# A SCATTERING EXPERIMENT WITH PARTIALLY COHERENT LIGHT

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Received 18 September 1989

A scattering experiment is described in which the spatial coherence properties of the incident light can be changed. The effect of spatial coherence on the angular distribution of the scattered radiation is studied. The results agree with the predictions of a recent analysis of this phenomenon [J. Jannson, T. Jannson and E. Wolf, Optics Letters 13 (1988) 1060].

## 1. Introduction

A recent theoretical analysis [1] has shown that the angular distribution of radiation scattered by a deterministic or by a spatially random medium depends on the degree of spatial coherence of the radiation incident on the medium. This gives rise to the possibility of discriminating between laser radiation and other types of radiation by utilizing the differences between their spatial coherence properties.

In this paper, we describe a scattering experiment which supports the conclusion of ref. [1]. In this experiment, a scattering medium is illuminated with radiation of different spatial coherence characteristics. We find that the angular distribution of the scattered light broadens when the degree of coherence of the field incident on the scatterer is reduced.

In ref. [1] a three-dimensional scatterer was considered whereas we use a planar scatterer. Accordingly, the theory of ref. [1] will first be adapted to our case (sec. 2). The experiment and the results will then be described (sec. 3).

## 2. Theory

A partially coherent field is incident on a planar linear scatterer located in the first focal plane of a converging lens. The (spectral) intensity distribution in the second focal plane is observed. We shall derive an expression for this intensity distribution, within the accuracy of the paraxial approximation. Our derivation parallels that of ref. [1] with some obvious changes.

Let $W^{(i)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$ be the cross spectral density [2] of the incident field between two typical points with position vectors $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ in the plane of the scatterer, at temporal frequency $\omega$. According to the theory of coherence in the space frequency domain [3], the cross spectral density can be expressed in the form

$$W^{(i)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = \langle U^{(i)}(\boldsymbol{r}_1, \omega)\, U^{(i)}(\boldsymbol{r}_2, \omega) \rangle_\omega , \tag{1}$$

where the angular brackets, with the subscript $\omega$, denote a statistical average taken over an ensemble $\{U^{(i)}(\boldsymbol{r}, \omega)\exp(-\mathrm{i}\omega t)\}$ of frequency-dependent monochromatic realizations. Let us denote by $U(\boldsymbol{R}, \omega)$ the field at a typical point, whose position vector is $\boldsymbol{R}$, in the back focal plane obtained when a given realization $U^{(i)}(\boldsymbol{r}, \omega)$ is incident on the scatterer. The field $U(\boldsymbol{R}, \omega)$ can be evaluated as follows [4],

$$U(\boldsymbol{R}, \omega) = -\mathrm{i}\frac{\omega}{cf}\int U^{(i)}(\boldsymbol{r}, \omega) \times T(\boldsymbol{r})\exp[-\mathrm{i}(\omega/cf)\boldsymbol{r}\cdot\boldsymbol{R}]\,\mathrm{d}^2 r . \tag{2}$$

Here $f$ is the focal length of the lens, $c$ is the speed of light in the medium surrounding the lens and $T(\boldsymbol{r})$ is the transmission function of the planar scatterer. We assume that $T(\boldsymbol{r})$ is frequency-independent at least over the range of temporal frequencies contained in the incident field. The cross spectral den-

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sity of the field in the back focal plane of the lens can be expressed in a form similar to eq. (1), viz.,

$$W(\boldsymbol{R}_1, \boldsymbol{R}_2, \omega) = \langle U^*(\boldsymbol{R}_1, \omega)\, U(\boldsymbol{R}_2, \omega) \rangle_\omega , \tag{3}$$

and can be easily evaluated. On inserting from eqs. (2) and (1) into eq. (3), we find that

$$W(\boldsymbol{R}_1, \boldsymbol{R}_2, \omega) = \frac{\omega^2}{c^2 f^2} \iint W^{(i)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)\, T^*(\boldsymbol{r}_1) T(\boldsymbol{r}_2) \times \exp[-i(\omega/cf)(\boldsymbol{r}_2 \cdot \boldsymbol{R}_2 - \boldsymbol{r}_1 \cdot \boldsymbol{R}_1)]\, d^2 r_1\, d^2 r_2 . \tag{4}$$

In particular, the spectral intensity at a point $\boldsymbol{R}$, say $I(\boldsymbol{R}, \omega)$, is obtained from eq. (4) by letting $\boldsymbol{R}_1 = \boldsymbol{R}_2 = \boldsymbol{R}$. One then obtains for $I(\boldsymbol{R}, \omega)$ the expression

$$I(\boldsymbol{R}, \omega) = \frac{\omega^2}{c^2 f^2} \iint W^{(i)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)\, T^*(\boldsymbol{r}_1) T(\boldsymbol{r}_2) \times \exp[-i(\omega/cf)(\boldsymbol{r}_2 - \boldsymbol{r}_1) \cdot \boldsymbol{R}]\, d^2 r_1\, d^2 r_2 . \tag{5}$$

This is the intensity distribution produced in the back focal plane by a deterministic scatterer and is seen to depend on the spatial coherence characteristics of the illuminating field. Suppose now that the intensity is averaged over an ensemble of scatterers. Eq. (5) then becomes

$$I(\boldsymbol{R}, \omega) = \frac{\omega^2}{c^2 f^2} \iint W^{(i)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) C(\boldsymbol{r}_1, \boldsymbol{r}_2) \times \exp[-i(\omega/cf)(\boldsymbol{r}_2 - \boldsymbol{r}_1) \cdot \boldsymbol{R}]\, d^2 r_1\, d^2 r_2 , \tag{6}$$

where

$$C(\boldsymbol{r}_1, \boldsymbol{r}_2) = \langle T^*(\boldsymbol{r}_1)\, T^*(\boldsymbol{r}_2) \rangle \tag{7}$$

is the correlation function of the transmission function $T(\boldsymbol{r})$. Here, the average is taken over the ensemble of scatterers. The intensity distribution is again seen to depend on the cross-spectral density of the field incident on the scatterer.

We shall now evaluate eq. (6) for the case in which $W^{(i)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega)$ and $C(\boldsymbol{r}_1, \boldsymbol{r}_2)$ have the forms

$$W^{(i)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \omega) = I^{(i)}(\omega) \exp\left(-\frac{r_1^2 + r_2^2}{4\sigma_I^2}\right) \exp\left(-\frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2}{2\sigma_\mu^2}\right), \tag{8}$$

$$C(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{A}{2\pi\sigma_T^2} \exp\left(-\frac{(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2}{2\sigma_T^2}\right), \tag{9}$$

i.e. we assume that the illuminating field across the scatterer is a (secondary) gaussian Schell model source [5,6], with frequency-independent variances $\sigma_I^2$ and $\sigma_\mu^2$ of the intensity and of the degree of spatial coherence, respectively. The correlation function of the scatterer is also assumed to be gaussian with variance $\sigma_T^2$. On inserting from eqs. (8) and (9) into eq. (6) we obtain, after some straightforward calculations, the following expression for the spectral intensity at the point $\boldsymbol{R}$,

$$I(\boldsymbol{R}, \omega) = I_M(\omega) \exp[-R^2/2\sigma^2(\omega)] , \tag{10}$$

where

$$I_M(\omega) = \frac{8\pi A I^{(i)}(\omega)\, \omega^2 \sigma_I^4 \sigma_\mu^2}{c^2 f^2 (\sigma_\mu^2 \sigma_T^2 + 4\sigma_I^2 \sigma_T^2 + 4\sigma_I^2 \sigma_\mu^2)} , \tag{11}$$

$$\sigma^2(\omega) = \frac{c^2 f^2}{2\omega^2} \left( \frac{1}{4\sigma_I^2} + \frac{1}{\sigma_\mu^2} + \frac{1}{\sigma_T^2} \right) . \tag{12}$$

The formula (10) shows that the intensity distribution in the back focal plane of the lens is a gaussian curve. It is seen from eq. (12) that, for a given value of $\sigma_T$, the variance $\sigma^2(\omega)$ depends on both $\sigma_I$ and $\sigma_\mu$. In particular, if we fix the value of $\sigma_I$, the variance $\sigma^2(\omega)$ reaches a minimum for coherent illumination ($\sigma_\mu \to \infty$), whereas it diverges in the incoherent limit ($\sigma_\mu \to 0$).

As far as comparison with the experiment is concerned, we note that both the maximum value and the variance of $I(\boldsymbol{R}, \omega)$ depend on $\omega$. Now, in most experiments, one actually integrates $I(\boldsymbol{R}, \omega)$ with respect to $\omega$. Because of this integration, the measured intensity can depend on $\boldsymbol{R}$ through a law which is somewhat different from that indicated by eq. (10). However, such an effect can be disregarded if the bandwidth of the radiation used in the experiment is sufficiently small. This is the case for our experiment. Accordingly, we assume that the integrated intensity is described, to a good approximation, by eqs. (10)–(12), with $\omega$ replaced by the mean frequency of the incident radiation.

## 3. Experimental results

The setup used in our experiment is shown sche-

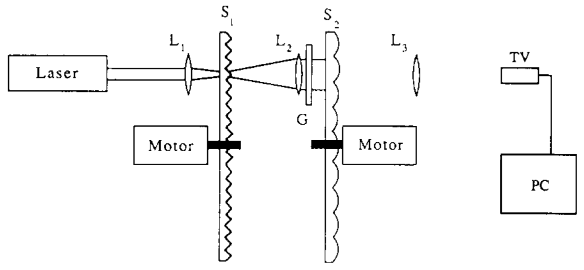


Fig. 1. Experimental set-up. $L_1$, $L_2$, $L_3$: converging lenses; $S_1$, $S_2$: scatterers; G: gaussian amplitude filter; TV, PC: television camera connected to a personal computer.

matically in fig. 1. A quasi-monochromatic, spatially incoherent source is synthesized by focusing a HeNe laser beam through the converging lens $L_1$ on a rotating disk of ground glass $S_1$. The glass is finely grounded in order to produce a transverse correlation length comparable to the wavelength of the incident radiation. The (secondary) source obtained when the disk rotates can be thought of as a spatially incoherent one. The intensity distribution across $S_1$ is a (two-dimensional) gaussian function whose width can be changed by moving the lens $L_1$ along the laser beam axis. The source $S_1$ is placed in the front focal plane of a second converging lens $L_2$. According to the van Cittert-Zernike theorem [7], the degree of spatial coherence at the output of $L_2$ is the Fourier transform of the intensity distribution across $S_1$ and is, therefore also a gaussian function. We note that the lens $L_2$ eliminates the quadratic phase factor that would otherwise appear in the expression of the degree of spatial coherence. Close to $L_2$ there is a gaussian amplitude filter G. As a consequence, the partially coherent field emerging from G has a gaussian intensity profile and a gaussian degree of spatial coherence. In this way, we obtain the type of field distribution described by eq. (8). This part of the apparatus is quite similar to that used in one of the first experiments on gaussian Schell model sources (also called Collett-Wolf Schell model sources) [8].

A second rotating glass disk $S_2$ is close to G and constitutes the planar scatterer, alluded to in the previous section. $S_2$ is made of the same type of glass that is sometimes used for office doors or windows. One of its surfaces is covered by a myriad of small bumps or tiny hills whose height is about 50 μm and whose base has a diameter of about 1 mm. When illuminated by a coherent collimated beam, such an object scatters radiation in a cone whose angular semi-aperture is about 2 degrees. We evaluated numerically the correlation function $C(\boldsymbol{r}_1, \boldsymbol{r}_2)$ pertaining to $S_2$ starting from a set of measurements of its thickness along several arrays of points. The results showed that the correlation function is approximately described by eq. (9), with a standard deviation $\sigma_T = 4$ μm.

The rotating scatterer $S_2$ is placed in the front focal plane of a third converging lens $L_3$. The intensity distribution across the back focal plane is detected by a television camera connected to a data acquisition board in a personal computer.

We first measured the time averaged intensity distribution along a diameter of the luminous spot, formed in the back focal plane of the lens $L_3$ (whose focal length is 50 mm), when the rotating scatterer $S_2$ is illuminated with coherent collimated radiation. This is obtained by removing the ground glass $S_1$ and using lenses $L_1$ and $L_2$ to expand and collimate the laser beam. The diameter of the expanded and collimated laser beam is about 10 times greater than the standard deviation of the filter G ($\sigma_I = 4$ mm); consequently the intensity of the light illuminating the filter is essentially constant all across the region in which the transmission function is significantly different from zero. We assume that the time average performed in the measurement with the rotating scatterer simulate an average over an ensemble of scatterers. The experimental situation is then described by eqs. (8) and (9), with $\sigma_T = 4$ μm, $\sigma_I = 4$ mm and $\sigma_\mu \to \infty$. On inserting these values into eq. (12) and using the mean wavelength (633 nm) of the red HeNe laser radiation, we find that $\sigma = 1.25$ mm. This is in good agreement with the experimental results as can be seen from curve (a) of fig. 2 where the intensity measured along a diameter of the spot is plotted.

Let us next consider the case of partially coherent illumination. The laser spot produced by $L_1$ on the ground glass was adjusted until the standard deviation of the degree of coherence obtained on G became comparable to the one pertaining to $S_2$. The chosen value of $\sigma_\mu$ was 3.2 μm. In this case, the value of $\sigma$ predicted by eq. (12) is 2 mm. This agrees with

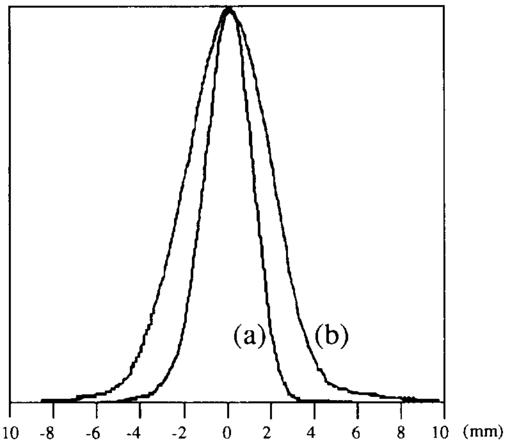


Fig. 2. Intensity recorded in the back focal plane of lens $L_3$ along the diameter of the scattered spot for $\sigma_T=4$ μm, $\sigma_I=4$ mm and $\lambda=633$ nm. Curve (a): coherent illumination of $S_2$. Curve (b): partially coherent illumination with $\sigma_\mu=3.2$ μm. Arbitrary units are used on the vertical axis.

the experimental result shown in curve (b) of fig. 2.

## 4. Conclusions

It has been shown in ref. [1] that scattering could in principle be used to discriminate between laser radiation and other types of radiation, utilizing the differences in their spatial coherence properties. We performed a simple experiment to confirm this result in the case of planar scatterers. The angular distribution of radiation obtained when the coherence area of the illuminating light is comparable to the correlation area of the scatterer is considerably different from that produced by coherent collimated light.

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