Coherence and the spatial distribution of intensity

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We address the following problem: Can two wave fields with different coherence properties produce the same optical intensity everywhere in the space? Limiting ourselves to paraxial propagation, we prove that in the one-dimensional case the answer is negative. On the other hand, in the two-dimensional case we show through examples that the answer is affirmative. Some consequences are discussed.

1. INTRODUCTION

It is well known that wave fields in different states of coherence can give the same optical intensity distribution across a selected plane. One of the most famous examples is given by the Collett–Wolf sources.[1-3] These are Gaussian-correlated, partially coherent sources that can produce the same far-zone optical intensity pattern as that of a lowest order coherent laser beam. The study of these sources has given rise to many results.[4-11] The existence of these sources shows that knowledge of the optical intensity across a certain plane is by no means sufficient for a determination of the coherence properties of a wave field. This remark is somewhat trivial, inasmuch as the optical intensity at a certain temporal frequency corresponds only to the diagonal elements of the correlation function, namely, the cross-spectral density.[12] But now let the optical intensity produced by a wave field in a certain state of coherence be fixed throughout the space. Can another wave field with different coherence properties give rise to the same intensity everywhere? In the present paper we try to answer this question and to elucidate some of the consequences.

In order to make clear the motivations of our study, we add a few remarks. A conceptually simple way to specify the coherence properties of a radiation field (at least to second-order level) is to give the cross-spectral density, at any selected temporal frequency, across an arbitrary plane orthogonal to the mean direction of propagation of the field. Once this is done, direct and inverse propagation formulas permit one to evaluate the cross-spectral density at any pair of points. From an experimental point of view, this procedure is generally hard to carry out. Not only is the measurement of the cross-spectral density an error-sensitive and time-consuming operation, but the measurements themselves fill a four-dimensional space because the correlation function depends, at each temporal frequency, on a pair of points. Stated in different terms, a partially coherent field has a huge number of degrees of freedom. Incidentally, this is why, still today, we cannot record a partially coherent field in a simple way, except in the limiting case of full coherence, for which holographic methods apply. On the other hand, the optical intensity alone can be measured rather easily. Therefore it is interesting to inquire about the constraints that the spatial distribution of the optical intensity imposes on the coherence properties of the field. Suppose that the intensity distribution throughout the space fully determines the cross-spectral density. Then, this characterization could afford an alternative approach to the problem of measuring the coherence functions of a field. On the contrary, suppose that the intensity distribution alone does not determine completely the cross-spectral density. In this case we could find fields that at no point in space can be distinguished on an intensity basis and yet are physically different. Their difference would play an important role in interferometric, diffractive, and scattering experiments.

In most of this paper we shall limit ourselves to the hypothesis of paraxial propagation. The conditions under which such a hypothesis applies for partially coherent fields are discussed in Ref. 13. While it is not extremely general, the paraxial propagation encompasses a large class of cases of practical interest. Roughly speaking, the basic condition to be met is that the field should propagate within an angular range that is not too large. This can be safely assumed, e.g., when the field propagates in the form of a more or less collimated beam and also when the field propagates through most optical systems. Here we are interested in free-space propagation.

A distinction will be made between one-dimensional and two-dimensional cases. To avoid misunderstanding, we find it useful to specify the meanings of these terms. Let z be a Cartesian coordinate along the mean direction of propagation of the field. We say that a certain case is of the one-dimensional type if all the quantities of interest are insensitive to one of the transverse coordinates x and y. Assuming y to be the irrelevant coordinate, this means, e.g., that the correlation function at two points (x_1, y_1, z) and (x_2, y_2, z) depends on x_1, x_2, and z only. Needless to say, a two-dimensional case is one in which both transverse coordinates are to be considered.

After a brief recall of some basic quantities of coherence theory (Section 2), we shall prove that in the one-

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dimensional case the distribution of the optical intensity throughout the space uniquely determines the cross-spectral density everywhere (Section 3). Such a proof does not hold in the two-dimensional case. In such a case, in fact, we shall be able to construct classes of fields possessing different coherence properties all of which give the same optical intensity everywhere (Section 4).

2. PRELIMINARIES

At any given temporal frequency ν the spatial coherence properties of a field are described by the cross-spectral density function.[12] For two typical points with position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ the cross-spectral density $W(\mathbf{r}_1, \mathbf{r}_2;\ \nu)$ can be evaluated through the following average[14]:

$$W(\mathbf{r}_1, \mathbf{r}_2; \nu) = \langle V(\mathbf{r}_1; \nu) V^*(\mathbf{r}_2; \nu)\rangle, \tag{2.1}$$

where V is the optical disturbance and the average is to be made over a suitable ensemble of monochromatic fields. In Eq. (2.1) the asterisk denotes the complex conjugate. When $\mathbf{r}_1 = \mathbf{r}_2 \equiv \mathbf{r}$, Eq. (2.1) reduces to

$$I(\mathbf{r}; \nu) = \langle |V(\mathbf{r}; \nu)|^2\rangle, \tag{2.2}$$

i.e., to the so-called optical intensity at frequency ν. In terms of the cross-spectral density function, one may define the quantity

$$\mu(\mathbf{r}_1, \mathbf{r}_2; \nu) = W(\mathbf{r}_1, \mathbf{r}_2; \nu)/[I(\mathbf{r}_1; \nu) I(\mathbf{r}_2; \nu)]^{1/2}, \tag{2.3}$$

namely, the degree of spatial coherence, which is normalized in such a way as to yield

$$0 \le |\mu(\mathbf{r}_1, \mathbf{r}_2; \nu)| \le 1 \tag{2.4}$$

for all values of $\mathbf{r}_1$, $\mathbf{r}_2$, and ν.[12] The limiting values 0 and 1 in inequality (2.4) indicate that the light fluctuations at frequency ν at the points $\mathbf{r}_1$ and $\mathbf{r}_2$ are uncorrelated and completely correlated, respectively.

Before concluding this section, we deduce the propagation formulas for the cross-spectral density in the paraxial regime. Here and in what follows, we omit, for the sake of brevity, the explicit dependence on ν. We denote by $W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ and $W(\mathbf{r}_1, \mathbf{r}_2, z)$ the cross-spectral density functions across the plane $z = 0$ and across a plane $z = \text{const}$, respectively, of a suitable reference frame. In the case of completely coherent fields the propagation problem can be solved by means of the Fresnel diffraction formula[15,16]

$$U(\mathbf{r}, z) = -\frac{i}{\lambda z}[\exp(ikz)] \int U(\boldsymbol{\rho}, 0) \exp\left[i\frac{\pi}{\lambda z}(\mathbf{r} - \boldsymbol{\rho})^2\right] \mathrm{d}\boldsymbol{\rho}, \tag{2.5}$$

where λ is the wavelength of the radiation and $k = 2\pi/\lambda$. In the more general case of partially coherent fields an expression similar to that of Eq. (2.5) can be deduced with the use of Eq. (2.1). In fact, owing to their monochromaticity, the fields $V(\mathbf{r})$ in Eq. (2.1) are perfectly coherent and therefore propagate according to Eq. (2.5). By combining Eqs. (2.1) and (2.5), we deduce the paraxial propagation formula for the cross-spectral density:

$$W(\mathbf{r}_1, \mathbf{r}_2, z) = \frac{1}{(\lambda z)^2} \iint W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \times \exp\left\{i\frac{\pi}{\lambda z}[(\mathbf{r}_1 - \boldsymbol{\rho}_1)^2 - (\mathbf{r}_2 - \boldsymbol{\rho}_2)^2]\right\} \mathrm{d}\boldsymbol{\rho}_1 \mathrm{d}\boldsymbol{\rho}_2. \tag{2.6}$$

3. ONE-DIMENSIONAL CASE

For a certain partially coherent field belonging to the one-dimensional class, let the optical intensity $I(x, z)$ be known at any point (x, z). Denote by $W_0(\xi_1, \xi_2)$ the cross-spectral density across the plane $z = 0$. The relationship between W_0 and I is derived from Eqs. (2.2) and (2.6) under the one-dimensional constraint:

$$I(x, z) = \frac{1}{\lambda|z|} \iint W_0(\xi_1, \xi_2) \times \exp\left\{\frac{2\pi i}{\lambda z}\left[\frac{\xi_1^2 - \xi_2^2}{2} - x(\xi_1 - \xi_2)\right]\right\} \mathrm{d}\xi_1 \mathrm{d}\xi_2. \tag{3.1}$$

Equation (3.1) holds for both $z > 0$ and $z < 0$.

The problem in which we are interested is an inversion problem: Can W_0 be evaluated from knowledge of I? We shall show below that an affirmative answer can be given. More precisely, we shall prove that a unique solution exists. On the other hand, we will not dwell on the construction of an explicit inversion formula.

To begin, we assume that $z \neq 0$ and introduce the new variables

$$\sigma = (\xi_1 + \xi_2)/2, \qquad \tau = (\xi_2 - \xi_1)/\lambda z. \tag{3.2}$$

In terms of these variables Eq. (3.1) is written as

$$I(x, z) = \iint L_0(\sigma, \lambda z \tau/2) \exp[-2\pi i(\sigma - x)\tau] \mathrm{d}\sigma \mathrm{d}\tau, \tag{3.3}$$

where L_0 is the cross-spectral density at $z = 0$ referred to a different frame. More explicitly, we put

$$L_0(s, t) = W_0(s - t, s + t). \tag{3.4}$$

Let us denote by $\tilde{I}(p, z)$ the one-dimensional Fourier transform of the function $I(x, z)$ with respect to the first variable, namely,

$$\tilde{I}(p, z) = \int I(x, z) \exp(-2\pi i p x) \mathrm{d}x. \tag{3.5}$$

After inserting Eq. (3.3) into Eq. (3.5), we obtain

$$\tilde{I}(p, z) = \int L_0(\sigma, \lambda z p/2) \exp(-2\pi i \sigma p) \mathrm{d}\sigma. \tag{3.6}$$

Similarly, we denote by $\tilde{L}_0(p, t)$ the Fourier transform of $L_0(s, t)$ with respect to the variable s, or

$$\tilde{L}_0(p, t) = \int L_0(s, t) \exp(-2\pi i p s) \mathrm{d}s. \tag{3.7}$$

It is seen by comparison of Eqs. (3.6) and (3.7) that the following relation holds:

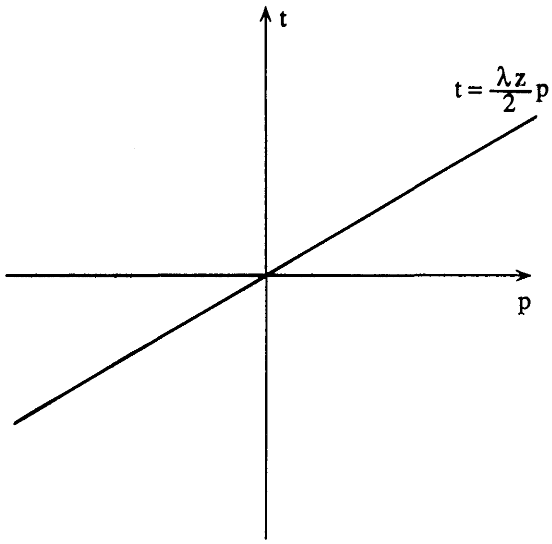

Fig. 1. Linear relationship between t and p for any fixed value of z.

$$\tilde{I}(p,z) = \tilde{L}_0(p, \lambda z p/2)\,. \tag{3.8}$$

Although this result has been derived under the hypothesis $z \neq 0$, it is not difficult to show that it holds true for $z = 0$ too. In fact, letting $t = 0$ in Eq. (3.7) and using Eq. (3.4), we obtain

$$\tilde{L}_0(p,0) = \int W_0(s,s)\exp(-2\pi i p s)\mathrm{d}s\,. \tag{3.9}$$

If we recall further that $W_0(s,s) = I(s,0)$, we see that Eq. (3.9) can be written as

$$\tilde{L}_0(p,0) = \tilde{I}(p,0)\,, \tag{3.10}$$

which is the same as Eq. (3.8) with $z = 0$.

We now examine the consequences of Eq. (3.8). Let us consider the (p,t) plane (see Fig. 1), and let z have any fixed value. From supposed knowledge of $I(x,z)$ we pass to $\tilde{I}(p,z)$ by a Fourier transformation. This in turn gives the values of $\tilde{L}_0$ along the line $t = \lambda z p/2$ through the origin [see Eq. (3.8)]. When z spans the whole axis (from $-\infty$ to $+\infty$), such a line sweeps the (p,t) plane by rotating around the origin. Therefore knowledge of $I(x,z)$ throughout the space entails a unique determination of $\tilde{L}_0(p,t)$ across the (p,t) plane. Ultimately, by Fourier inversion the function $L_0(s,t)$ and hence [see Eq. (3.4)] the cross-spectral density W_0 are uniquely determined.

We have reached the following conclusion: In the one-dimensional case it is impossible for fields with different correlation functions to produce the same optical intensity distribution everywhere in the space. As a corollary, no partially coherent field can give everywhere the same intensity as that from a fully coherent one.

Let us summarize and add a few comments. In the one-dimensional case two wave fields with different states of coherence can be distinguished on the basis of the intensity distributions that they produce in space. In principle, there exists a unique solution to the following inverse problem: Given the optical intensity $I(x,z)$, find the cross-spectral density at the plane $z = 0$. We can ask whether these results can be turned into some practical method that evaluates the coherence properties of a field from knowledge of the intensity alone. This question is beyond the aim of the present paper, but some simple remarks can be made. The ideal inversion procedure would require knowledge of $I(x,z)$ at any plane $z =$ const. In practice, $I(x,z)$ can be measured in a finite range of values of z, say from $z = z_1$ to $z = z_2$. According to Eq. (3.8) this amounts to knowledge of $\tilde{L}_0(p,t)$ within the striped region of Fig. 2. In other words, a filtered version of $\tilde{L}_0(p,t)$ would be available. Although such limited knowledge of $\tilde{L}_0(p,t)$ does not permit an exact inversion, except perhaps in the presence of some prior knowledge of, or constraint on, W_0, a somewhat blurred version of W_0 could be recovered. It is seen that a typical inverse problem with partial information is opened up by the present results.[17]

4. TWO-DIMENSIONAL CASE

We now turn to the general, two-dimensional case, and we ask again: Can two wave fields possessing different coherence properties give the same optical intensity at all points of the space? In the one-dimensional case this possibility is excluded by the proof given in Section 3. It is easy to show that such a proof cannot be extended to the two-dimensional case. This result, in itself, does not mean that one could not find a different proof. In order to discard such a hypothesis, we shall resort to a different argument. We shall simply show that the answer to the opening question is affirmative by constructing some classes of fields with the required property.

The basic tool that we are going to use is a coherent field with the following distribution in the plane $z = 0$:

$$V_n^{(+)}(r,\theta,0) = F_n(r,0)\exp(in\theta)\,, \tag{4.1}$$

where r and θ are polar coordinates, n is a positive integer, and $F_n(r,0)$ is an arbitrary, circularly symmetric function. Of course, we assume that F_n is endowed with sufficient regularity properties, so that the diffraction integral in which it is going to be inserted makes sense. Note that

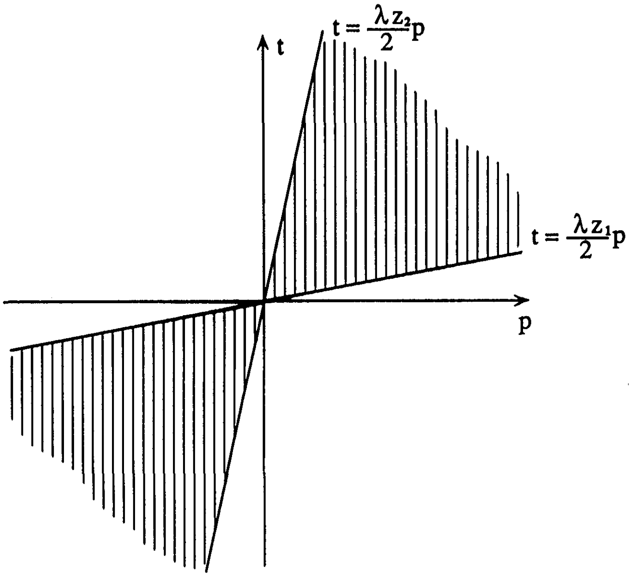

Fig. 2. Accessible region in the (p,t) plane where the optical intensity is known for a finite interval of z values ($z_1 \leq z \leq z_2$).

the inclusion of the subscript n after the F does not necessarily mean that $F_n(r, 0)$ shows any dependence on n. As a consequence, it could be omitted. On the other hand, we shall see in a moment that the effect of propagation on the radial part of the field does depend on n. Therefore we also use the index n in the plane $z = 0$.

At any plane $z = \text{const.} \neq 0$ the field corresponding to Eq. (4.1) can be evaluated by means of the Fresnel diffraction integral [Eq. (2.5) above]. The resulting field at $z =$ const, say $V_n^{(+)}(r, \theta, z)$, has the form (see Appendix A)

$$V_n^{(+)}(r, \theta, z) = F_n(r, z)\exp(in\theta), \tag{4.2}$$

where

$$F_n(r, z) = \frac{k}{|z|}(-i)^{n+1}\left\{\exp\left[ik\left(z + \frac{r^2}{2z}\right)\right]\right\}\int_0^\infty F_n(\rho, 0) \times \left[\exp\left(i\frac{k}{2z}\rho^2\right)\right]J_n\left(\frac{k\rho r}{z}\right)\rho\,\mathrm{d}\rho. \tag{4.3}$$

Here J_n is the Bessel function of the first kind of order n. We now introduce another field distribution across the plane $z = 0$, namely,

$$V_n^{(-)}(r, \theta, 0) = F_n(r, 0)\exp(-in\theta). \tag{4.4}$$

The only difference between the fields described by Eqs. (4.1) and (4.4) is the sign in the exponential function. On evaluating the field corresponding to Eq. (4.4) at a typical plane $z = \text{const} \neq 0$, we find (see Appendix A)

$$V_n^{(-)}(r, \theta, z) = F_n(r, z)\exp(-in\theta). \tag{4.5}$$

If we compare Eqs. (4.2) and (4.5), we see that the two fields $V_n^{(+)}$ and $V_n^{(-)}$ produce the same optical intensity everywhere because they differ only by a phase factor. We note that each is perfectly coherent. If we now construct an ensemble of superpositions of $V_n^{(+)}$ and $V_n^{(-)}$ with uncorrelated coefficients, the spatial distribution of the optical intensity will remain of the same form as $|V_n^{(+)}|^2$ or $|V_n^{(-)}|^2$, whereas the coherence properties will change. Let us proceed to an explicit evaluation of the pertaining cross-spectral density. We denote by $V_n(r, \theta, z)$ a typical member of the ensemble, or

$$V_n(r, \theta, z) = a_n^{(+)}V_n^{(+)}(r, \theta, z) + a_n^{(-)}V_n^{(-)}(r, \theta, z), \tag{4.6}$$

where $a_n^{(+)}$ and $a_n^{(-)}$ are uncorrelated random coefficients. Using Eqs. (2.1), (4.2), and (4.5), we find that the cross-spectral density at any plane $z = $ const is given by

$$\begin{aligned} W_n(r_1, \theta_1, r_2, \theta_2; z) = F_n(r_1, z)F_n^*(r_2, z) &\times \{(P_n + M_n)\cos[n(\theta_1 - \theta_2)] \\ &+ i(P_n - M_n)\sin[n(\theta_1 - \theta_2)]\}, \end{aligned} \tag{4.7}$$

where

$$P_n = \langle|a_n^{(+)}|^2\rangle, \qquad M_n = \langle|a_n^{(-)}|^2\rangle. \tag{4.8}$$

If we set

$$T_n = P_n + M_n, \qquad \epsilon_n = (P_n - M_n)/(P_n + M_n), \tag{4.9}$$

Eq. (4.7) can be written as

$$\begin{aligned} W_n&(r_1, \theta_1, r_2, \theta_2; z) \\ &= T_nF_n(r_1, z)F_n^*(r_2, z)\{\exp[i\delta_n(\theta_1 - \theta_2)]\} \\ &\quad\times \{\cos^2[n(\theta_1 - \theta_2)] + \epsilon_n^2\sin^2[n(\theta_1 - \theta_2)]\}^{1/2}, \end{aligned} \tag{4.10}$$

where

$$\delta_n(\theta_1 - \theta_2) = \tan^{-1}\{\epsilon_n\tan[n(\theta_1 - \theta_2)]\}. \tag{4.11}$$

In particular, setting $r_1 = r_2 = r$ and $\theta_1 = \theta_2 = \theta$, we find that the optical intensity is axially symmetric and has the expression

$$I_n(r, z) = T_n|F_n(r, z)|^2. \tag{4.12}$$

On inserting Eqs. (4.10) and (4.12) into Eq. (2.3) above, we find the degree of spatial coherence:

$$\begin{aligned} \mu_n&(r_1, \theta_1, r_2, \theta_2; z) \\ &= (\exp\{i[\Phi_n(r_1, z) - \Phi_n(r_2, z) + \delta_n(\theta_1 - \theta_2)]\}) \\ &\quad\times \{\cos^2[n(\theta_1 - \theta_2)] + \epsilon_n^2\sin^2[n(\theta_1 - \theta_2)]\}^{1/2}, \end{aligned} \tag{4.13}$$

where Φ_n is the argument of the function F_n.

The meaning of the present results can be seen from Eqs. (4.12) and (4.13). The former shows that the optical intensity does not depend on ϵ_n, whereas the latter shows that μ_n is a function of ϵ_n. In this way we have a one-parameter family of fields all of which possess the same intensity everywhere while possessing different coherence properties. The parameter ϵ_n can vary from -1 to 1. The extreme values ($|\epsilon_n| = 1$) correspond to the two coherent cases in which only one of the two fields $V_n^{(+)}$ and $V_n^{(-)}$ is present. The modulus of μ_n is then equal to 1. Any other value of ϵ_n gives a partially coherent field. In particular, if $\epsilon_n = 0$, we derive from Eq. (4.13) the relation

$$|\mu_n(r_1, \theta_1, r_2, \theta_2; z)| = |\cos[n(\theta_1 - \theta_2)]|. \tag{4.14}$$

In this case any pair of points with an angular separation equal to an odd multiple of $\pi/2n$ has a vanishing degree of spatial coherence.

A simple example of the previous class is obtained when $V_n^{(+)}$ and $V_n^{(-)}$ at $z = 0$ have the explicit expression

$$V_l^{(\pm)}(r, \theta, 0) = Ar\exp(-r^2/w_0^2)\exp(\pm i\theta), \tag{4.15}$$

where A and w_0 are constants. As is well known, these are two particular Laguerre–Gauss laser modes[18] taken at their waist, with spot size w_0. It can be noted that the linear combinations

$$V_1^{(+)}(r, \theta, 0) + V_1^{(-)}(r, \theta, 0) = 2Ar\exp(-r^2/w_0^2)\cos\theta, \tag{4.16}$$

$$V_1^{(+)}(r, \theta, 0) - V_1^{(-)}(r, \theta, 0) = 2iAr\exp(-r^2/w_0^2)\sin\theta \tag{4.17}$$

have the same structure as the two Hermite–Gauss modes that are generally denoted by TEM_{10} and TEM_{01}, respectively. Any superposition of the fields (4.15) with uncorrelated coefficients gives rise to the following intensity distribution across the plane $z = 0$:

$$I_1(r, 0) = T_1A^2r^2\exp(-2r^2/w_0^2), \tag{4.18}$$

as one can see from Eqs. (4.1), (4.4), (4.9), (4.12), and (4.15). On the contrary, the modulus of the degree of spatial coherence depends on the relative weight of the two modes, which is given by Eq. (4.14) with $n = 1$.

A pattern of the form (4.18) is often encountered in laboratory practice with lasers and is generally referred to as a doughnut mode. It is believed that such a pattern is due to a mixture of TEM_{10} and TEM_{01} modes oscillating at slightly different frequencies.[19] In order to preserve the circular symmetry, one must assume that the two modes have the same mean power. It is to be noted that in this case one refers to the time-averaged intensity, which is obtained by integration of the frequency-dependent optical intensity over the spectral range of the radiation field, inasmuch as contributions at different frequencies add up incoherently. The previous results show that, in principle, a doughnut mode could be interpreted as an incoherent superposition of the fields (4.15) with any relative weight. Practical details of the laser cavity may suggest which interpretation of the pattern (4.18) is more appropriate. In any case the important point to be made is that the only way to distinguish between a pure Laguerre–Gauss mode [either $V_1^{(+)}$ or $V_1^{(-)}$] and a superposition of modes is to measure the degree of coherence.

We can generalize the previous results by summing Eq. (4.6) over a certain (possibly infinite) number of values of n, i.e., by constructing a field of the form

$$V(r, \theta, z) = \sum_n V_n(r, \theta, z). \tag{4.19}$$

Any of the fields V_n is subject to the same regularity conditions as those for the field discussed after Eq. (4.1). In addition, of course, the sum in Eq. (4.19) must be convergent if an infinite number of terms is implied. We assume that the coefficients $a_n^{(\pm)}$ behave on ensemble average as

$$\langle a_n^{(+)} a_m^{(+)*} \rangle = \delta_{nm} P_n, \qquad \langle a_n^{(-)} a_m^{(-)*} \rangle = \delta_{nm} M_n,$$
$$\langle a_n^{(+)} a_m^{(-)*} \rangle = 0 \qquad (n, m = 0, \pm 1, \pm 2, \ldots). \tag{4.20}$$

In this case the cross-spectral density and the optical intensity can be written as

$$W(r_1, \theta_1; r_2, \theta_2; z) = \sum_n W_n(r_1, \theta_1; r_2, \theta_2; z), \tag{4.21}$$

$$I(r, \theta) = \sum_n I_n(r, \theta), \tag{4.22}$$

respectively, where W_n and I_n are given by Eqs. (4.10) and (4.12), respectively. The coherence features of the field depend on two sets of parameters, namely, T_n and ϵ_n [see Eqs. (4.9)]. On the other hand, the optical intensity depends only on T_n. As a consequence, we can change at will any of the ϵ_n values between -1 and 1, thus producing coherence variations while leaving unchanged the optical intensity. It is to be noted, however, that none of the present fields is completely coherent unless the sum in Eq. (4.19) reduces to a single term, say $n = m$, and $|\epsilon_m| = 1$.

The previous cases might suggest that fields with equal intensity and different coherence are necessarily endowed with axial symmetry. This is not true, as the next simple example shows. Let us consider the following field distributions across the plane $z = 0$:

$$V^{(+)}(x, y, 0) = A_x \cos(\beta x) + i A_y \cos(\beta y), \tag{4.23}$$

$$V^{(-)}(x, y, 0) = A_x \cos(\beta x) - i A_y \cos(\beta y), \tag{4.24}$$

where A_x, A_y, and β are real constants. It will be noted that

$$V^{(-)}(x, y, 0) = V^{(+)*}(x, y, 0). \tag{4.25}$$

The effect of propagation on the fields (4.23) and (4.24) is simply to multiply them by a phase factor. More explicitly, we have

$$V^{(\pm)}(x, y, z) = \{\exp[iz(k^2 - \beta^2)^{1/2}]\} V^{(\pm)}(x, y, 0), \tag{4.26}$$

which can be seen immediately by an elementary plane-wave expansion.[20] It should be noted that in this case the paraxial approximation is not used. Because of Eqs. (4.23)–(4.26), both $V^{(+)}$ and $V^{(-)}$ produce the same intensity distribution across any plane $z =$ const or, which is the same, they are nondiffracting fields.[21] In addition, each is perfectly coherent.

Let us now construct an ensemble of superpositions of $V^{(+)}$ and $V^{(-)}$ with uncorrelated zero-mean coefficients, say $a^{(+)}$ and $a^{(-)}$, respectively. The cross-spectral density across any plane $z =$ const is easily found and turns out to be

$$\begin{aligned} W(x_1, y_1; x_2, y_2) &= (P + M)[A_x^2 \cos(\beta x_1)\cos(\beta x_2) + A_y^2 \cos(\beta y_1)\cos(\beta y_2)] \\ &\quad - i(P - M)A_x A_y[\cos(\beta x_1)\cos(\beta y_2) - \cos(\beta y_1)\cos(\beta x_2)], \end{aligned} \tag{4.27}$$

where

$$P = \langle |a^{(+)}|^2 \rangle, \qquad M = \langle |a^{(-)}|^2 \rangle. \tag{4.28}$$

The corresponding intensity is

$$I(x, y) = (P + M)[A_x^2 \cos^2(\beta x) + A_y^2 \cos^2(\beta y)]. \tag{4.29}$$

From Eqs. (4.27) and (4.29) the degree of spatial coherence can be easily evaluated. For the sake of simplicity, we shall limit discussion to the case $P = M$ and $A_x^2 = A_y^2$. We then obtain

$$\mu(x_1, y_1; x_2, y_2) = \frac{\cos(\beta x_1)\cos(\beta x_2) + \cos(\beta y_1)\cos(\beta y_2)}{\{[\cos^2(\beta x_1) + \cos^2(\beta y_1)][\cos^2(\beta x_2) + \cos^2(\beta y_2)]\}^{1/2}}. \tag{4.30}$$

The superposition field described by Eqs. (4.27), (4.29), and (4.30) is no longer fully coherent. As an example, let $x_1 = y_1 = 0$. Then, for all pairs (x_2, y_2) satisfying

$$y_2 = x_2 + (2n + 1)(\pi/\beta), \tag{4.31}$$

with integer n, the degree of coherence (4.30) vanishes.

5. CONCLUSIONS

We proved that in the one-dimensional case the distribution of the optical intensity throughout the space fully determines the coherence properties of a field. This suggests that we could envisage some methods for evaluation of the spatial correlation function of the field starting from measurements of the optical intensity alone. On the other hand, we showed that the previous conclusion does not hold in the two-dimensional case, for which fields endowed with different coherence properties can produce the same optical intensity everywhere. In particular, we found that in some cases a partially coherent field cannot be distinguished from a fully coherent one as far as the optical intensity is concerned. This result should be taken into account when optical beams (e.g., laser beams) are characterized. In this case one is inclined to think that the distribution of optical intensity of the beam fully specifies the spatial properties of the field. Yet, different coherence features could be exhibited by beams with equally distributed optical intensity, which would be revealed by diffraction and interference experiments.

The analysis presented in this paper gives only partial answers to the problem of the mutual constraints between coherence and the spatial distribution of optical intensity. However, we hope that the results presented here will stimulate further study on this theme. We add a final remark. It is known that certain techniques exist for retrieval of the phase of coherent light fields starting from knowledge of the intensity.[22-26] As we saw in Section 4, in certain cases two different coherent fields can give rise to the same intensity everywhere. This underlines the problems of phase ambiguities that may be encountered in the above-mentioned techniques.

APPENDIX A

Starting from the Fresnel diffraction integral written in polar coordinates,[16]

$$V(r,\theta,z) = \frac{-i}{\lambda z}[\exp(ikz)]\int_0^{2\pi} d\phi \int_0^{\infty} V(\rho,\theta,0) \times \exp\left(\left\{i\frac{k}{2z}[r^2+\rho^2-2r\rho\cos(\phi-\theta)]\right\}\right)\rho\, d\rho, \tag{A1}$$

and substituting from Eq. (4.1) above into Eq. (A1), we obtain

$$V_n^{(+)}(r,\theta,z) = \frac{-i}{\lambda z}\exp\left\{\left[ik\left(z+\frac{r^2}{2z}\right)\right]\right\} \times \int_0^{\infty} F_n(\rho,0)\left[\exp\left(i\frac{k}{2z}\rho^2\right)\right]\rho\, d\rho \times \int_0^{2\pi}\exp\left[in\phi - i\frac{kr\rho}{z}\cos(\phi-\theta)\right]d\phi. \tag{A2}$$

By introducing the new variable $\tau = \phi - \theta + \pi/2$, we can write Eq. (A2) as

$$V_n^{(+)}(r,\theta,z) = \frac{1}{\lambda z}[\exp(in\theta)](-i)^{n+1}\left\{\exp\left[ik\left(z+\frac{r^2}{2z}\right)\right]\right\} \times \int_0^{\infty} F_n(\rho,0)\left[\exp\left(i\frac{k}{2z}\rho^2\right)\right]\rho\, d\rho \times \int_0^{2\pi}\exp\left[in\tau - i\frac{kr\rho}{z}(\sin\tau)\right]d\tau. \tag{A3}$$

The second integral in Eq. (A3) can be calculated[27] and turns out to be equal to the Bessel function of the first kind of order n, evaluated in $kr\rho/z$, times 2π; hence

$$V_n^{(+)}(r,\theta,z) = [\exp(in\theta)](-i)^{n+1}\frac{2\pi}{\lambda z}\left\{\exp\left[ik\left(z+\frac{r^2}{2z}\right)\right]\right\} \times \int_0^{\infty} F_n(\rho,0)\left[\exp\left(i\frac{k}{2z}\rho^2\right)\right]J_n\left(\frac{kr\rho}{z}\right)\rho\, d\rho. \tag{A4}$$

The expression (A4) is clearly of the form (4.2) above, with $F_n(r,z)$ given by Eq. (4.3).

On the other hand, if we consider the field (4.4) and proceed in the same manner as before, we obtain the expression (4.5).

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