5] results concerning a Young's interference experiment, which show that the state of coherence of the radiation incident on the apertures can modify the spectrum in the region of superposition, suggest a lot of applications of this simple interferometric scheme. In particular it is possible to determine the correlation function on the plane of the double aperture from measurements of power spectra instead of fringe visibility [6]. Once the spectral degree of coherence is known for a fixed separation of the apertures, the intensity distribution on the surface of a planar and quasi-homogeneous source (if the normalized spectrum is the same at every source point) can be recovered by means of a spatial inverse Fourier transform [7,8]; this technique is alternative to the other, based on the measurements of the degree of spectral coherence at a fixed frequency for different separations of the two antennas (which correspond to the Young's apertures), which is used in radioastronomy for stars [1]. Recently, it has also been shown that this experiment can be used to determine the separation of a pair of sources (in astronomy, this method can be applied to a pair of stars) [9,10].

In this paper, we shall consider a Young's interference experiment again; in agreement with the theoretical predictions of James and Wolf [2], we show through a simple experiment that the size of an incoherent source can be evaluated in a fairly accurate way starting from spectral measurements.

2. Theoretical analysis of the experiment

We refer to the scheme illustrated in Fig. 1. A spatially incoherent source lies in the plane $\xi\eta$. It consists of a slit, $2a$ wide, aligned to the η axis (orthogonal to the plane of Fig. 1) illuminated with uniform spatially incoherent light. In a plane xy, parallel to the plane $\xi\eta$, there is a thin lens, whose focal length is f, and, immediately afterwards, there is an opaque mask with two parallel slits aligned to the y axis. Let

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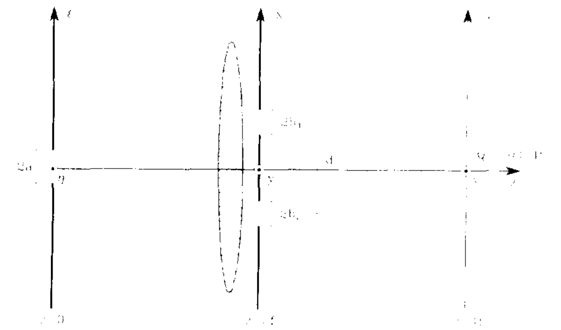

Fig. 1. Illustration of the scheme used.

$2b_1$ and $2b_2$ be the widths of the two slits and d the distance between their axes. The plane xy is at a distance $2f$ from the plane $\xi\eta$. Both the source and the Young slits are assumed so long that the dependence of the quantities of interest on the coordinates η and y can be neglected. The power spectrum of the radiation emerging from the Young slits is recorded in the plane uv, at a distance $2f$ from the plane xy.

Let $W_M(x_1,x_2;\nu)$ be the cross spectral density [1] at frequency ν of the field illuminating the mask. This function can be written in the form

$$W_M(x_1,x_2;\nu) = \sqrt{G_M(x_1;\nu)G_M(x_2;\nu)}\,\mu_M(x_1,x_2;\nu), \tag{1}$$

where G_M and μ_M denote the power spectrum and the spectral degree of coherence, respectively, on the plane $z = 2f$ (Mask plane). By making use of the van Cittert-Zernike theorem, we obtain for these quantities the following expressions

$$G_M(x;\nu) = G_M(\nu) = \frac{a}{f} s_0(\nu) I_0, \tag{2}$$

$$\mu_M(x_1,x_2;\nu) = \exp\left[\frac{ik}{4f}\left(x_1^2 - x_2^2\right)\right] \mathrm{sinc}\left[\frac{a\,(x_2 - x_1)\,\nu}{cf}\right], \tag{3}$$

where $\mathrm{sinc}(t) = \sin(\pi t)/(\pi t)$. In Eq. (3), $s_0(\nu)$ is the normalized spectrum [1] of the source, which we assume to be independent of the position and I_0 is the optical intensity at the source.

In order to evaluate the power spectra on the observation plane, at a distance u from the axis, when only a single slit is open (say $G_{S1}(u;\nu)$ and $G_{S2}(u;\nu)$) as well as the power spectrum produced by both slits ($G_B(u;\nu)$), propagation laws for the cross spectral density must be used, that is [11]

$$G_{Sj}(u;\nu) = \iint \tau_{Sj}^*(x_1)\tau_{Sj}(x_2) W_M(x_1,x_2;\nu) \times K^*(u,x_1;\nu)K(u,x_2;\nu)\,\mathrm{d}x_1\,\mathrm{d}x_2 \quad (j = 1,2), \tag{4}$$

$$G_B(u;\nu) = \iint \tau_B^*(x_1)\tau_B(x_2) W_M(x_1,x_2;\nu) \times K^*(u,x_1;\nu)K(u,x_2;\nu)\,\mathrm{d}x_1\,\mathrm{d}x_2, \tag{5}$$

where the asterisk denotes the complex conjugate. Here $\tau_{S1}(x)$, $\tau_{S2}(x)$ and $\tau_B(x)$ are the transmission functions of the mask, when a single slit (the $2b_1$ one or the $2b_2$ one wide, respectively) or both slits are open and are given by the expressions

$$\tau_{Sj}(x) = \mathrm{rect}\left[\frac{x + (-1)^j d/2}{2b_j}\right] \quad (j = 1,2), \tag{6}$$

$$\tau_B(x) = \mathrm{rect}\left(\frac{x - d/2}{2b_1}\right) + \mathrm{rect}\left(\frac{x + d/2}{2b_2}\right), \tag{7}$$

where, as usual, $\mathrm{rect}(t)$ equals one for $|t| \leq 1/2$ and equals zero otherwise. $K(u,x;\nu)$ is the propagation kernel and, when the paraxial approximation is used, takes the form

$$K(u,x;\nu) = \sqrt{\frac{-i}{\lambda z}} \exp(ikz) \exp\left[\frac{ik}{2z}(u - x)^2\right]. \tag{8}$$

By inserting Eqs. (1), (3), (6)-(8) into Eqs. (4) and (5), after some calculations, similar to those developed in Ref. [5], the following expressions are obtained for the power spectra G_{S1}, G_{S2} and G_B

$$G_{Sj}(u;\nu) = \frac{G_M(\nu)}{2a\pi}\left(\sum_{i=1}^{2} T(2b_j, h_i)\right), \quad (j = 1,2), \tag{9}$$

$$G_B(u;\nu) = \frac{G_M(\nu)}{2a\pi}\left(\sum_{i=1}^{2}[T(2b_1,h_i) + T(2b_2,h_i) + T(d + b_1 + b_2, h_i) + T(d - b_1 - b_2, h_i) - 2H(d, d - b_1 + b_2, h_i)]\right), \tag{10}$$

where

$$h_i = \frac{\pi\nu}{2cf}\left[a + (-1)^i u\right] \quad (i = 1,2), \tag{11}$$

and the functions T and H are defined as follows

$$T(\alpha, s) = \alpha \, \mathrm{Si}(2\alpha s) - \sin^2(\alpha s)/s, \tag{12}$$

$$H(\gamma, \alpha, s) = \gamma \, \mathrm{Si}(2\alpha s) - \sin^2(\alpha s)/s. \tag{13}$$

Here, Si stands for the sine integral function [12].

Making use of the values of the experimental spectra observed when one or both slits are open (from now denoted by G^E_{Sj} and G^E_B, respectively), the source dimension $2a$ can, in principle, be recovered from Eqs. (9) and (10). In particular, letting $u = 0$ (on axis spectrum) and choosing d such that G^E_B intersects the sum of G^E_{S1} and G^E_{S2} at least at one frequency (say ν_0) where the radiation power spectrum is appreciably different from zero, the problem is reduced to the determination of ν_0. In fact, once ν_0 is known, by solving the following equation (in this case $h_1 = h_2 = h$)

$$T(d + b_1 + b_2, h) + T(d - b_1 - b_2, h) - 2H(d, d - b_1 + b_2, h) = 0, \tag{14}$$

with respect to h, a is evaluated from Eq. (11). Incidentally, we note that if the Young slits were so narrow as to verify the conditions

$$2b_j \ll \frac{2cf}{a\nu}, \quad 2b_j \ll d \quad (j = 1,2), \tag{15}$$

for any frequency ν at which the radiation power spectrum is appreciably different from zero, Eqs. (9) and (10), could be replaced by the approximate formulas

$$G_{Sj}(u;\nu) = \frac{2\nu}{cf} G_M(\nu) \frac{\sin^2(\pi\nu u b j/cf)}{(\pi\nu u/cf)^2} \quad (j = 1,2), \tag{16}$$

$$\begin{aligned} G_B(u;\nu) &= G_{S1}(u;\nu) + G_{S2}(u;\nu) + \frac{2\nu}{cf} G_M(\nu) \frac{\mathrm{sinc}(a\nu d/cf)}{(\pi\nu u/cf)^2} \\ &\times \left\{ 2\cos\left(\frac{\pi\nu u d}{cf}\right) \sin\left(\frac{\pi\nu u b_1}{cf}\right) \sin\left(\frac{\pi\nu u b_2}{cf}\right) + \sin\left(\frac{\pi\nu u d}{cf}\right) \sin\left[\frac{\pi\nu u (b_1 - b_2)}{cf}\right] + \frac{\pi\nu u (b_2 - b_1)}{cf} \sin\left[\frac{\pi\nu u (d - b_1 + b_2)}{cf}\right] \right\}, \end{aligned} \tag{17}$$

and Eq. (14) reduces to

$$\mathrm{sinc}\left(\frac{a\nu d}{cf}\right) = 0. \tag{18}$$

3. Experimental results

The experimental set-up reproduces the scheme of Fig. 1. The slit at plane $z = 0$, which represents the source used in our experiment, is illuminated by means of a 360 W tungsten halogen lamp. A dc power supply (82 V) was used for operating the lamp. The power spectra were recorded using a spectrometer (produced by EG&G Princeton Applied Research) with an array detector of 1024 silicon diodes. A magnified version of the fringes formed at $z = 4f$ is produced by a 10× microscope objective on the entrance slit of the spectrometer. In this way the condition that the entrance aperture is much narrower than the mean period of the (polychromatic) Young fringes is satisfied. We measured the power spectra at the axial point Q ($u = 0$)(see Fig. 1). The experimental spectra obtained with only one of the Young slits open at a time, that is G^E_{S1} and G^E_{S2}, are shown in Fig. 2. The curve in Fig. 3 is the experimental power spectrum produced by both slits, G^E_B. The focal length was $f = 30$ cm, the dimensions of the two slits $2b_1 = 209 \pm 5$ μm and $2b_2 = 248 \pm 5$ μm and the distance between their axes $d = 2.86 \pm 0.08$ mm.

By comparing the power spectrum G^E_B with the sum of the spectra G^E_{S1} and G^E_{S2} (Fig. 4), it can be seen that they intersect in a point. So, as we observed at the end of last section, we can simply determine the unknown source width. To this end, we calculated the value of the wavelength (λ_0) of the intersection point by means of polynomial fits (order equal to 8) of the two experimental spectra and a statistical error was associated to it, dependent on the standard deviations of the fit coefficients [13].

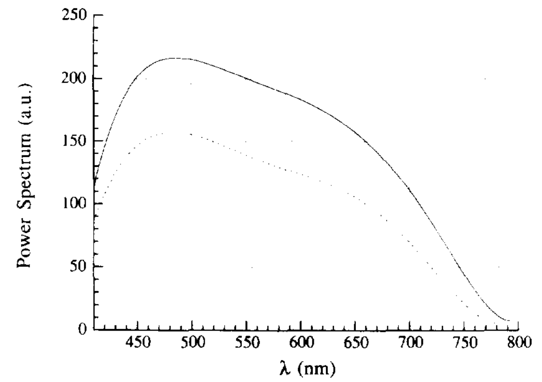

Fig. 2. Experimental power spectra, produced by a single Young slit (G^E_{S1} and G^E_{S2}). The dotted (full) line is the spectrum recorded with only the right (left) slit (looking towards the source) open.

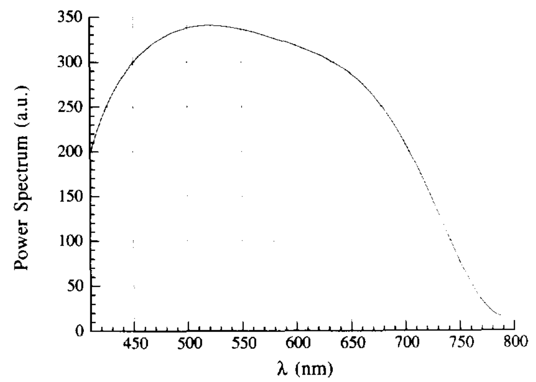

Fig. 3. Experimental power spectrum when both slits are open (G^E_B).

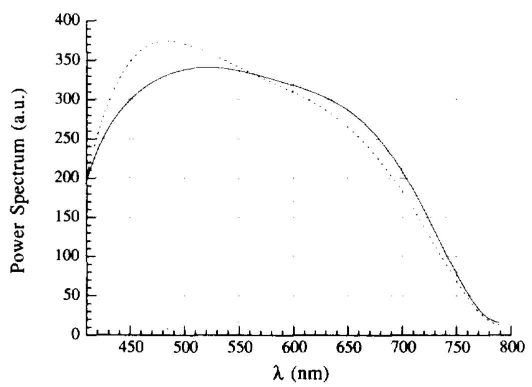

Fig. 4. The full line is the experimental power spectrum produced by both slits G^E_B; the dotted line is the sum of the experimental spectra with a single slit open, $G^E_{S1} + G^E_{S2}$.

We obtained $\lambda_0 = 562 \pm 9$ nm and $a = 59 \pm 1$ μm. In order to test the goodness of this result, we used it to make a comparison between experimental and theoretical spectra. We proceeded as follows. The experimental curves G^E_{S1} and G^E_{S2} were taken as estimates of $G_{S1}(0;\nu)$ and $G_{S2}(0;\nu)$, respectively. By inserting these two functions into Eq. (9) and using the experimental value of a, it was possible to obtain an estimate of $G_M(\nu)$ and, then, to calculate the expected spectrum $G_B(0;\nu)$, by means of Eq. (10). The agreement between experimental results and theoretical predictions is completely satisfactory within the experimental errors. On the other hand, the experimental value of a agrees also very well with the measurement (58 ± 1 μm), obtained by photographs of the slit taken with an electron microscope.

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