ELSEVIER

Fusion Engineering and Design 26 (1995) 361–364

Fusion Engineering and Design

Full-wave theory of a quasi-optical launching system for lower hybrid waves: the step density case

F. Frezza [a], F. Gori [b], M. Santarsiero [b], F. Santini [c], G. Schettini [a]

[a] *Dipartimento di Ingegneria Elettronica, Università "La Sapienza", Roma, Italy*
[b] *Dipartimento di Fisica, Università "La Sapienza", Roma, Italy*
[c] *Associazione EURATOM–ENEA sulla Fusione, CRE, Frascati, Italy*

Abstract

Numerical studies on the use of a quasi-optical grill to couple lower hybrid waves to a plasma, for current drive in tokamaks, are currently under way. Study of the coupling has been carried out in a rigorous way, through the solution of scattering from cylinders with parallel axes in the presence of a plane of discontinuity for electromagnetic constants. Here we restrict our attention to the case of a plasma with a step density profile and report some results on launched spectra, coupled power and directivity.

1. Introduction

When a beam of r.f. radiation is to be injected into a plasma for current drive purposes at the lower hybrid frequency [1], one is faced with the problem that a propagating plane wave is completely reflected by the plasma. The solution is to use a coupling mechanism via slow waves. This is generally accomplished by a sophisticated arrangement of waveguides [2]. An alternative approach that has been proposed not long ago [3] is to produce slow waves by scattering at a grating made of, for example, conducting cylinders.

A rigorous analysis of such a system is not easy because it entails the solution of a heavy scattering problem. The main difficulty arises from the presence of the plane reflecting surface. Indeed, solutions of the scattering problem from circular-section cylindrical structures in homogeneous media are available [4,5]. In the presence of plane interfaces, solutions that hold in the limit of wires [6] are known. It is not possible to apply this approximation to the present case, because the radii of the cylinders are comparable with the operating wavelength.

It is possible to obtain a rigorous solution of the problem for an arbitrary plasma density profile by using the plane wave expansion of cylindrical functions [7]. The general features of the method have been presented elsewhere [8].

In the next section the solution in the case of a constant density plasma is obtained; this approach leads to remarkable numerical simplifications with respect to the general formulation [8]. Numerical results will be given in Section 3, while the last section is dedicated to future developments and conclusions.

SSDI 0920-3796(94)00202-9

2. N cylinders in front of a plane plasma surface

Owing to the various geometrical features of the interacting waves and bodies, the imposition of the right boundary conditions is not a trivial task. To solve this problem it is customary to expand the diffracted field in terms of cylindrical functions, i.e. the product of a Hankel function H_n of integer order and a sinusoidal angular factor ($\exp(\mathrm{i}n\vartheta)$).

The problem consists in calculating the coefficients of the expansion of the field diffracted by every cylinder, in the presence of a plane surface with reflection coefficient $\Gamma(n_\|)$, in terms of the cylindrical functions. Such coefficients can be determined by imposing the right electromagnetic boundary conditions on the conducting surfaces; with this aim it is convenient to express the field in terms of cylindrical functions centred on the different cylinders.

Since the reflection properties of a plane of discontinuity for electromagnetic constants are generally known for incident plane waves [2,4], in order to obtain a rigorous solution it is essential to use the analytic plane wave expansion of the above-mentioned cylindrical functions [7,8]. However, dealing with a constant plasma density n_0, the reflection coefficient Γ is independent of $n_\|$ ($\Gamma = [(1-n_0/n_c)/(1+n_0/n_c)]^{1/2}$, with n_c the critical density) and it is possible to use a simplified formulation. In this case, it can be shown that the field is given by the diffraction of an incident wave on a structure formed by the real cylinders together with an arrangement of virtual cylinders specularly placed beyond the plasma surface, without taking into account the discontinuity surface (the image method).

In this paper we consider a plane wave with wavevector $\boldsymbol{k}^{\mathrm{i}}$ as the incident field. The linear polarization with the magnetic vector parallel to the axes of the cylinders has been chosen to launch a lower hybrid slow wave properly. The notation used throughout the paper is shown in Fig. 1.

The magnetic field H_{tot} can be expressed as the sum of the following fields: H_{i}, field of the incident plane wave; H_{r}, field due to the reflection of H_{i} from the plane surface; H_{d}, field diffracted by

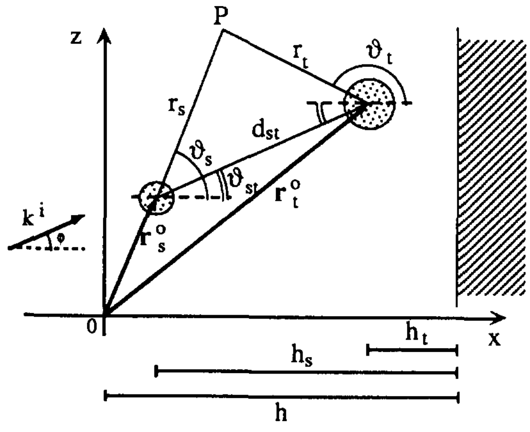

Fig. 1. Geometry and notation used in this paper.

the cylinders; H_{dr}, field due to the reflection of H_{d} from the plane surface. The H_{i} and H_{r} fields can be expanded in terms of Bessel functions J_n [4], while the diffracted field can be expressed as a sum of cylindrical functions with unknown coefficients c_{sn} ($s = 1, \ldots, N$ where N represents the total number of cylinders). Moreover, the use of the image method allows us to write

$$H_{\mathrm{dr}}(x_s, z_s) = \Gamma H_{\mathrm{d}}(2h_s - x_s, z_s) \tag{1}$$

By imposing that the tangential component of the electric field on the cylindrical surfaces disappears, we find the following linear system:

$$\begin{aligned}
&\mathrm{i}^m \exp(\mathrm{i}k^{\mathrm{i}}{}_\| z_s^0)\{\exp(\mathrm{i}k^{\mathrm{i}}{}_\perp x_s^0)\exp(-\mathrm{i}m\varphi) \\
&\quad + \Gamma \exp(-\mathrm{i}k^{\mathrm{i}}{}_\perp x_s^0)\exp[\mathrm{i}m(\varphi-\pi)]\} \\
&\quad + \sum_{t=1}^{N}\sum_{n}(\{H_{m-n}(kd_{st}{}^{(1)})\exp[-\mathrm{i}(m-n)\vartheta_{st}] \\
&\quad + H_{m+n}(kd_{st}{}^{(2)})\exp[-\mathrm{i}(m+n)\vartheta_{st}]\}(1-\delta_{s,t}) \\
&\quad + G_n(ka_s)\delta_{m,n}\delta_{s,t})\mathrm{i}^n\exp(-\mathrm{i}n\varphi)\, c_{tn} = 0
\end{aligned} \tag{2}$$

where a_s is the radius of the sth cylinder, $\delta_{i,j}$ is the Kronecker symbol, $d_{st}{}^{(1)}$ is the distance between the sth and tth real cylinder, $d_{st}{}^{(2)}$ is the distance between the sth real cylinder and the tth virtual cylinder,

$$\alpha = \sin^{-1}(n_\|) \qquad G_n(\xi) \equiv \frac{J_n'(\xi)}{H_n'(\xi)} \tag{3}$$

The solution of such a system leads to the evaluation of the unknown coefficients c_{sn}; therefore the total magnetic field H_{tot} is fully determined.

To calculate the power flux towards the plasma core, i.e.

$$\Phi_x = \frac{kZ_0}{4\pi}\left(\int_{|n_\parallel|<1} |\sigma_H|^2(1-|\Gamma|^2)n_\perp \, dn_\parallel - 2\int_{|n_\parallel|>1} |\sigma_H|^2 \, \mathrm{Im}(\Gamma)\, |n_\perp| \, dn_\parallel\right) \quad (4)$$

we need the Fourier spectrum σ_H of the launched waves. The analytical evaluation of σ_H can be performed using the plane wave expansion of the cylindrical functions [7]:

$$H_n(kr)\exp(in\vartheta)\big|_{x=h} = \int_{-\infty}^{\infty} F_n(n_\parallel, kh)\exp(ikn_\parallel z)\, dn_\parallel \quad (5)$$

In order to optimize the coupling efficiency, the power of the incident wave has to be defined. Since an ideal plane wave carries an infinite power, we consider only the power due to the portion of the plane wave corresponding to the width of the array of cylinders along the z axis. A more realistic modeling will be done by means of an incident Gaussian beam in a forthcoming paper.

3. Numerical results

The method outlined in the previous section has been applied to the evaluation of the diffracted field, the launched spectrum, and the coupled power in different experimental configurations.

As an example we consider the layout shown in Fig. 2, where an alignment of N identical cylinders in front of a plasma, having a density $n_0 = 2n_c$, is sketched.

The shape of the coupled power spectrum is reported in Fig. 3 for $N = 20$, $ka = 0.85$, $kd = 2.9$ and $\varphi = 45°$; as can be seen, only the -1 and $+1$ orders carry a significant amount of power.

In Figs. 4 and 5 are reported the power reflection coefficient and the selectivity vs. the total number (N) of cylinders for different values of the periodicity of the grill. The selectivity has been defined as the ratio of the power coupled in the -1 order to the total coupled power.

In Fig. 6 the reflected power and selectivity of a single-layer quasi-optical grill vs. the angle of

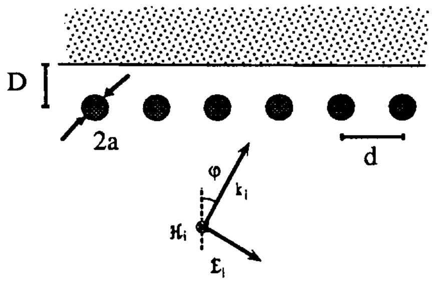

Fig. 2. A single-layer quasi-optical grill.

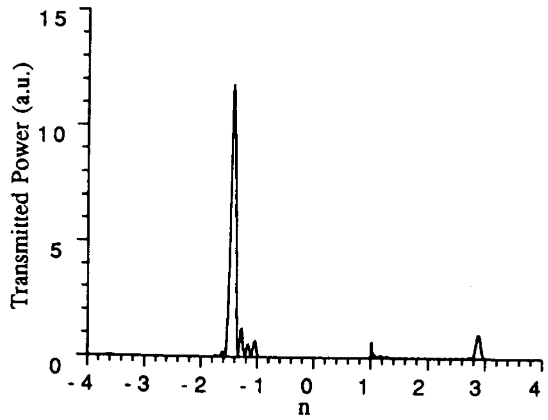

Fig. 3. Power spectrum ($N = 20$; $ka = 0.85$; $kd = 2.9$; $\varphi = 45°$; $n_0 = 2n_c$; $kD = 1.1$) (a.u., arbituary units).

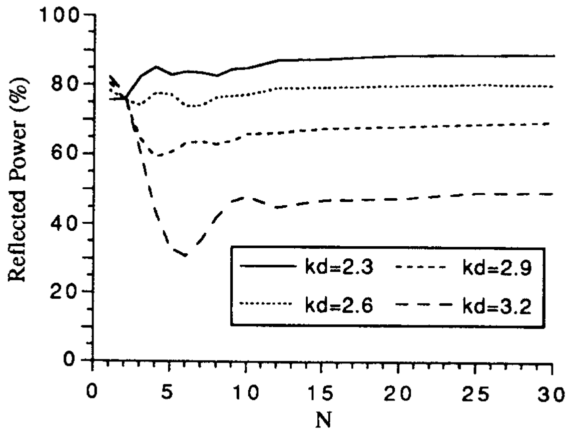

Fig. 4. Reflected power ($ka = 0.85$; $\varphi = 45°$; $n_0 = 2n_c$; $kD = 1.1$).

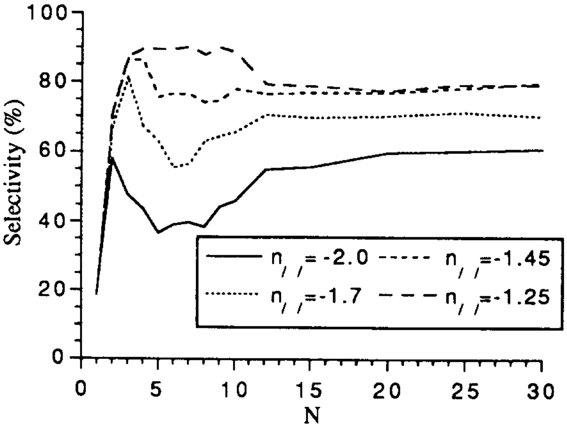

Fig. 5. Selectivity ($ka = 0.85$; $\varphi = 45°$; $n_0 = 2n_c$; $kD = 1.1$).

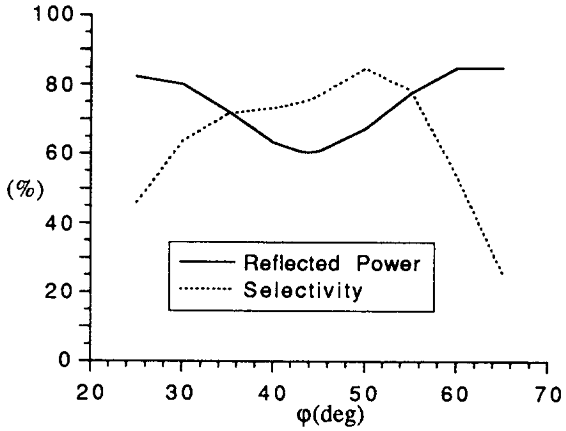

Fig. 6. Coupling parameters ($N = 5$; $ka = 0.85$; $kd = 2.9$; $n_0 = 2n_c$; $kD = 1.1$).

incidence are shown, while in Fig. 7 the same parameters are plotted vs. the normalized density n_0/n_c.

4. Conclusions

The numerical results of this work are relevant to the optimization of the single-layer quasi-optical grill; the use of rods shaped in a different way could give better coupling results [9]; the double-array configuration can be studied as a particular case of that outlined in Section 2.

The aim of our analysis is a more realistic modeling, which needs the use of incident fields

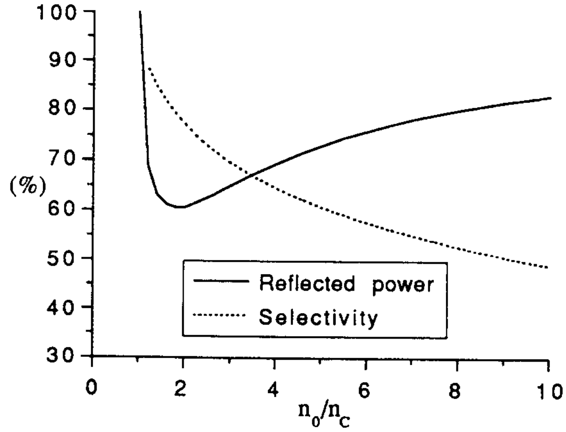

Fig. 7. Coupling parameters ($N = 5$; $ka = 0.85$; $kd = 2.9$; $\varphi = 45°$; $kD = 1.1$).

different from plane waves and the presence of metallic side walls. For instance, a Gaussian beam can be assumed as the incident wave; such a beam can actually be transmitted from an r.f. generator to the grill in the form, for example, of an HE_{11} mode of a corrugated waveguide. On the contrary, metallic walls can be simulated by means of a suitable set of wires [10], thus allowing us to use the method outlined in this paper.

References

[1] R.A. Cairns, Radiofrequency Heating of Plasmas, Adam Hilger, Bristol, 1991.
[2] M. Brambilla, Nucl. Fusion 16 (1976) 47.
[3] M.I. Petelin, E.V. Suvorov, Soc. Tech. Phys. Lett. 15 (1989) 882.
[4] C.A. Balanis, Advanced Engineering Electromagnetics, Wiley, New York, 1989.
[5] H.A. Ragheb and M. Hamid, Int. J. Electron. 59 (1985) 407.
[6] J.R. Wait, Can. J. Phys. 32 (1954) 571.
[7] G. Cincotti, F. Gori, M. Santarsiero, F. Frezza, F. Furnò and G. Schettini, Opt. Commun. 95 (1993) 192.
[8] G. Cincotti, F. Gori, M. Santarsiero, R. Serrecchia, F. Frezza, G. Schettini and F. Santini, Proc. 10th Top. Conf. on RF Power in Plasmas, Boston, MA, 1–3 April 1993, p. 285.
[9] E.V. Surorov and N.F. Kovalev, personal communication.
[10] J.H. Richmond, IEEE Trans. MTT-13 (1965) 408.