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Spectral changes in a Young interference pattern

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We detected experimentally the spectral changes in a Young interference pattern recently predicted by James and Wolf. In order to facilitate the experiment, we changed slightly the interferometric scheme analyzed by James and Wolf. After discussion of the modified set-up, we present some experimental results.

In a recent paper [1], James and Wolf have shown that the spectrum detected at the center of a Young interference pattern may be different from the source spectrum. This effect arises from two contributions. The first one is due to the basic laws of diffraction. It appears even if only one of the two Young pinholes is open. Therefore, it is not directly connected to the interference phenomenon. The second contribution, on the other hand, arises from interference and depends on the state of coherence of the radiation incident on the pinholes. In the following, we shall be mainly interested in the effect produced by the second contribution. Such an effect may be described by saying that the spectrum obtained when both pinholes are open may differ from the spectrum produced with a single pinhole.

The origin of this effect can be qualitatively explained as follows. It may well happen that the various spectral components of the radiation used in the interferometer have different coherence properties at the two pinholes. As a consequence, they may contribute differently to the interference pattern and this gives rise to spectral changes with respect to the spectrum where only one pinhole is open. In this note we only consider the situation where the observation point is assumed to be on the axis of symmetry of the interferometer [#1]. In this case, the spectral changes are not due to the dependence of the fringe period on the temporal frequency. As is well known, such a dependence is responsible for the coloured appearance of a Young interference pattern obtained with white light.

[#1] The off-axis case has also been treated by James and Wolf [2].

In their analysis, James and Wolf considered the case in which the two pinholes are illuminated by the partially coherent field generated by a uniform, circular, quasi-homogeneous source. They showed that with narrow-band radiation only small spectral changes occur, whereas drastic changes can take place with broad-band radiation. As can example, James and Wolf give spectral curves corresponding to several coherence states when the radiation is generated by a blackbody source at 3000 K. In this case, the interval of wavelengths across which the most significant changes take place ranges, roughly speaking, from 0.5 to 2 μm.

In this paper, we shall describe an experiment aimed at detecting the spectral changes described above. From the experimental point of view, the use of radiation with a very broad band entails some difficulty, because the very low level of available optical intensity requires the use of high sensitivity detectors, such as photomultipliers, whose spectral response is typically limited to a few tenths of a micrometer. The reason why a rather large frequency interval is required to detect significant spectral changes in the scheme of James and Wolf is that the degree of spectral coherence of the radiation generated by a uniform, incoherent circular source does not change very rapidly with respect to the frequency. On the other hand, the adoption of a different source geometry can lead to a more rapidly varying degree of coherence. In this way, the effect to be demonstrated can be detected with radiation whose bandwidth is somewhat narrower than that of the blackbody spectrum considered by James and Wolf. In our experiment, we use a source made up two very long, parallel strips. They radiate as two quasi-homogeneous sources which are mutually incoherent. As we shall see, this proves to be sufficient to produce large spectral changes in a wavelength interval of about 0.4 μm.

In order to take into account the differences that we introduce in the basic scheme, a few simple modifications have to be made in the analysis of James and Wolf. We shall refer to the scheme illustrated in fig. 1. A quasi-homogeneous source [3] lies in the plane $\xi\eta$. The source is made up of two mutually incoherent uniformly radiating strips parallel to one another and aligned to the η axis. The width of the strips and the distance between their axes are denoted by a and D_1, respectively. In a plane xy parallel to the plane $\xi\eta$ there is an opaque mask with two parallel slits aligned to the y axis. From now on, they will be termed Young slits. The width of the slits and the distance between their axes are denoted by b and D_2, respectively. Let L_1 be the distance between the two planes $\xi\eta$ and xy. Both the source strips and the Young slits are assumed to be very long so that all the quantities of interest are independent, to a good approximation, of the coordinates η and y. The plane containing the axes η, y and z (see fig. 1) is a symmetry plane for the system. At a distance L_2 from the mask there is the observation plane uv. We assume the plane uv to be in the far zone with respect to the mask plane. The latter, in turn, is assumed to be in the far zone with respect to the source plane. The power spectrum of the radiation arising from the Young slits is detected along the v axis. We now proceed to evaluate such a power spectrum.

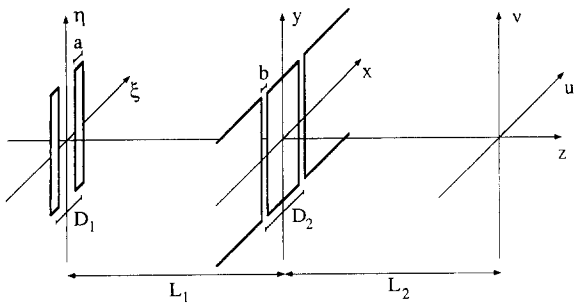

Fig. 1. Illustration of the geometry and notations used in this paper.

Let $W_M(x_1, x_2; \nu)$ be the cross spectral density at frequency ν of the field illuminating the mask. It can be written in the form [4]

$$W_M(x_1, x_2; \nu) = [G_M(x_1; \nu) G_M(x_2; \nu)]^{1/2} \mu_M(x_1, x_2; \nu) , \quad (1)$$

where G_M and μ_M denote the power spectrum and the degree of spectral coherence, respectively. We now turn to the power spectrum at the origin $u=0$ in the detection plane. We shall distinguish between the power spectrum that is observed when only a single slit is open [#2], $G_S(0; \nu)$, and the power spectrum produced by both slits, $G_B(0; \nu)$. The two spectra can be evaluated by means of the propagation laws for the cross spectral density and are given by the following expressions,

$$G_S(0; \nu) = \iint \tau_S^*(x_1) \tau_S(x_2) W_M(x_1, x_2; \nu) \times K^*(0, x_1; \nu) K(0, x_2; \nu) \, dx_1 \, dx_2 , \quad (2)$$

$$G_B(0; \nu) = \iint \tau_B^*(x_1) \tau_B(x_2) W_M(x_1, x_2; \nu) \times K^*(0, x_1; \nu) K(0, x_2; \nu) \, dx_1 \, dx_2 . \quad (3)$$

Here the asterisk denotes the complex conjugate. The functions τ_S and τ_B stand for the transmission functions for passage through a single and double slit, respectively, and K is a propagation kernel. Let us first discuss the form to be given to W_M. As the source consists of two quasi-homogeneous sources which are mutually incoherent, we can assume that G_M does not vary appreciably when we move across each slit and from one slit to the other (we recall that the Young slits are symmetrically located with respect to the ξz plane). The degree of spectral coherence of the field generated by our quasi-homogeneous source on the plane of the Young slits can be evaluated [3] by Fourier transforming the power spectrum distri-

[#2] For symmetry reasons, it does not matter which of the two slits is open.

bution across the source. Then, the complete expression for W_M turns out to be

$$W_M(x_1, x_2; \nu) = G_M(\nu) \operatorname{sinc}\left(\frac{a\nu(x_1 - x_2)}{cL_1}\right) \times \cos\left(\frac{\pi D_1 \nu(x_1 - x_2)}{cL_1}\right), \tag{4}$$

where, as usual, $\operatorname{sinc}(t) = \sin(\pi t)/\pi t$ and c stands for the speed of light. Because of the transversally one-dimensional character of our problem, the propagation kernel K takes the form

$$K(u, x; \nu) = \left(\frac{\nu}{cL_2}\right)^{1/2} \exp\left(-2\pi i \frac{ux\nu}{cL_2} - \tfrac{1}{4}\pi i\right), \tag{5}$$

where the small angle approximation has been used. It is seen that, as far as the integrals in (2) and (3) are concerned, K is a constant. Finally, τ_S and τ_B are given by the expressions

$$\tau_S(x) = \operatorname{rect}\left(\frac{x - \frac{1}{2}D_2}{b}\right), \tag{6}$$

$$\tau_B(x) = \operatorname{rect}\left(\frac{x - \frac{1}{2}D_2}{b}\right) + \operatorname{rect}\left(\frac{x + \frac{1}{2}D_2}{b}\right), \tag{7}$$

where $\operatorname{rect}(t)$ equals unity for $|t| \leqslant \frac{1}{2}$ and equals zero otherwise. On inserting eqs. (4)–(7) into eqs. (2) and (3) the spectra G_S and G_B can be evaluated. After some lengthy but straightforward calculations, we find that

$$G_S(0; \nu) = G_M(\nu) \frac{L_1}{\pi a L_2} [F(b, t_1) - F(b, t_2)], \tag{8}$$

$$\begin{aligned} G_B(0; \nu) = G_M(\nu) \frac{L_1}{\pi a L_2} \times \{ & [F(b + D_2, t_1) - F(b + D_2, t_2)] \\ & + [F(b - D_2, t_1) - F(b - D_2, t_2)] \\ & + 2[F(b, t_1) - F(b, t_2)] \\ & - [F(D_2, t_1) - F(D_2, t_2)] \}, \end{aligned} \tag{9}$$

where

$$t_1 = \frac{\pi\nu(D_1 - a)}{2cL_1}, \quad t_2 = \frac{\pi\nu(D_1 + a)}{2cL_1}, \tag{10}$$

and

$$F(\alpha, s) = \sin^2(\alpha s)/s - \alpha \operatorname{Si}(2\alpha s). \tag{11}$$

Here, Si stands for the sine integral function [5]. We shall shortly discuss eqs. (8) and (9). Before doing that, however, it is useful to consider some limiting case. This will make it easier to compare the present results to those of James and Wolf.

Let us suppose that the width of the Young slit is very small. More precisely, let us assume $b \ll 2cL_1/\nu a$. In this case, it can be shown that eqs. (8) and (9) can be replaced by the approximate formulas

$$G_S(0; \nu) = G_M(\nu) \frac{\nu b^2}{cL_2}, \tag{12}$$

$$G_B(0; \nu) = G_M(\nu) \frac{2\nu b^2}{cL_2} \times \left[1 + \operatorname{sinc}\left(\frac{a\nu D_2}{cL_1}\right) \cos\left(\frac{\pi \nu D_1 D_2}{cL_1}\right)\right], \tag{13}$$

In addition, if the two-strip source of fig. 1 is replaced by a single strip centered at the η axis, eq. (13) becomes

$$G_B(0; \nu) = G_M(\nu) \frac{2\nu b^2}{cL_2} \left[1 + \operatorname{sinc}\left(\frac{a\nu D_2}{cL_1}\right)\right]. \tag{14}$$

Eqs. (12) and (14) are similar to eqs. (12) and (13) of ref. [1]. The main differences are as follows. First, the diffraction induced factor in front of the square brackets is proportional to ν whereas a ν^2 factor appears in the corresponding formulas of ref. [1]. This is because our system is one-dimensional in the transverse direction. Second, we have a sinc function in eq. (14) instead of a function of the form $2J_1(...)/(...)$, where J_1 is the Bessel function of the first kind and first order. This is due to the passage from the circular source considered by James and Wolf to a strip source. Apart from these differences, our results agree closely with those of ref. [1].

On comparing eqs. (13) and (14) we can see the effect due to the use of a source made up of two strips instead of one. Such an effect is accounted for by the cosine factor within the square brackets in eq. (13). This factor stems from the structure of the degree of spectral coherence of the field produced by the double strip source and plays an important role in the experiment to be described. Thanks to this factor, we can make the degree of spectral coherence vary appreciably even in a limited frequency range. As a consequence, the power spectrum $G_S(0; \nu)$ produced

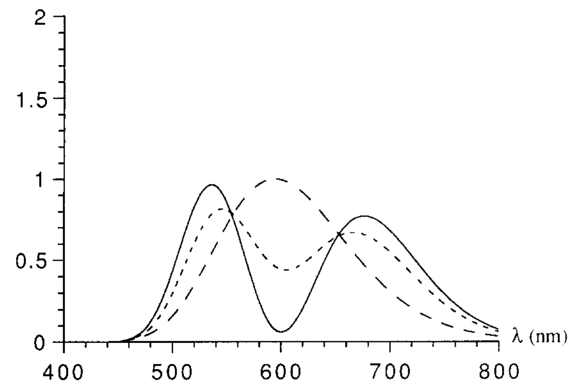

Fig. 2. Comparison between the theoretical spectra G_S (long-dash line) and G_B (full line for $a=0.01$ mm; short-dash line for $a=0.03$ mm) obtained from eqs. (8) and (9) with a source spectrum G_M of Gaussian form and with $D_1=0.3$ mm, $L_1=L_2=1$ m, $D_2=10$ mm, $b=0.1$ mm.

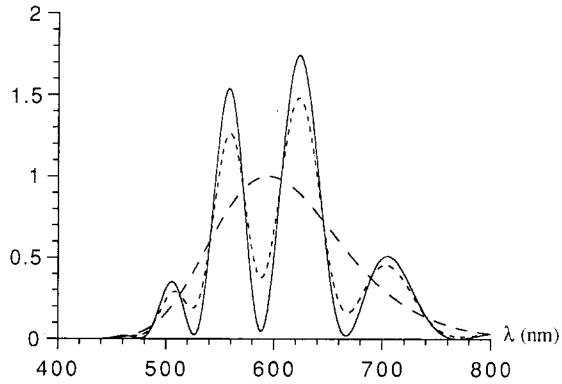

Fig. 3. Same as fig. 2 except that $D_1=1$ mm.

by a single slit can be very different from the power spectrum $G_B(0;\nu)$ due to both slits.

We now come back to eqs. (8) and (9). Basically, they describe the same phenomena as eqs. (12) and (13) but they can be used also when the Young slits have non-negligible width. Because of their structure, the significance of eqs. (8) and (9) is not immediately evident and is best illustrated by some examples. In fig. 2, a plot of eq. (8) is given by the long-dash line for a Gaussian power spectrum [#3] $G_M(\nu)$ centered at $\nu=5\times10^{14}$ Hz with a r.m.s. width $\sigma=0.5\times10^{14}$ Hz. We let $L_1=L_2=1$ m, $D_1=0.3$ mm, $D_2=10$ mm and $b=0.1$ mm. The full and short-dash lines are plots of the expressions given by eq. (9) for $a=0.01$ mm and $a=0.03$ mm, respectively. We see that an increase of the width of the source strips reduces the spectral change effects. Fig. 3 is analogous to fig. 2 except that $D_1=1$ mm. On comparing figs. 2 and 3 we see that an increase of the distance between the two source strips gives rise to more rapidly varying spectral modulations. To avoid misunderstanding, we point out that figs. 2 and 3 refer to variation of spectral intensity with wavelength and not with position. As a consequence, they have nothing to do with ordinary interference curves.

We present now some experimental results supporting the predictions of James and Wolf, taking into account the previously discussed differences. For the sake of brevity, the word spectrum will be used to mean normalized power spectrum [6].

The experimental set-up that we used basically reproduces the scheme of fig. 1. The two-strip source is obtained through a pair of slits back illuminated by an incandescent lamp. The lamp is fairly close to the slits, in order to ensure that they are incoherently illuminated. The Young slits are located in the far zone with respect to the source by means of a converging lens. Similarly, lenses are used to put the input plane of the spectrometer in the far zone of the Young mask. The entrance slit of the spectrometer is centered at the v axis (see fig. 1) and is much narrower than the mean period of the (polychromatic) Young fringes. Because of the low intensity available at the output of the spectrometer, a photomultiplier (Philips 56TVP) is used as a detector. The spectral response of the corresponding photocathode (S 20 response) covers, in a non-uniform way, the range of wavelengths from 300 to 700 nm. As a result, the detected spectrum is the product of the spectrum of the radiation incident on the spectrometer times the spectral response of the photocathode. We are interested in the comparison between the spectrum G_S produced by a single Young slit and the spectrum G_B produced when both slits are open. As a consequence, it is immaterial that the measured spectrum is affected by the spectral response of the detector.

In order to compare the experimental results with the formulas derived above, we proceeded as follows. First, the spectrum produced by a single Young

[#3] In our figures, the wavelength is used on the horizontal axis whereas the power spectrum is expressed as a Gaussian function of the frequency.

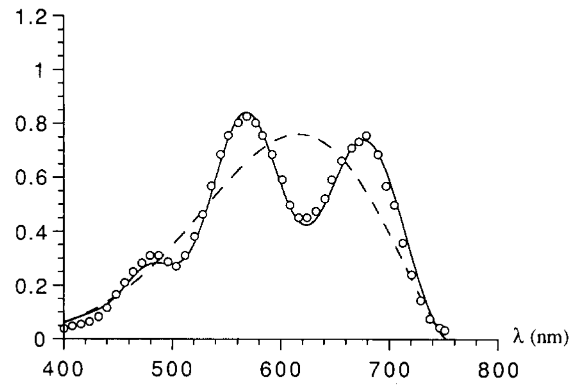

Fig. 4. Comparison between expected G_B spectrum (full line) and experimental values of G_B (circles) with $D_1=0.33$ mm, $a=0.024$ mm, $b=0.11$ mm and $D_2=3.4$ mm. The dashed line represents a best fitting curve for the spectrum G_S as determined by experiment.

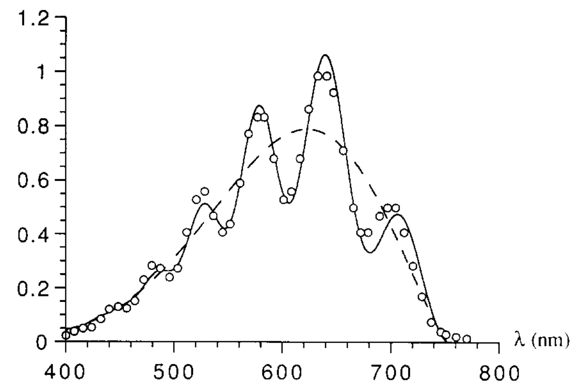

Fig. 5. Same as fig. 4 except that $D_1=0.68$ mm and $a=0.026$ mm.

slit was measured at a number of frequencies and a best fitting curve was taken as a numerical estimate of the function $G_S(0,\nu)$. An estimate of $G_M(\nu)$ was then obtained by using eq. (8) and the spectrum $G_B(0,\nu)$ was evaluated by means of eq. (9).

Two examples are shown in figs. 4 and 5. Experimental spectra G_S are drawn by dashed lines. Full lines give the expected G_B spectra, whereas the circles correspond to the experimental values of the G_B spectra. In fig. 4, the slits used to synthesize the two-strip source had a width $a=0.024$ mm and their centers were spaced by $D_1=0.33$ mm. In fig. 5, we had $a=0.026$ mm and $D_1=0.68$ mm. For both cases, the Young slits had a width $b=0.11$ mm with their centers at a distance $D_2=3.4$ mm. The experimental uncertainties are due to several factors, such as errors in the measurement of geometrical quantities, slight differences between widths of the slits used for the source and similar differences between the Young slits, precision of the spectrometer. An estimate of the error size is given by the diameters of the circles. It is seen that the agreement between the experimental results and the theoretical predictions is satisfactory.

The effect discussed in the present Letter belongs to a rather large class of phenomena that have received attention in recent years. They can be traced back to the fact that the customarily used phrase "power spectrum of a radiation field" has actually an ambiguous meaning, the power spectrum being, in general, a function of the observation point [4,7,8]. This has far reaching consequences, as was first brought into evidence by Wolf [6,9] and was subsequently confirmed in several instances, both theoretically and experimentally [10–22].

The present effect should not be confused with the coloured appearance of the fringes in an interferometer illuminated with broadband light. As a matter of fact, it is detected at the very location (the center of the interference pattern) where the so-called "white" fringe is expected to appear.

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