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Full length article

Spectrum invariance on paraxial propagation

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We investigate conditions ensuring spectral invariance on paraxial propagation of the radiation field produced by a partially coherent source. We find a sufficient condition that reduces to Wolf's scaling law in the case of quasi homogeneous sources.

1. Introduction

Spectrum variations in the course of propagation of partially coherent fields have been known for some time [1–3]. Not long ago, Wolf [4] found a condition, namely the scaling law, ensuring that the normalized spectrum in the far field of a quasi homogeneous source does not depend on the observation point. Wolf also showed that violations of the scaling law have far reaching consequences [5,6] some of whom have been subsequently verified experimentally [7–10]. In a recent paper [11], it has been shown that for quasi homogeneous sources possessing the same power spectrum at any source point the scaling law is sufficient to guarantee spectral invariance not only in the far field but also in the near field whenever paraxial propagation can be assumed.

In this paper, we investigate whether spectral invariance in the course of paraxial propagation can be obtained for sources not necessarily described by the quasi homogeneous model. We find a simple sufficient condition that guarantees spectral invariance. When applied to quasi homogeneous sources, such a condition reduces to the scaling law. However, it holds true as well for other classes of sources that we shall illustrate through examples.

2. Paraxial propagation of the cross spectral density

Let us consider a planar, partially coherent secondary source. We denote by $W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k)$ the cross spectral density [2] across the source plane. Here, $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are the position vectors of two typical source points and k is the wavenumber. We choose the plane of the source as the plane $z=0$ of a suitable reference frame and we denote by $W_z(\boldsymbol{r}_1, \boldsymbol{r}_2, k)$ the cross spectral density, in a plane $z=\text{const}>0$, between two points whose position vectors are $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ (see fig. 1).

In the paraxial regime [#1], the following propagation law holds,

$$W_z(\boldsymbol{r}_1, \boldsymbol{r}_2, k) = \left(\frac{k}{2\pi z}\right)^2 \iint W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) \exp\left(-\frac{ik}{2z}\left[(\boldsymbol{r}_1-\boldsymbol{\rho}_1)^2-(\boldsymbol{r}_2-\boldsymbol{\rho}_2)^2\right]\right) d^2\rho_1\, d^2\rho_2\,. \quad (2.1)$$

Here and in the following, infinite limits of integration are assumed unless otherwise indicated. The power spectrum at a point $\boldsymbol{r}$, say $S_z(\boldsymbol{r}, k)$, is obtained from eq. (2.1) by letting $\boldsymbol{r}_1=\boldsymbol{r}_2=\boldsymbol{r}$. The resulting expression for S_z can be written as

[#1] Conditions under which paraxial propagation can be assumed are discussed in ref. [11].

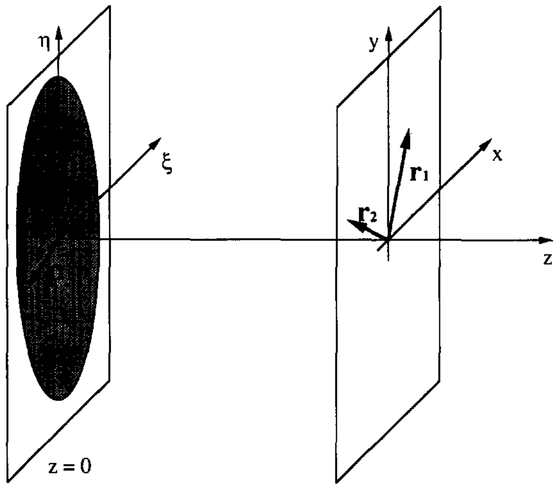

Fig. 1. Illustration of the notations used in this paper.

$$S_z(\boldsymbol{r}, k) = \left(\frac{k}{2\pi z}\right)^2 \iint W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) \exp\left(-\frac{\mathrm{i}k}{z}\left[\tfrac{1}{2}(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)\cdot(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) - \boldsymbol{r}\cdot(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)\right]\right) \mathrm{d}^2\rho_1\, \mathrm{d}^2\rho_2 , \tag{2.2}$$

where some terms have been rearranged in the exponential function. Eq. (2.2) can be written in an alternative way by introducing the variables

$$\boldsymbol{\sigma} = \tfrac{1}{2}(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2), \quad \boldsymbol{\tau} = k(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) . \tag{2.3}$$

On substituting from eqs. (2.3) into eq. (2.2) we obtain

$$S_z(\boldsymbol{r}, k) = \left(\frac{1}{2\pi z}\right)^2 \iint W_0(\boldsymbol{\sigma} + \boldsymbol{\tau}/2k, \boldsymbol{\sigma} - \boldsymbol{\tau}/2k, k) \exp[-(\mathrm{i}/z)\boldsymbol{\tau}\cdot(\boldsymbol{\sigma} - \boldsymbol{r})]\, \mathrm{d}^2\sigma\, \mathrm{d}^2\tau . \tag{2.4}$$

Eq. (2.4) will be the starting point of our analysis. Before we proceed, it is important to distinguish between the power spectrum and the normalized spectrum. The latter, say $s_z(\boldsymbol{r}, k)$, is defined as

$$s_z(\boldsymbol{r}, k) = S_z(\boldsymbol{r}, k) \Big/ \int_0^\infty S_z(\boldsymbol{r}, k)\, \mathrm{d}k . \tag{2.5}$$

Obviously, the normalized spectrum satisfies the condition

$$\int_0^\infty s_z(\boldsymbol{r}, k)\, \mathrm{d}k = 1 , \tag{2.6}$$

for any choice of z and $\boldsymbol{r}$. In other words, the normalized spectrum describes only the shape of the spectral curve regardless of the power content of the radiation.

3. The requirement of spectral invariance on propagation

We require that the normalized spectrum is independent from $\boldsymbol{r}$ across any plane $z = \text{const.} \geqslant 0$, or

$$s_z(\mathbf{r}, k) = \Sigma_z(k) . \tag{3.1}$$

Let us define the function

$$I_z(\mathbf{r}) = \int_0^\infty S_z(\mathbf{r}, k)\, \mathrm{d}k . \tag{3.2}$$

This function can be thought of as the optical intensity at the point $\mathbf{r}$ in the plane $z=$const. On inserting from eqs. (3.1) and (3.2) into eq. (2.5) we obtain the following requirement for spectral invariance,

$$S_z(\mathbf{r}, k) = \Sigma_z(k) I_z(\mathbf{r}) . \tag{3.3}$$

Therefore, at any plane $z=$const., the power spectrum has to factorize into the product of two functions. One of these functions, namely $I_z(\mathbf{r})$, depends on z and $\mathbf{r}$ whereas the other depends on z and k. It is to be noted that eqs. (3.1) and (3.3) must hold even in the limit $z\to 0$. Accordingly, the power spectrum satisfies the condition in the source plane

$$S_0(\boldsymbol{\rho}, k) = \Sigma_0(k) I_0(\boldsymbol{\rho}) . \tag{3.4}$$

We see from eq. (3.4) that any source satisfying the spectral invariance requirement (3.1) must possess the same normalized power spectrum (although not necessarily the same power spectrum) at any source point. This is to be contrasted to the case in which spectral invariance is required in the far field only. In that case, the invariance property can be exhibited by sources whose normalized spectrum changes across the source plane.

We can further prove that the function $\Sigma_z(k)$ is actually independent from z, i.e.

$$\Sigma_z(k) = \Sigma_0(k) . \tag{3.5}$$

To prove this, let us integrate eq. (2.2) in the plane $z=$const. We then obtain

$$\int S_z(\mathbf{r}, k)\, \mathrm{d}^2 r = \iint W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) \exp[-(\mathrm{i}k/2z)\,(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)\cdot(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)]\, \mathrm{d}^2\rho_1\, \mathrm{d}^2\rho_2 \times \int \exp\,[2\pi\mathrm{i}(\mathbf{r}/\lambda z)\cdot(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)]\, \mathrm{d}^2(r/\lambda z) , \tag{3.6}$$

where $\lambda = (2\pi)/k$. The rightmost integral in eq. (3.6) equals the Dirac function $\delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$. As a consequence, eq. (3.6) becomes

$$\int S_z(\mathbf{r}, k)\, \mathrm{d}^2 r = \int W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_1, k)\, \mathrm{d}^2\rho_1 = \int S_0(\boldsymbol{\rho}_1, k)\, \mathrm{d}^2\rho_1 . \tag{3.7}$$

On inserting from eqs. (3.3) and (3.4) into eq. (3.7) we obtain

$$\Sigma_z(k) \int I_z(\mathbf{r})\, \mathrm{d}^2 r = \Sigma_0(k) \int I_0(\boldsymbol{\rho})\, \mathrm{d}^2 \rho . \tag{3.8}$$

This proves that the two functions $\Sigma_z(k)$ and $\Sigma_0(k)$ are proportional to each other. Actually, the proportionality factor is unity because the integral of both $\Sigma_z(k)$ and $\Sigma_0(k)$ with respect to k must be unity (see eqs. (2.6) and (3.1)).

4. A sufficient condition for spectral invariance

We can write the spectral invariance requirement (3.3) in the form

$$\left(\frac{1}{2\pi z}\right)^2 \iint F_0(\boldsymbol{\sigma}, \boldsymbol{\tau}, k) \exp[-(\mathrm{i}/z)\boldsymbol{\tau}\cdot(\boldsymbol{\sigma} - \mathbf{r})]\, \mathrm{d}^2\sigma\, \mathrm{d}^2\tau = \Sigma_0(k) I_z(\mathbf{r}) , \tag{4.1}$$

where eqs. (2.4) and (3.5) have been used and where

$$F_0(\boldsymbol{\sigma}, \boldsymbol{\tau}, k) = W_0(\boldsymbol{\sigma}+\boldsymbol{\tau}/2k, \boldsymbol{\sigma}-\boldsymbol{\tau}/2k, k) . \tag{4.2}$$

In order to satisfy eq. (4.1) for any value of z, we can require that $F_0(\boldsymbol{\sigma}, \boldsymbol{\tau}, k)$ factorizes into the product of $\Sigma_0(k)$ times a function of $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$, say $H_0(\boldsymbol{\sigma}, \boldsymbol{\tau})$,

$$F_0(\boldsymbol{\sigma}, \boldsymbol{\tau}, k) = \Sigma_0(k) H_0(\boldsymbol{\sigma}, \boldsymbol{\tau}) . \tag{4.3}$$

Eq. (4.3) expresses a sufficient condition for the source in order to produce a normalized spectrum that is invariant through propagation in the half-space $z \geqslant 0$.

Taking into account eqs. (2.3), (4.2) and (4.3) we see that the cross spectral density across the source has the form

$$W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) = \Sigma_0(k) H_0[\tfrac{1}{2}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2), k(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)] . \tag{4.4}$$

In particular, the power spectrum across the source is given by

$$S_0(\boldsymbol{\rho}, k) = \Sigma_0(k) H_0(\boldsymbol{\rho}, 0) . \tag{4.5}$$

As a consequence, the degree of spectral coherence [2], namely

$$\mu_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) = W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) / [S_0(\boldsymbol{\rho}_1, k) S_0(\boldsymbol{\rho}_2, k)]^{1/2} , \tag{4.6}$$

is given by

$$\mu_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) = H_0[\tfrac{1}{2}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2), k(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)] / [H_0(\boldsymbol{\rho}_1, 0) H_0(\boldsymbol{\rho}_2, 0)]^{1/2} . \tag{4.7}$$

Let us recall that the scaling law [4], found for quasi homogeneous sources, requires that the degree of spectral coherence μ_0 depends on the argument $k(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$ only. We see from eq. (4.7) that the present condition for spectral invariance, namely eq. (4.4), implies for μ_0 a rather more general form. However, as far as the dependence of μ_0 on k is concerned, eq. (4.7) is of the same form of the scaling law and can be considered a generalized scaling law.

5. Examples

(a) Let us first consider the class of quasi homogeneous sources [12]. In this case, the cross spectral density across the source can be approximated by

$$W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) = S_0(\tfrac{1}{2}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2), k) \mu_0(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2, k) , \tag{5.1}$$

where $S_0(\boldsymbol{\rho}, k)$ varies much more slowly with $\boldsymbol{\rho}$ than $\mu_0(\boldsymbol{\rho}', k)$ varies with $\boldsymbol{\rho}'$. On inserting from eq. (3.4) into eq. (5.1) we obtain

$$W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k) = \Sigma_0(k) I_0(\tfrac{1}{2}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)) \mu_0(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2, k) . \tag{5.2}$$

We have seen that the sufficient condition (4.4) for spectral invariance is satisfied if μ_0 depends on k through the variable $k(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$ only (see eq. (4.7)). Accordingly, in eq. (5.2) μ_0 must have the form

$$\mu_0(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2, k) = g_0[k(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)] , \tag{5.3}$$

where $g_0(0)=1$. Eq. (5.3) expresses the Wolf's scaling law. The present result agrees with that of ref. [11]. Note however that now only the normalized power spectrum is required to be the same at any source point whereas the power spectrum itself can differ from one source point to another (see eq. (3.4)).

(b) A factorization property of the form (5.1) can hold under more general conditions than those required for quasi homogeneous sources [13]. More precisely, we can suppose the cross spectral density across the source to be given by the formula

$$W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=S_0(\tfrac{1}{2}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2),k)M_0(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2,k)\,, \tag{5.4}$$

where, without loss of generality, M_0 has been assumed to be unity for $\boldsymbol{\rho}_1=\boldsymbol{\rho}_2$. Using eq. (3.4) we see that W_0 has the form

$$W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=\Sigma_0(k)I_0(\tfrac{1}{2}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2),k)M_0(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2,k)\,. \tag{5.5}$$

The difference between eqs. (5.2) and (5.5) lies in the fact that M_0 is not necessarily equal to the degree of spectral coherence. In fact, we have

$$\mu_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=\frac{I_0(\tfrac{1}{2}(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2))}{[I_0(\boldsymbol{\rho}_1)I_0(\boldsymbol{\rho}_2)]^{1/2}}M_0(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2,k)\,. \tag{5.6}$$

As a consequence, M_0 coincides with μ_0 only if the quasi homogeneous hypothesis applies, i.e. if $I_0(\boldsymbol{\rho})$ is a slow function of $\boldsymbol{\rho}$ and $M_0(\boldsymbol{\rho}',k)$ is a fast function of $\boldsymbol{\rho}'$. The spectral invariance condition (4.4) requires that M_0 is a function of $k(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$. Let us denote such a function by L_0, i.e

$$M_0(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2,k)=L_0[k(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)]\,. \tag{5.7}$$

As an example, let us consider a gaussian Schell-model source [14]. The pertaining cross spectral density is given by

$$W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=\Sigma_0(k)\exp\left(-\frac{\rho_1^2+\rho_2^2}{4\sigma_s^2(k)}\right)\exp\left(-\frac{(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2}{2\sigma_\mu^2(k)}\right). \tag{5.8}$$

At any temporal frequency, both the power spectrum and the degree of spectral coherence across the source are gaussianly shaped with variances $\sigma_s^2(k)$ and $\sigma_\mu^2(k)$ respectively. We can easily show that eqs. (5.5) and (5.7) can be satisfied through a suitable choice of σ_s^2 and σ_μ^2. In fact, eq. (5.8) can be written as

$$W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=\Sigma_0(k)\exp\left[-\frac{1}{2\sigma_s^2(k)}\left(\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{2}\right)^2\right]\exp\left[-\left(\frac{1}{2\sigma_\mu^2(k)}+\frac{1}{8\sigma_s^2(k)}\right)(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2\right]. \tag{5.9}$$

In order to satisfy eqs. (5.5) and (5.7) it is sufficient to let

$$\sigma_s^2(k)=\sigma_s^2,\quad 1/\sigma_\mu^2(k)+1/4\sigma_s^2=k^2/a^2\,, \tag{5.10}$$

where a is a suitable constant. This means that the variance σ_s^2 of the power spectrum is independent from k whereas the variance σ_μ^2 of the degree of spectral coherence is the following function of k,

$$\sigma_\mu^2(k)=4\sigma_s^2a^2/(4\sigma_s^2k^2-a^2)\,. \tag{5.11}$$

Eq. (5.11) requires that $2\sigma_s k>a$ for the range of values of k for which the normalized spectrum $\Sigma_0(k)$ is different from zero. It is immediately seen that this condition is compatible with vastly different coherence characteristics of the source. When a is much smaller than $2\sigma_s k$ the quasi homogeneous limit is obtained. On the other hand, when a approaches $2\sigma_s k$, the source becomes highly coherent.

(c) As a last example, we consider a cross spectral density that satisfies condition (4.4) but that cannot be factorized in the form expressed by eq. (5.5). In ref. [15] the existence of a class of sources that emit fields that are shape-invariant through propagation was shown. Their cross spectral densities can be expanded in terms of the Hermite–Gauss functions, very well known from the laser theory. These expansions, for two-dimensional sources, are of the form [16]

$$W(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=\sum_{n,h=0}^{\infty}\beta_{nh}(k)G_{nh}(\boldsymbol{\rho}_1,k)G_{nh}(\boldsymbol{\rho}_2,k)\,, \tag{5.12}$$

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where

$$G_{nh}(\boldsymbol{\rho}, k)=\frac{1}{v_0}\sqrt{\frac{2}{\pi 2^{n+h} n! h!}}\, H_n(\sqrt{2}\,\xi/v_0)\, H_h(\sqrt{2}\,\eta/v_0)\exp[-(\xi^2+\eta^2)/v_0^2]\,, \tag{5.13}$$

and ξ and η are the cartesian components of the position vector $\boldsymbol{\rho}$ on the source plane. In eq. (5.13) the function H_n is nth Hermite polynomial and the parameter v_0 represents the spot-size of the zeroth-order mode [17] and contains implicitly the dependence on k.

For the first two functions of this class, namely $W^{(0)}$ and $W^{(1)}$, the expansion coefficients are given by

$$\beta_{nh}^{(0)}(k)=\beta_0(k)\,[q(k)]^{n+h}\,, \tag{5.14}$$

$$\beta_{nh}^{(1)}(k)=(n+h)\beta_0(k)\,[q(k)]^{n+h-1}\,, \tag{5.15}$$

respectively, where $\beta_0(k)$ is an arbitrary positive function of k and

$$0<q(k)<1, \quad \forall k\,. \tag{5.16}$$

The closed forms of these two functions are respectively given by

$$W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)=\frac{\beta_0}{v_0^2}\frac{2}{\pi(1-q^2)}\exp[-p^2(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)^2-m^2(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2]\,, \tag{5.17}$$

$$W^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)=\frac{\beta_0}{v_0^2}\frac{4}{\pi(1-q^2)^2}\,[q+p^2(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)^2-m^2(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2]\exp[-p^2(\boldsymbol{\rho}_1+\boldsymbol{\rho}_2)^2-m^2(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)^2], \tag{5.18}$$

where we introduced for convenience the notations

$$p^2=\frac{1}{2v_0^2}\left(\frac{1-q}{1+q}\right), \quad m^2=\frac{1}{2v_0^2}\left(\frac{1+q}{1-q}\right), \tag{5.19}$$

and we omitted the explicit dependence on k.

We already considered the case of $W^{(0)}$ in the previous example. As a matter of fact, this cross spectral density exactly corresponds to the one produced by a gaussian Schell-model source, if one simply sets

$$p^2=\frac{1}{8\sigma_s^2}, \quad m^2=\frac{1}{8\sigma_s^2}+\frac{1}{2\sigma_\mu^2}, \tag{5.20}$$

as can be seen by comparison of eq. (5.9) and eq. (5.17). From the previous example it follows that, by requesting (see eqs. (5.10))

$$p^2=\text{const.}, \quad m^2=k^2/2a^2\,, \tag{5.21}$$

the cross spectral density produced by this kind of source automatically satisfies eq. (4.4). One way to satisfy eqs. (5.21) is by imposing, for example,

$$\frac{1+q}{1-q}=\frac{k}{k_0}, \quad \frac{1}{2v_0^2}=\frac{1}{w_0^2}\frac{k}{k_0}\,, \tag{5.22}$$

where k_0 and w_0 are arbitrary positive constants (see eqs. (5.19)).

On the contrary, if we substitute from eqs. (5.22) into eq. (5.18), the resulting expression of $W^{(1)}$ is not in the form of eq. (4.4), because of the presence of the linear term in q.

Let us consider instead a linear combination of $W^{(0)}$ and $W^{(1)}$ of the form

$$W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)=\alpha(q)\,W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)+W^{(1)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\,, \tag{5.23}$$

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and let us demonstrate that such a cross spectral density can be expressed in the form (4.4) by suitably choosing the function $\alpha(q)$.

By imposing the relation

$$\alpha(q)=2(c-q)/(1-q^2)\,, \tag{5.24}$$

where c is an arbitrary constant, and substituting from eqs. (5.17), (5.18), (5.19), (5.22) and (5.24) into eq. (5.23), we obtain

$$W_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=\frac{k_0}{2\pi w_0^2}\left(\frac{k}{k_0}+1\right)^4\frac{\beta_0(k)}{k}\left[c+\left(\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{w_0}\right)^2-\left(\frac{k}{k_0}\right)^2\left(\frac{\boldsymbol{\rho}_1-\boldsymbol{\rho}_2}{w_0}\right)^2\right]$$
$$\times\exp\left[-\left(\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{w_0}\right)^2-\left(\frac{k}{k_0}\right)^2\left(\frac{\boldsymbol{\rho}_1-\boldsymbol{\rho}_2}{w_0}\right)^2\right], \tag{5.25}$$

which is exactly of the form (4.4), with

$$\Sigma_0(k)=\frac{k_0}{2\pi w_0^2}\left(\frac{k}{k_0}+1\right)^4\frac{\beta_0(k)}{k}\,, \tag{5.26}$$

and

$$H_0(\boldsymbol{\sigma},\boldsymbol{\tau})=[c+4(\boldsymbol{\sigma}/w_0)^2-(\boldsymbol{\tau}/k_0w_0)^2]\exp[-4(\boldsymbol{\sigma}/w_0)^2-(\boldsymbol{\tau}/k_0w_0)^2]\,. \tag{5.27}$$

To gaurantee the nonnegativity of the integral operator whose kernel is given by W_0, we must request that $\alpha(q)>0$ for all the wavelengths that belong to the spectrum of the source. Recalling eq. (5.24), the previous condition implies $c>q$. Taking into account eq. (5.16), if $c>1$ this condition is automatically satisfied.

Using eqs. (4.5) and (4.7), the power spectrum and the degree of spatial coherence of this source assume the following expressions:

$$S_0(\boldsymbol{\rho},k)=\Sigma_0(k)\,[c+4(\boldsymbol{\rho}/w_0)^2]\exp[-4(\boldsymbol{\rho}/w_0)^2]\,, \tag{5.28}$$

$$\mu_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)=\left[c+\left(\frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{w_0}\right)^2-\left(\frac{k}{k_0}\right)^2\left(\frac{\boldsymbol{\rho}_1-\boldsymbol{\rho}_2}{w_0}\right)^2\right]\left\{\left[c+4\left(\frac{\boldsymbol{\rho}_1}{w_0}\right)^2\right]\left[c+4\left(\frac{\boldsymbol{\rho}_2}{w_0}\right)^2\right]\right\}^{-1/2}\exp\left\{\left[1-\left(\frac{k}{k_0}\right)^2\right]\left(\frac{\boldsymbol{\rho}_1-\boldsymbol{\rho}_2}{w_0}\right)^2\right\}. \tag{5.29}$$

In fig. 2 we show the behaviour of $\mu_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)$ as a function of ξ_1 for different values of ξ_2 (letting $\eta_1=\eta_2=0$),

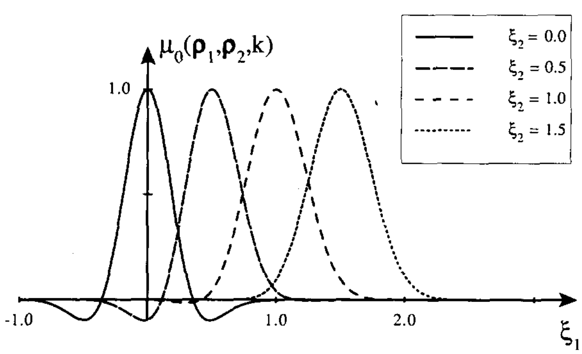

Fig. 2. Degree of spectral coherence $\mu_0(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,k)$ given by eq. (5.29) as a function of ξ_1 for different values of ξ_2 (letting $\eta_1=\eta_2=0$), with $k/k_0=3$ and $w_0=1$.

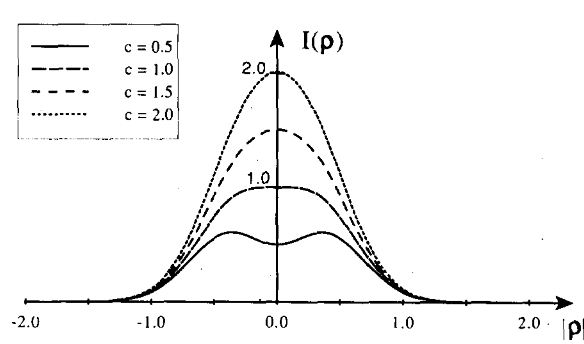

Fig. 3. Distribution of the optical intensity $I_0(\boldsymbol{\rho})=S_0(\boldsymbol{\rho},k)/\Sigma_0(k)$, as a function of $|\boldsymbol{\rho}|$, for different values of c, with $w_0=1$.

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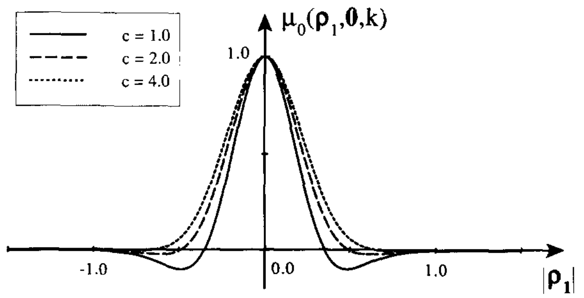

Fig. 4. Degree of spectral coherence $\mu_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k)$ for different values of c (with $k/k_0=3$, $w_0=1, \boldsymbol{\rho}_2=0$).

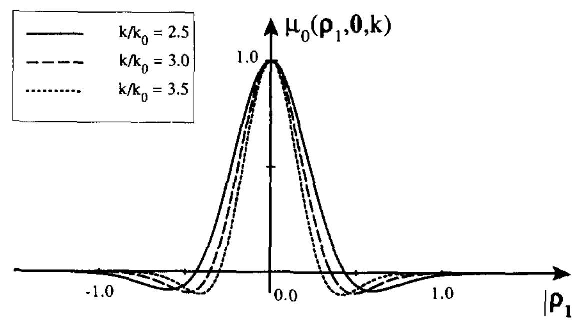

Fig. 5. Degree of spectral coherence $\mu_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k)$ for different wavenumbers (with $c=1$, $w_0=1, \boldsymbol{\rho}_2=0$).

while keeping fixed the value of k/k_0. As it is seen, the degree of coherence does not depend on $(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)$ only, as was the case for the previous examples.

The distribution of the optical intensity $I_0(\boldsymbol{\rho})=S_0(\boldsymbol{\rho}, k)/\Sigma_0(k)$ is represented in fig. 3 for different values of the constant c: by increasing c the distribution tends to assume a gaussian profile, as the contribution of $W^{(0)}$ becomes predominant in the linear combination (5.23). For the same reason also $\mu_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k)$ tends to a gaussian curve as c increases (fig. 4).

At last, in fig. 5 we show the dependence of $\mu_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, k)$ on k. As it is seen, the width of the distribution decreases by increasing k or, equivalently, by decreasing the wavelength of the radiation.

6. Conclusions

It is known that partially coherent sources emit fields whose spectra generally change on propagation. In this paper we introduced a condition which, in paraxial approximation, ensures spectral invariance of fields emitted by partially coherent sources. This condition can be satisfied by a wide class of sources, as it was shown by examples, and reduces to the scaling law in the special case of quasi homogeneous sources.

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