QUASI-OPTICAL LAUNCHERS FOR LOWER HYBRID WAVES: A FULL WAVE APPROACH

F. FREZZA, G. SCHETTINI
Dip. di Ingegneria Elettronica, Università 'La Sapienza',

F. GORI, M. SANTARSIERO
Dip. di Fisica, Università, 'La Sapienza'

F. SANTINI
Associazione Euratom–ENEA sulla Fusione, C.R.E. Frascati, Frascati

Rome, Italy

ABSTRACT. Theoretical investigations on the use of an advanced launcher to couple lower hybrid waves to a plasma, for current drive in tokamaks, have been developed. A study of the coupling has been carried out in a rigorous way, through the solkution of the scattering by an array of cylinders with parallel axes in the presence of a plane of discontinuity for the electromagnetic constants. The general features of the proposed method are presented together with results on the launched spectra and coupled power. The numerical results of this work are relevant to the optimization of the single layer qusi-optical grill launching RF power towards a constant density plasma: it is shown that the power coupled in the plasma can reach values of about 25–30% in the −1 diffracted order and about 40% in all other orders. As a consequence it is expected that the coupling values for a suitable double layer configuration should approach those for total transmission.

1. INTRODUCTION

When a beam of radiofrequency radiation is to be injected into a plasma for coupling to a slow lower hybrid wave [1], one is faced with the problem that a propagating plane wave is completely reflected by the plasma. The solution is to use a coupling mechanism via evanescent waves.

The most popular coupling device is represented by waveguide grills. Since their introduction as launchers in toroidal plasmas [2], in the middle of the seventies, the waveguide phased arrays have shown a number of useful features, among them the absence of any structure inside the toroidal chamber and the very high flexibility regarding both the launched spectrum and the directivity of the antenna. These properties allow a large number of experimental configurations, from pure heating to full plasma current drive, to be realized. On the other hand, the need for the use of single mode waveguides with reduced section, in conjunction with a very little periodicity to properly launch slow waves, involves high values of dissipation. Moreover, the success of ever increasing big size experiments has implied the need for a very large number of waveguides: the next step devices require launchers with thousands of elements. Nevertheless, antennas of this kind have been used in a large number of successful experiments [3].

About ten years later, a simplified version of phased arrays has been proposed [4] and successfully tested [5]: the multijunction. In these devices a fixed phase relation between adjacent waveguides has been introduced, allowing the feeding of the terminal part of the antenna from a single oversized waveguide. As a consequence the dissipation and the RF plant complexity were reduced giving more experimental reliability but decreasing the spectral flexibility. Further simplifications in these structures have recently been proposed, giving rise to the concept of hyperguide [6].

A very different conceptual approach is that of quasi-optical grills (QOG) [7], in which the excitation of the plasma lower hybrid wave is produced by means of the scattering of an RF beam by a grating made of conducting rods. If such devices could produce reasonable values of coupling power and directivity, they would certainly show a drastic reduction in dissipation and RF plant complexity.

In this paper we present the analysis of the diffraction of a propagating plane wave by N circular section rods arbitrarily placed close to a plasma. A rigorous analysis of such a system is not easy because it entails the solution of a complex scattering problem. The main difficulty arises from the presence of the plasma reflecting surface. Indeed, solutions of the scattering problem from cylindrical structures in homogeneous media are available [8]. In the presence of plane interfaces, solutions are known that hold in the limit of wires or perfectly reflecting surfaces [9]. It is not possible to apply these approximations to the present case, because the radii of the cylinders are comparable with the operating wavelength and the plasma surface is not perfectly reflecting.

The introduction of the plane wave expansion of cylindrical functions [10] has allowed the solution of the problem in a very general way, as we shall see in the next section. A significant reduction of the numerical complexity of the problem has been achieved in the case of a constant density plasma (Section 3). We remark that this case has been an important testing bench, also for the developing of waveguide grills [11]. Section 4 will be devoted to the evaluation of the coupled power, while in Section 5 numerical results will be presented. In the last section, conclusions and further developments will be discussed.

2. THE GENERAL SOLUTION

The problem under investigation is the scattering of an electromagnetic, linearly polarized plane wave, with wavevector $\boldsymbol{k}^i$, impinging on a group of perfectly conducting parallel cylinders, placed near a plane plasma surface, parallel to the cylinders (Fig. 1), characterized by the reflection coefficient $\Gamma(n_\parallel)$. Here and in the following, $n_\parallel$ stands for the ratio between the component of the wavevector parallel to the confinement toroidal magnetic field, $\boldsymbol{B}_t$, and the modulus k of the wavevector itself. The structure is assumed infinite in the y direction so the

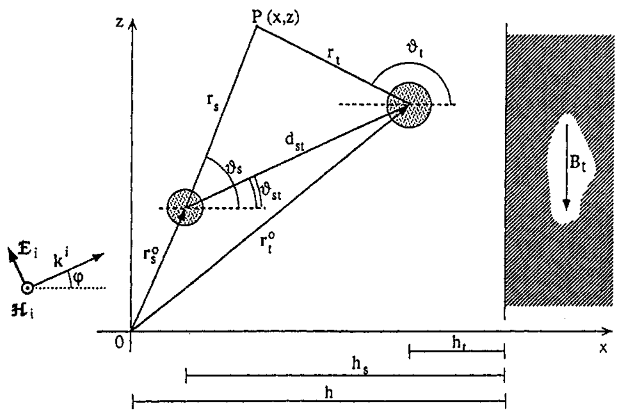

FIG. 1. N cylinders arbitrarily placed near a plasma surface.

problem can be considered a two dimensional one. The linear polarization with the magnetic vector parallel to the axes of the cylinders (H polarization) has been chosen to properly launch a lower hybrid slow wave [1].

To obtain the solution in a vacuum [8], it is customary to expand the diffracted field in terms of cylindrical functions, i.e. the product of a Hankel function of integer order H_n times a sinusoidal angular factor ($\exp\{in\vartheta\}$). The expansion coefficients can be determined by imposing the electromagnetic boundary conditions on the conducting cylinders; to this aim it is convenient to express the field in terms of cylindrical functions centred on the various cylinders.

In the presence of a plane interface, owing to the various geometrical features of the interacting waves and bodies, the imposition of the right boundary conditions is a quite difficult task. In particular, since the reflection properties of a plane of discontinuity for electromagnetic constants are known for incident plane waves [2, 12], in order to obtain the rigorous solution it is essential to use the analytic plane wave expansion of the above mentioned cylindrical functions [10].

With reference to Fig. 1, we denote by subscript t the co-ordinates of the reference frame centred on the axis of the tth cylinder ($t = 1, \ldots, N$), while the co-ordinates of the axis of the same cylinder in the principal frame (x, z) will be denoted by $\boldsymbol{r}_t^0$ (hence, ϑ_t^0, x_t^0, z_t^0, etc.). In order to impose the boundary conditions we express the magnetic field in terms of cylindrical waves in the reference frame of each of the cylinders.

It is convenient to express the magnetic field $\mathcal{H}_{tot}$ as the sum of the following fields:

- $\mathcal{H}_i$: field of the incident plane wave;
- $\mathcal{H}_r$: field due to the reflection of $\mathcal{H}_i$ from the plane surface;
- $\mathcal{H}_d$: field diffracted from the cylinders;
- $\mathcal{H}_{dr}$: field due to the reflection of $\mathcal{H}_d$ from the plane surface.

The fields $\mathcal{H}_d$ and $\mathcal{H}_{dr}$ will be given in terms of a superposition of assigned functions weighted with unknown coefficients; this allows us to take into account all the multiple reflections.

By using the expression of a plane wave in terms of the Bessel functions of the first kind, J_n [8], the incident field $\mathcal{H}_i$ can be written as

$$\mathcal{H}_i(x,z) = \mathcal{H}_0 e^{ik_\perp^i x + ik_\parallel^i z} = \mathcal{H}_0 e^{ik_\perp^i x_t^0 + ik_\parallel^i z_t^0} e^{ik_\perp^i x_t + ik_\parallel^i z_t}$$

$$= \mathcal{H}_0 e^{ik_\perp^i x_t^0 + ik_\parallel^i z_t^0} \sum_{m=-\infty}^{\infty} i^m J_m(kr_t) e^{im(\vartheta_t - \varphi)} \qquad (1)$$

where $\mathcal{H}_0$ is the amplitude of the plane wave, (x_t, z_t) are the co-ordinates of a generic point in the reference frame centred on the tth cylinder, and the symbols $\perp$ and $\parallel$ stand for the orthogonal and parallel parts of a vector, with respect to $\boldsymbol{B}_t$, respectively. With the same procedure, the $\mathcal{H}_r$ field takes the form

$$\mathcal{H}_r(x, z) = \Gamma(n_\parallel^i)\ \mathcal{H}_0\, e^{-ik_\perp^i(x_t^0 - 2h) + ik_\parallel^i z_t^0}$$

$$\times \sum_m i^m J_m(kr_t)\ e^{im(\vartheta_t - \varphi')} \tag{2}$$

where use has been made of the plasma reflection coefficient Γ (the plasma surface is at $x = h$), and of the propagation angle of the reflected plane wave, $\varphi' = \pi - \varphi$.

The field diffracted by the sth cylinder, under the action of the overall field, may be expressed as a sum of cylindrical functions, weighted by unknown coefficients c_{sm}. Therefore, we obtain the following expression for the diffracted field:

$$\mathcal{H}_d(x, z) = \mathcal{H}_0 \sum_{s=1}^{N} \sum_m i^m c_{sm} H_m(kr_s)\ e^{im(\vartheta_s - \varphi)} \tag{3}$$

where the $\mathcal{H}_0$ and i^m terms have been made evident.

By using Graf formula [13], the distance d_{st} between cylinders and the angle ϑ_{st} (Fig. 1), it is possible to express the H_m functions relevant to the sth cylinder with $s \neq t$ in terms of Bessel and Hankel functions relevant to the tth cylinder. Therefore, with some algebra we obtain:

$$\mathcal{H}_d(x, z) = \mathcal{H}_0 \sum_{\substack{s=1 \\ s\neq t}}^{N} \sum_{m,n} i^n c_{sn}\, e^{-in\varphi} H_{n-m}(kd_{st})$$

$$\times\ e^{i(n-m)\vartheta_{st}} J_m(kr_t)\, e^{im\vartheta_t}$$

$$+\ \mathcal{H}_0 \sum_m i^m c_{tm} H_m(kr_t)\, e^{im(\vartheta_t - \varphi)} \tag{4}$$

The expression for the $\mathcal{H}_{dr}$ field is slightly more complicated, because we have to treat plane and cylindrical geometries together. The key for solving the problem has been the introduction of the Fourier spectrum of cylindrical functions in an $x = h$ plane, with h different from zero. This spectrum is defined as

$$H_n(kr)\, e^{in\vartheta}\Big|_{x=h} = \int_{-\infty}^{\infty} F_n(n_\parallel, kh)\, e^{ikn_\parallel z} dn_\parallel \tag{5}$$

The explicit expression for the above functions is [10]:

$$F_n(n_\parallel, kh) = \begin{cases} (-1)^n \dfrac{(\sqrt{n_\parallel^2 - 1} - n_\parallel)^{-n}\, e^{-kh\sqrt{n_\parallel^2 - 1}}}{i\pi\sqrt{n_\parallel^2 - 1}} & -\infty < n_\parallel < -1 \\[2ex] \dfrac{e^{ikh\sqrt{1 - n_\parallel^2} - in\cos^{-1} n_\parallel}}{\pi\sqrt{1 - n_\parallel^2}} & -1 < n_\parallel < 1 \\[2ex] \dfrac{(\sqrt{n_\parallel^2 - 1} + n_\parallel)^n\, e^{-kh\sqrt{n_\parallel^2 - 1}}}{i\pi\sqrt{n_\parallel^2 - 1}} & 1 < n_\parallel < \infty \end{cases} \tag{6}$$

By using Eqs (5) and (6) it is possible to express the diffracted field as a continuous superposition of plane waves. In the reflection from the plasma surface each of these waves is multiplied by a value of the Γ factor selected by a fixed value of $n_\parallel$. The reflected plane waves, in turn, can be given in terms of J_n Bessel functions centred on the axes of the cylinders, as we have seen in Eq. (1). After some algebra we obtain the following expression:

$$\mathcal{H}_{dr}(x, z) = \mathcal{H}_0 \sum_{s=1}^{N} \sum_{m,n} i^{m+n} c_{sm}\, e^{-im\varphi} J_n(kr_t)\, e^{in\vartheta_t}$$

$$\times \int \Gamma\left(\frac{k_\parallel}{k}\right) F_m\left(\frac{k_\parallel}{k}, kh_s\right) e^{ik_\parallel(z_t^0 - z_s^0)}\, e^{ik_\perp h_t}\, e^{-in\alpha'}\, \frac{dk_\parallel}{k} \tag{7}$$

where α' refers to the direction of propagation of the generic reflected plane wave ($\alpha' = \pi - \sin^{-1}(k_\parallel/k)$).

By imposing the vanishing of the tangential component of the total electric field on the cylindrical surfaces [12],

$$\mathcal{E}_{tot}(a_t) = 0, \quad \forall\vartheta_t, \quad (t = 1, \ldots, N)$$

where a_t is the radius of the tth cylinder; taking advantage of the orthogonality properties of the Bessel functions, we obtain the following system:

$$\sum_{s=1}^{N} \sum_n \{[H_{n-m}(kd_{st})\, e^{i(n-m)\vartheta_{st}}(1 - \delta_{s,t})$$

$$+\ i^m A_{nm}^{st}]\, G_m(ka_t) + \delta_{s,t}\delta_{m,n}\}\, i^n e^{-in\varphi} c_{sn}$$

$$= -i^m e^{ik_\parallel^i z_t^0}\, [e^{ik_\perp^i x_t^0}\, e^{-im\varphi}$$

$$+\ \Gamma(n_\parallel^i)\, e^{ik_\perp^i(2h - x_t^0)}\, e^{im(\varphi - \pi)}]\, G_m(ka_t)$$

$$m = 0, \pm 1, \pm 2, \ldots; \quad t = 1, \ldots, N \tag{8}$$

where $\delta_{i,j}$ is the Kronecker symbol,

$$A_{nm}^{st} = \int \Gamma(n_\parallel) F_n(n_\parallel, kh_s)\ e^{ik_\parallel(z_t^0 - z_s^0)}\, e^{ik_\perp h_t}\, e^{-im(\pi - \alpha)} dn_\parallel \tag{9}$$

and

$$G_n(\xi) = \frac{J_n'(\xi)}{H_n'(\xi)} \tag{10}$$

where $J_n'(\xi)$ and $H_n'(\xi)$ are the derivatives of the Bessel and Hankel functions with respect to the argument, respectively.

The solution of such a system would require, in principle, to know an infinite number of coefficients c_{sn}. Fortunately, this is not the real case, and a good approximation to these coefficients can be done by cutting off the series in the previous system (8), i.e. on the assumption that the terms over a certain maximum order N_T can be set equal to zero. A good truncation rule is $N_T > 3ka$, where a is the largest radius of the cylinders [14]. Therefore, the total magnetic field $\mathcal{H}_{\rm tot}$ is fully determined.

3. THE CONSTANT DENSITY PLASMA

Let us start with the expression for the reflection coefficient Γ. We consider an H polarized plane wave impinging on the vacuum–plasma interface at a certain angle, which may be complex.

The reflection coefficient is defined as

$$\Gamma = \frac{\mathcal{E}_{\rm r}(x=h)}{\mathcal{E}_{\rm i}(x=h)} \tag{11}$$

where $\mathcal{E}_{\rm i}$ and $\mathcal{E}_{\rm r}$ are the amplitudes of the incident and reflected plane waves, respectively.

The expression of the total field in a vacuum, due to the superposition of the two aforementioned waves, is given by

$$\begin{cases} \mathcal{E}_x(x,z) = \mathcal{E}_{ix}\,{\rm e}^{{\rm i}(k_z z + k_x x)} - \mathcal{E}_{rx}\,{\rm e}^{{\rm i}(k_z z - k_x x)} \\ \mathcal{E}_y(x,z) = 0 \\ \mathcal{E}_z(x,z) = -[\mathcal{E}_{iz}\,{\rm e}^{{\rm i}(k_z z + k_x x)} + \mathcal{E}_{rz}\,{\rm e}^{{\rm i}(k_z z - k_x x)}] \\ \mathcal{H}_x(x,z) = 0 \\ \mathcal{H}_y(x,z) = Y_0[\mathcal{E}_{\rm i}\,{\rm e}^{{\rm i}(k_z z + k_x x)} - \mathcal{E}_{\rm r}\,{\rm e}^{{\rm i}(k_z z - k_x x)}] \\ \mathcal{H}_z(x,z) = 0 \end{cases} \tag{12}$$

where Y_0 is the vacuum characteristic admittance.

To impose the continuity conditions on the plasma surface it is convenient to use the Fourier transform, with respect to z, of the tangential field components inside the plasma, i.e. $\tilde{\mathcal{E}}_z^{\rm pl}(x,k_z)$, $\tilde{\mathcal{H}}_y^{\rm pl}(x,k_z)$. By imposing the boundary conditions, we obtain:

$$\tilde{\mathcal{E}}_z^{\rm pl}(0,k_z') = -2\pi(\mathcal{E}_{iz} + \mathcal{E}_{rz})\delta(k_z' - k_z) \tag{13a}$$

$$\tilde{\mathcal{H}}_y^{\rm pl}(0,k_z') = 2\pi Y_0(\mathcal{E}_{\rm i} - \mathcal{E}_{\rm r})\delta(k_z' - k_z) \tag{13b}$$

where δ is the Dirac function. The plasma admittance is given by

$$Y_{\rm pl}(k_z) = -\frac{\tilde{\mathcal{H}}_y^{\rm pl}(0,k_z)}{\tilde{\mathcal{E}}_z^{\rm pl}(0,k_z)} \tag{14}$$

Then, Eqs (12) and (13) yield:

$$\mathcal{E}_{\rm i} - \mathcal{E}_{\rm r} = (\mathcal{E}_{iz} + \mathcal{E}_{rz})\frac{Y_{\rm pl}(k_z)}{Y_0} \equiv (\mathcal{E}_{\rm i} + \mathcal{E}_{\rm r})\xi(k_z) \tag{15}$$

where

$$\xi(k_z) = \frac{Y_{\rm pl}(k_z)}{Y_0}\sqrt{1 - \left(\frac{k_z}{k}\right)^2} \tag{16}$$

For plasma edge conditions corresponding to a step size profile with constant density n_0, the plasma admittance assumes the value [11]

$$Y_{\rm pl} = Y_0\frac{\sqrt{1 - \dfrac{n_0}{n_{\rm c}}}}{\sqrt{1 - n_\parallel^2}} \tag{17}$$

where $n_{\rm c}$ is the critical plasma density.

As a consequence the reflection coefficient $\Gamma = (1 - \xi)/(1 + \xi)$ becomes

$$\Gamma = \frac{1 - \sqrt{1 - \dfrac{n_0}{n_{\rm c}}}}{1 + \sqrt{1 - \dfrac{n_0}{n_{\rm c}}}} \tag{18}$$

Equation (18) shows that Γ has no $n_\parallel$ dependence.

We may intuitively think that this case can be treated as an extension of the image method. This is indeed the case as we will show. Let us start from Eq. (7): by using Eq. (1), it is possible to avoid the use of the J_n Bessel functions; therefore, the field $\mathcal{H}_{\rm dr}$ turns out to be given by

$$\mathcal{H}_{\rm dr}(x,z) = \Gamma\mathcal{H}_0\sum_{s=1}^{N}\sum_n {\rm i}^n c_{sn}\,{\rm e}^{-{\rm i}n\varphi} \times \int F_n\left(\frac{k_\parallel}{k}, kh_s\right){\rm e}^{{\rm i}k_\parallel z_s}\,{\rm e}^{{\rm i}k_\perp(h_s - x_s)}\frac{{\rm d}k_\parallel}{k} \tag{19}$$

Taking into account the propagation property of an angular spectrum of waves, i.e.

$$F_n\left(\frac{k_\parallel}{k}, kx\right) = F_n\left(\frac{k_\parallel}{k}, kh\right){\rm e}^{{\rm i}k_\perp(x-h)} \tag{20}$$

we obtain for expression (19):

$$\mathcal{H}_{\rm dr}(x,z) = \Gamma\mathcal{H}_0\sum_{s=1}^{N}\sum_n {\rm i}^n c_{sn}\,{\rm e}^{-{\rm i}n\varphi} \times \int F_n\left(\frac{k_\parallel}{k}, k(2h_s - x_s)\right){\rm e}^{{\rm i}k_\parallel z_s}\frac{{\rm d}k_\parallel}{k} \tag{21}$$

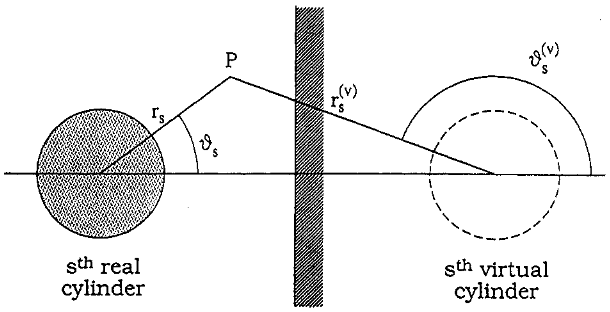

FIG. 2. Real cylinder and its image.

Finally, by using Eq. (5) and the notations introduced in Fig. 2, we obtain

$$\mathcal{H}_{\rm dr}(x, z) = \Gamma \mathcal{H}_0 \sum_{s=1}^{N} \sum_{n} i^n c_{sn} H_n\,(kr_s^{(v)})\, e^{in(\vartheta_s^{(v)} - \varphi)} \quad (22)$$

Comparing Eqs (22) and (3), we see that the field $\mathcal{H}_{\rm dr}$ is proportional to that generated by a set of cylinders specularly placed beyond the plasma surface (Fig. 2).

As we said before, Eq. (22) shows that the problem may be solved by introducing a distribution of N virtual cylinders in the image half-space, i.e. the half-space beyond the plasma surface. The only differences from the ordinary image method are that the Γ coefficient may be complex and its modulus may be different from one.

On imposing the vanishing of the tangential component of the electric field on the cylindrical surfaces, the following linear system is obtained:

$$\sum_{s=1}^{N} \sum_{n} \{[H_{n-m}(kd_{st})\, e^{i(n-m)\vartheta_{st}}(1-\delta_{s,t})$$

$$+ \Gamma H_{m+n}(kd_{st}^{(v)})\, e^{i(m+n)\vartheta_{st}^{(v)}}]$$

$$\times\, G_m(ka_t) + \delta_{m,n}\delta_{s,t}\}\; i^n\, e^{-in\varphi}\, c_{sn}$$

$$= -i^m\, e^{ik_\parallel^i z_t^0}[e^{ik_\perp^i x_t^0}\, e^{im\varphi}$$

$$+ \Gamma\, e^{-ik_\perp^i x_t^0}\, e^{im(\varphi-\pi)}]\; G_m(ka_t)$$

$$m = 0, \pm 1, \pm 2, \ldots; \quad t = 1, \ldots, N \quad (23)$$

where a_s is the radius of the sth real cylinder, $d_{st}^{(v)}$ and $\vartheta_{st}^{(v)}$ are the distance and the angle between the sth real cylinder and the tth virtual one, respectively. Here, the plasma surface has been assumed to coincide with the yz plane of the reference frame.

Treating such a system along the lines discussed in Section 2, we can estimate the total magnetic field $\mathcal{H}_{\rm tot}$.

4. THE COUPLED POWER

Once the solution of the diffraction problem is obtained with the methods set forth in the previous sections, i.e. the numerical values of the c_{sn} coefficients have been calculated, we know the amplitudes of the waves going back and forth in the region between the launcher and the plasma (see Fig. 3) by using the expressions (3) and (7).

To calculate the power flux towards the plasma core, it is convenient to express the field in this region by means of the Fourier superposition of the two amplitude spectra of waves going forward ($\sigma(k_\parallel)$) and backward ($\rho(k_\parallel)$) [15] as follows:

$$\mathcal{E}_z^{\rm tot}(x, z) = -\frac{Z_0}{2\pi}\int_{-\infty}^{\infty} [\sigma(k_\parallel)\, e^{ik_\perp(x-h)}$$

$$+ \rho(k_\parallel)\, e^{-ik_\perp(x-h)}]\, e^{ik_\parallel z}\, \frac{k_\perp}{k}\, dk_\parallel$$

$$\mathcal{E}_x^{\rm tot}(x, z) = \frac{Z_0}{2\pi}\int_{-\infty}^{\infty} [\sigma(k_\parallel)\, e^{ik_\perp(x-h)}$$

$$- \rho(k_\parallel)\, e^{-ik_\perp(x-h)}]\, e^{ik_\parallel z}\, \frac{k_\parallel}{k}\, dk_\parallel \quad (24)$$

$$\mathcal{H}_y^{\rm tot}(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} [\sigma(k_\parallel)\, e^{ik_\perp(x-h)}$$

$$- \rho(k_\parallel)\, e^{-ik_\perp(x-h)}]\, e^{ik_\parallel z}\, dk_\parallel$$

where

$$\rho(k_\parallel) = \Gamma\left(\frac{k_\parallel}{k}\right)\sigma(k_\parallel)$$

and $Z_0 = 1/Y_0$. The quantities $\rho(k_\parallel)$ and $\sigma(k_\parallel)$ can be calculated by using Eqs (5) and (6).

By using the Poynting vector component relevant to the field given by expressions (24), we obtain the following

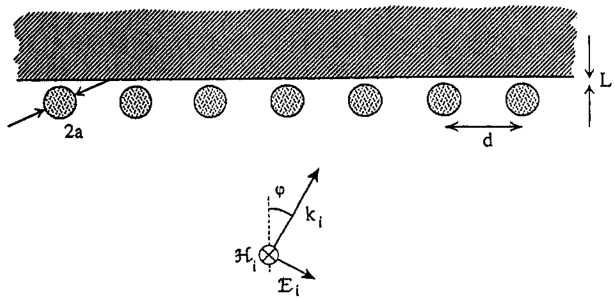

FIG. 3. Single layer quasi-optical grill.

expression for the power flow, per unit y length, in the x direction:

$$\Phi_x = \frac{kZ_0}{4\pi}\left\{\int_{|n_\parallel|<1} |\sigma|^2(1 - |\Gamma|^2)\, n_\perp dn_\parallel - 2\int_{|n_\parallel|>1} |\sigma|^2 \mathrm{Im}[\Gamma]\, |n_\perp| dn_\parallel\right\} \tag{25}$$

where Im[Γ] is the imaginary part of the plasma reflection coefficient.

The only incident power contributing to the transmitted plasma waves is that relevant to the portion of the plane wave corresponding to the width of the array of cylinders along the z axis, so we take it as the incident power for the evaluation of the coupling efficiency.

5. NUMERICAL RESULTS

The method outlined in the previous sections has been applied to the evaluation of the diffracted field, the launched spectrum and the coupled power in different experimental configurations.

The results shown in the following figures are relevant to the case of a step size profile with constant plasma density n_0. The directivity is defined as the power coupled for negative values of $n_\parallel$ with respect to the total coupled power.

A first configuration is that shown in Fig. 3, where an alignment of N identical cylinders in front of a plasma is sketched. The shape of the coupled power spectrum is shown in Fig. 4 for $N = 20$, $ka = 0.85$, $kd = 2.9$, $kL = 0.25$, $\varphi = 45°$, $n_0 = 2n_c$: evidently, only the -1 and $+1$ orders carry a significant amount of power.

In Figs 5 and 6 the power reflection coefficient and the directivity are shown versus the total number of cylinders (N) for different values of the grill periodicity.

In Fig. 7 the coupling parameters of a single layer quasi-optical grill versus the angle of incidence (φ) of the plane wave are shown.

In Fig. 8 the power reflection coefficient and the directivity versus the distance L are shown. Both parameters are increasing functions of the distance L. In Fig. 9 the same quantities are plotted versus the radius of the cylinders, with the distance L being kept fixed. We can infer that both the reflected power and the directivity reach optimum values in the region $0.8 < ka < 1.0$.

In Fig. 10 the same parameters are plotted versus the normalized density n_0/n_c, with results similar to those of a waveguide grill [11].

To reduce the angle of incidence corresponding to maximum coupling a different configuration has been

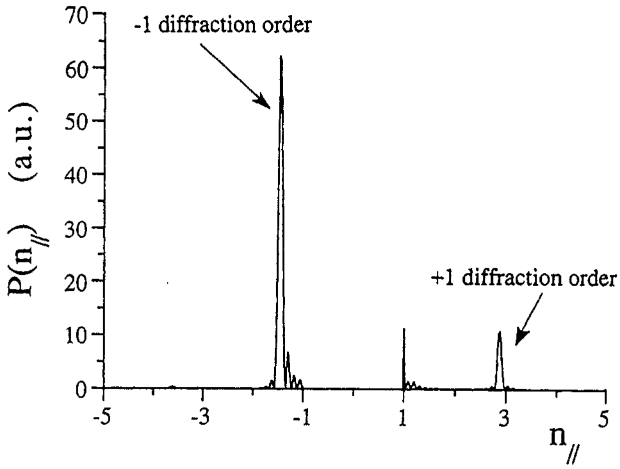

FIG. 4. Coupled power spectrum relevant to the layout of Fig. 3.

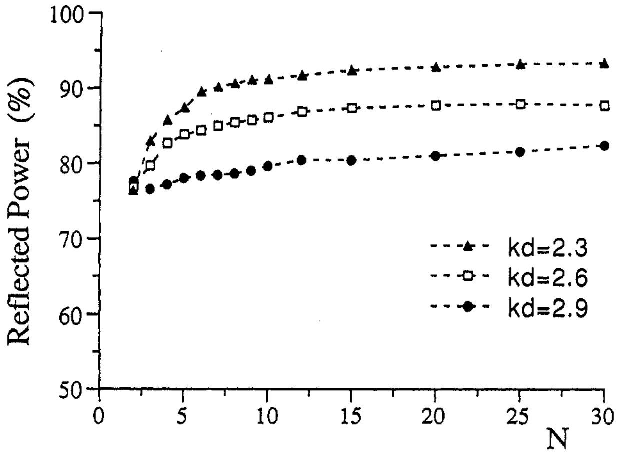

FIG. 5. Reflected power (%) versus N*;* ka = *0.85,* kL = *0.25,* $\varphi = 45°$, $n_0 = 2n_c$*. The different values of the periodicity,* kd = *2.9, 2.6, 2.3, correspond to central peak values of the −1 order given by* $n_{\parallel-1} = -1.45, -1.7, -2.0$*, respectively.*

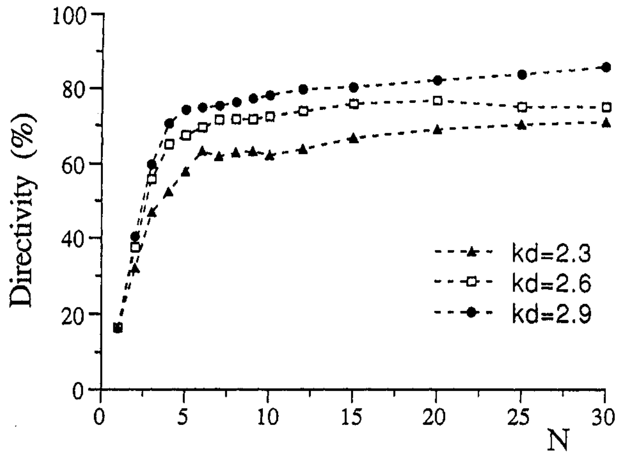

FIG. 6. Directivity (%) versus N*;* ka = *0.85,* kL = *0.25,* $\varphi = 45°$, $n_0 = 2n_c$*.*

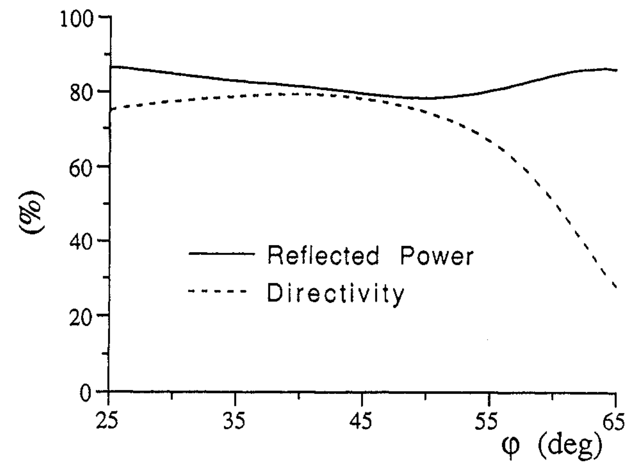

FIG. 7. Reflected power and directivity (%) versus angle of incidence φ; N = *10*, ka = *0.85*, kd = *2.9*, kL = *0.25*, $n_0 = 2n_c$.

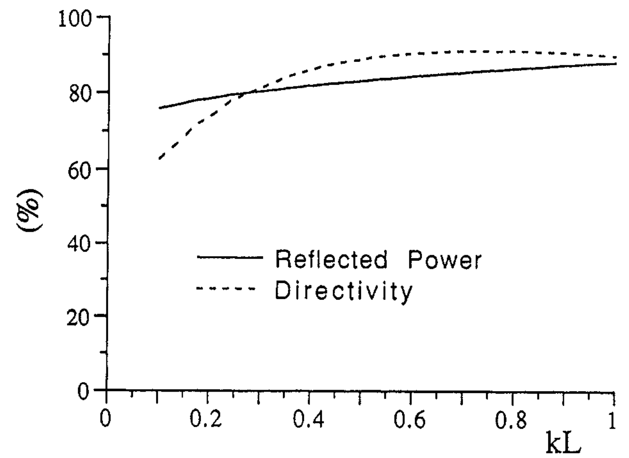

FIG. 8. Reflected power and directivity (%) versus distance kL; N = *10*, ka = *0.85*, kd = *2.9*, φ = *45°*, $n_0 = 2n_c$.

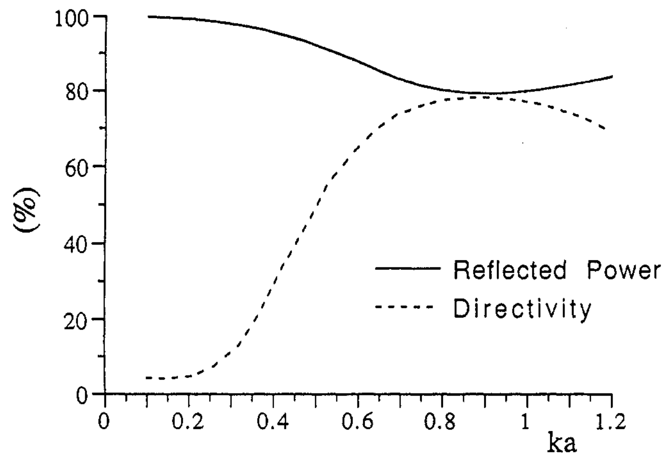

FIG. 9. Reflected power and directivity (%) versus radius ka; N = *10*, kd = *2.9*, kL = *0.25*, φ = *45°*, $n_0 = 2n_c$.

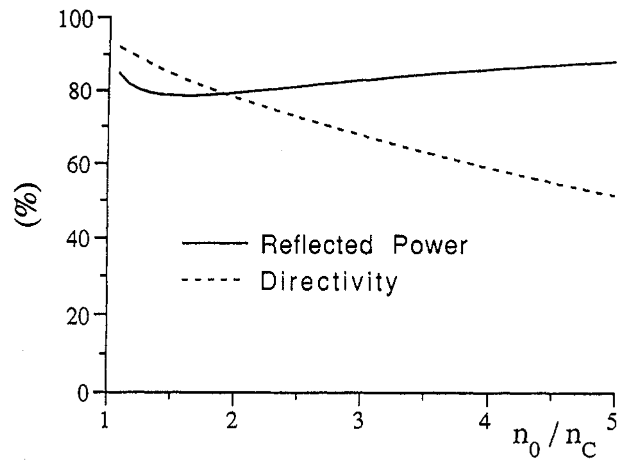

FIG. 10. Reflected power and directivity (%) versus normalized density n_0/n_c; N = *10*, ka = *0.85*, kd = *2.9*, kL = *0.25*, φ = *45°*, $n_0 = 2n_c$.

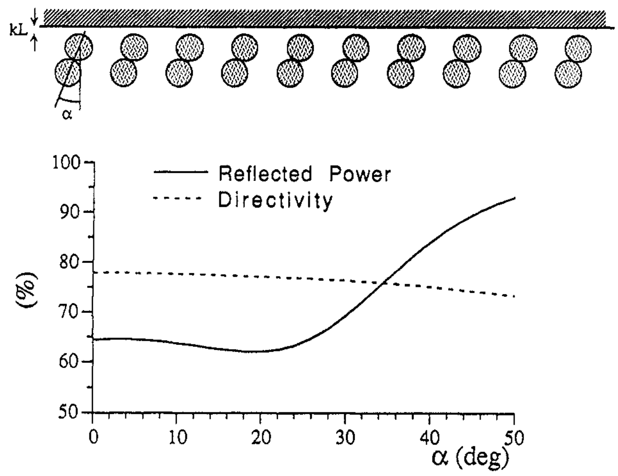

FIG. 11. Reflected power and directivity (%) of an array of strip like 'groups' of cylinders; two cylinders per group, ten groups, ka = *0.85*, kL = *0.25*, kd = *2.9*, φ = *25°*, $n_0 = 2n_c$.

tested. We have considered a scattering element formed by an alignment of two cylinders tilted at an angle α with respect to the x axis. In Fig. 11 the coupling parameters of an array of ten of such groups (with $ka = 0.85$, $kd = 2.9$, $kL = 0.25$, i.e. the same values as are used for Fig. 7), as functions of the tilting angle α, are shown for a fixed value of the angle of incidence of the plane wave $\varphi = 25°$. We see that a kind of co-operative effect is present and acceptable values of the reflection coefficient can be reached even with a narrow angle of incidence. By choosing the plane wave incidence angle to be $\varphi = 45°$, the tilting angle $\alpha = 0°$, and the other parameters as in Fig. 11, the reflected power can be lowered to about 55%.

6. CONCLUSIONS

The numerical results of this work are relevant to the optimization of the single layer quasi-optical grill: it has been shown that the coupled power can reach values of about 25–30% in the −1 diffracted order and about 40% in all the orders. The use of the resonant double array configuration can be studied as a particular case of that outlined in the previous sections; work is in progress on the subject.

Our analysis can be generalized by using incident fields different from plane waves and taking into account the presence of always present metallic side walls. In fact, a general beam can be represented by a discretized plane wave spectrum; so the described method is applicable. For example, a Gaussian beam can be assumed as the incident wave; such a beam can actually be transmitted from a radiofrequency generator to the grill, e.g. in the form of an HE_{11} mode of a corrugated waveguide. Other shapes of the incident beam can be obtained from an array of waveguides or a waveguide terminated with a suitable horn, too.

On the other hand, metallic walls can be simulated by means of a suitable set of wires [16], thus allowing us to use the method outlined in this paper. The same approach can be used to simulate rods with non-circular cross-section, which could yield better coupling results [17].

REFERENCES

[1] CAIRNS, R.A., Radiofrequency Heating of Plasmas, Adam Hilger, Bristol (1991).

[2] BRAMBILLA, M., Nucl. Fusion **16** (1976) 47.

[3] JOBES, F.C., et al., Phys. Rev. Lett. **55** (1985) 1295;
ALLADIO, F., et al., in Plasma Physics and Controlled Nuclear Fusion Research 1984 (Proc. 10th Int. Conf. London, 1984) Vol. 1, IAEA, Vienna (1985) 481;
LEUTERER, F., et al., Nucl. Fusion **31** (1991) 2315.

[4] GORMEZANO, C., et al., Nucl. Fusion **25** (1985) 419.

[5] LITAUDON, X., et al., Nucl. Fusion **32** (1992) 1883;
JACQUINOT, J., et al., Plasma Phys. Control. Fusion **35** (1993) A35;
NAGAMI, M., JT-60 TEAM, in Controlled Fusion and Plasma Physics (Proc. 16th Eur. Conf. Venice, 1989), Vol. 13B, Part III, European Physical Society, Geneva (1989) 1597.

[6] SCHILD, P., et al., paper presented at European Workshop on LH Heating and Current Drive, JET, Abingdon, UK (May 1992).

[7] PETELIN, M.I., SUVOROV, E.V., Sov. Tech. Phys. Lett. **15** (1989) 882.

[8] RAGHEB, H.A., HAMID, M., Int. J. Electron. **59** (1985) 407.

[9] WAIT, J.R., Can. J. Phys. **32** (1954) 571.

[10] CINCOTTI, G., et al., Opt. Commun. **95** (1993) 192.

[11] STEVENS, J.E., et al., Nucl. Fusion **21** (1981) 1259.

[12] BALANIS, C.A., Advanced Engineering Electromagnetics, J. Wiley, New York (1989).

[13] ABRAMOWITZ, M., STEGUN, I.A., Handbook of mathematical functions, Eq. 9.1.79, Dover, New York (1972).

[14] RAGHEB, H.A., HAMID, M., IEEE Trans. Antennas Propag. **28** (1987) 349.

[15] BRAMBILLA, M., Nucl. Fusion **18** (1978) 493.

[16] RICHMOND, J.H., IEEE Trans. Microwave Theory Tech. **13** (1965) 408.

[17] KOVALEV, N.F., et al., in RF Heating and CD of Fusion Devices (Proc. Europhys. Top. Conf. Brussels, 1992), Vol. 16E, European Physical Society, Geneva (1992) 89.

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