SUPERRESOLVING POSTPROCESSING FOR INCOHERENT IMAGERY

F. GORI, G. GUATTARI, C. PALMA and M. SANTARSIERO
Dipartimento di Fisica, Università di Roma "La Sapienza", P. le A. Moro 2, I-00185 Rome, Italy

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A superresolving pupil working with incoherent objects is emulated through an image postprocessing. The method simply requires the convolution of the image intensity distribution with a suitable correction function. Experimental results are given.

1. Introduction

A low-pass filtered signal, with bandwidth $2\nu_M$, is completely determined by a set of samples at a distance $(2\nu_M)^{-1}$ from each other. This distance is usually assumed as the resolution limit allowed by the filter. However, it is well known [1] that "a priori" information about the input signal can be used to lower the resolution limit. A frequently used "a priori" information is the knowledge that the input signal has a finite support. In this case, the image can be processed so as to obtain an enhancement of resolution in the interval inside which the input signal exists. To this aim, several methods have been envisaged, ranging from iterative procedures [2] to the use of eigenfunctions [3] or singular functions [4] of the operator describing the image formation. All these methods involve non-trivial mathematical algorithms. A potentially attractive alternative would be to make use of the very first method for superresolution, namely the superresolving pupil [5]. This would lead to a real time superresolution without the need for any image postprocessing. Unfortunately, the superresolving pupil requires that severe tolerance conditions are met in the fabrication process and this has prevented its practical use. This is ultimately due to the ill-posed nature of the superresolution problem [4]. Such a characteristic also manifests itself through numerical instabilities in any postprocessing method but, of course, it is easier to deal with these difficulties in a numerical processing rather than in an actual fabrication procedure.

In a previous paper [6], we showed that there exists a simple method for emulating the effect of a superresolving pupil through image postprocessing. The method applies to objects illuminated by light of any state of coherence and it was experimentally tested in the limiting case of coherent illumination [6]. In the general case of partially coherent illumination, the quantity to be processed is the cross-spectral density [7] across the image and the quantity to be recovered is the cross-spectral density across the object. Now, in many instances, one is interested in the recovery not of the whole cross-spectral density at the object plane but just of its diagonal elements, i.e. of the optical intensity. Nonetheless, the whole cross-spectral density across the image is to be processed. This is an intrinsic feature in case of partially coherent illumination and, of course, it is a rather inconvenient one because it requires to measure and process the cross-spectral density, i.e. a function of two position vectors, whereas the sought intensity is a function of one position vector only.

In this paper, we prove that if the object is illuminated with spatially incoherent light the knowledge of the image intensity distribution is sufficient for emulating the superresolving pupil. Furthermore, we explicitly calculate the correction function to be used for the image postprocessing and experimentally test the method through the application of the correction procedure to a number of measured data.

2. Principle of the method

Let us consider the image forming system schematically represented in fig. 1. Here, the y-, ν- and x-planes are the input (object), pupil and output (image) plane, respectively. All of them coincide with focal planes of the two (identical) lenses L, of focal length f. For simplicity, one-dimensional signals are considered, the extension to two-dimensional rectangular coordinates being only a matter of more cumbersome symbolism. We assume that the system is a perfect low-pass filter, with bandwidth $2\nu_M$ so that its incoherent transfer function (say $H(\nu)$) and its impulse response (say $S(x)$) can be written [8] [#1]

$$H(\nu)=2\nu_M(1-|\nu|/2\nu_M)\,,\quad \text{for } |\nu|\leqslant 2\nu_M\,,$$
$$=0\,,\quad \text{for } |\nu|\geqslant 2\nu_M\,, \tag{2.1}$$

and

$$S(x)=4\nu_M^2\,\mathrm{sinc}^2(2\nu_M x)\,, \tag{2.2}$$

respectively, where as usual $\mathrm{sinc}(x)=\sin(\pi x)/\pi x$. For such a system, the output intensity distribution $I_o(x)$ can be expressed through the input intensity $I_{in}(y)$ by means of the equation [8]

$$I_o(x)=\int I_{in}(y)\,S(x-y)\,\mathrm{d}y\,. \tag{2.3}$$

In eq. (2.3) and in the following, infinite limits of integration can be assumed because the spatial limitation of the object is accounted for by $I_{in}(y)$ itself.

[#1] Throughout this paper, the incoherent transfer function is assumed equal the true autocorrelation of the coherent pupil function rather than its normalized version. This should facilitate the comparison with the coherent case.

Let $p(\nu)$ be the amplitude transmission function of a superresolving pupil vanishing for $|\nu|>\nu_M$ and let $S_{sup}(x)$ be the corresponding incoherent impulse response, i.e. the Fourier transform of the autocorrelation function of $p(\nu)$ [8]. If such a pupil could be used in the system of fig. 1, the superresolved output intensity, say $(I_o)_{sup}$, would be

$$(I_o)_{sup}(x)=\int I_{in}(y)\,S_{sup}(x-y)\,\mathrm{d}y\,. \tag{2.4}$$

Fourier transformation of eqs. (2.3) and (2.4) gives, respectively,

$$\mathrm{F}\{I_o\}(\nu)=\mathrm{F}\{I_{in}\}(\nu)\,H(\nu)\,, \tag{2.5}$$

and

$$\mathrm{F}\{(I_o)_{sup}\}(\nu)$$
$$=\mathrm{F}\{I_{in}\}(\nu)\,[p(\nu)\star p(\nu)]\,, \tag{2.6}$$

where F{ } means Fourier transform and $\star$ denotes correlation operation. On inserting $\mathrm{F}\{I_{in}\}(\nu)$ as deducible from eq. (2.5) into eq. (2.6) and making an inverse Fourier transformation (to be denoted by $\mathrm{F}^{-1}\{\ \}$) we obtain

$$(I_o)_{sup}(x)=I_o(x)*K(x)\,, \tag{2.7}$$

where $*$ denotes convolution and

$$K(x)=\mathrm{F}^{-1}\{p(\nu)\star p(\nu)/H(\nu)\}\,. \tag{2.8}$$

Eq. (2.7) shows that the intensity distribution of the superresolved image can be obtained from that of the actual low-filtered image through a simple convolution operation. Therefore, such an operation constitutes an image postprocessing method for

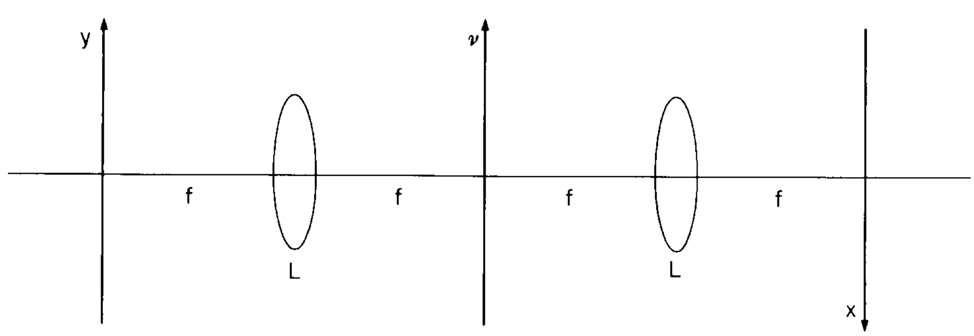

Fig. 1. Image forming system considered for describing the postprocessing method. L represents a converging lens of focal length f and the y-, ν- and x-planes represent the input (object), pupil and output (image) planes, respectively.

emulating the superresolving pupil with incoherent objects. It is to be noted that in this case, differently from the general case of partially coherent illumination [6], only the knowledge of the diagonal elements of the cross-spectral density, namely of the optical intensity $I_o(x)$, is required for the correction procedure [9]. Furthermore, due to the band-limited nature of I_o and to the sampling theorem, only a discrete set of samples of I_o (the effective measured data) can be used for the convolution in eq. (2.7) [6].

We finally observe that, as $H(\pm 2\nu_M)=0$, in order to make sure of the non-divergency of the quantity inside brackets at the right-hand side of eq. (2.8), the behavior at the same points $\pm 2\nu_M$ of the autocorrelation $p(\nu)\star p(\nu)$ should be examined closely. We will see in the next section that, for the pupils we are concerned with, F$\{K(x)\}$ is a well behaving function.

3. Evaluation of the correction function

In this section we want to evaluate the explicit correction function $K(x)$ to be used for the image post-processing. As in ref. [6], we follow the direct method of Toraldo di Francia [5] and we express the (bandlimited) impulse response $S_{\mathrm{sup}}(x)$ through a superposition of suitably weighted $\mathrm{sinc}(2\nu_M x)$-functions. As a consequence, the superresolving pupil transmission function $p(\nu)$ can be written

$$p(\nu)=\sum_{n=-N}^{N} p_n \times \exp(2\pi i n\nu/2\nu_M)\,\mathrm{rect}(\nu/2\nu_M)\,, \qquad (3.1)$$

where symmetrical weights can be assumed, i.e. $p_{-n}=p_n$. On inserting from eqs. (3.1) and (2.1) into eq. (2.8) and on calculating the autocorrelation function, we obtain

$$K(x)=\mathrm{F}^{-1}\left\{\mathrm{rect}(\nu/4\nu_M)\sum_{n=-N}^{N}\sum_{m=-N}^{N} p_n p_m^* \times \exp[\pi i(n+m)\,\nu/2\nu_M] \times \mathrm{sinc}[(n-m)(1-|\nu|/2\nu_M)]\right\}, \qquad (3.2)$$

where the asterisk denotes the complex conjugate. After some algebra for the calculation of the inverse Fourier transform at the right-hand side of eq. (3.2), the following expression is obtained for $K(x)$

$$K(x)=\sum_{n=-N}^{N}|p_n|^2\,4\nu_M \times \mathrm{sinc}(4\nu_M x+2n)+K_1(x)\,, \qquad (3.3)$$

where

$$K_1(x) = \sum_{\substack{n=-N\\ n\neq m}}^{N}\sum_{m=-N}^{N} p_n p_m^*\, 2\nu_M/(4\pi\nu_M x+\pi n+\pi m) \times \cos(4\pi\nu_M x+\pi n+\pi m)\,\mathrm{Si}[8\pi\nu_M x+2\pi n+2\pi m] + \sin(4\pi\nu_M x+\pi n+\pi m) \times[\gamma-\mathrm{Ci}(|8\pi\nu_M x+2\pi n+2\pi m|) + \ln(|8\pi\nu_M x+2\pi n+2\pi m|)]\,, \qquad (3.4)$$

for $4\pi\nu_M x + \pi n + \pi m - \pi|n-m| = 0$ or $4\pi\nu_M x + \pi n + \pi m + \pi|n-m| = 0$, and

$$K_1(x)=\sum_{\substack{n=-N\\ n\neq m}}^{N}\sum_{m=-N}^{N} p_n p_m^*\, 2\nu_M/\pi|n-m| \times \{\cos(4\pi\nu_M x+\pi n+\pi m) \times[\mathrm{Si}(4\pi\nu_M x+\pi n+\pi m+\pi|n-m|) - \mathrm{Si}(4\pi\nu_M x+\pi n+\pi m-\pi|n-m|)] + \sin(4\pi\nu_M x+\pi n+\pi m) \times[\mathrm{Ci}(|4\pi\nu_M x+\pi n+\pi m-\pi|n-m||) - \mathrm{Ci}(|4\pi\nu_M x+\pi n+\pi m+\pi|n-m||) + \ln(|4\nu_M x+n+m+|n-m||/|4\nu_M x +n+m-|n-m||)]\}\,, \qquad (3.5)$$

otherwise. In eqs. (3.4) and (3.5), Si and Ci denote sine- and cosine-integral, respectively, and $\gamma=0.5772156649$ is the Euler constant. The behavior of $K(x)/\nu_M$ as a function of $\nu_M x$ is shown in fig. 2. The values of N and p_n are the same as used in ref. [6], i.e. $N=5$ and $p_0=1$, $p_1=-0.25$, $p_2=0.75$, $p_3=-4.5$, $p_4=3$, $p_5=2$.

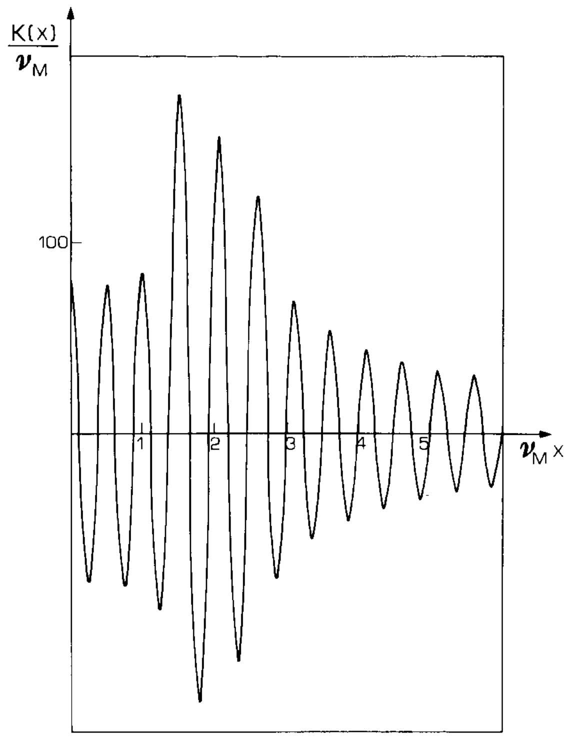

Fig. 2. Correction function used for the image postprocessing (eq. (3.3)), with $N=5$ and $p_0=1$, $p_1=-0.25$, $p_2=0.75$, $p_3=-4.5$, $p_4=3$, $p_5=2$.

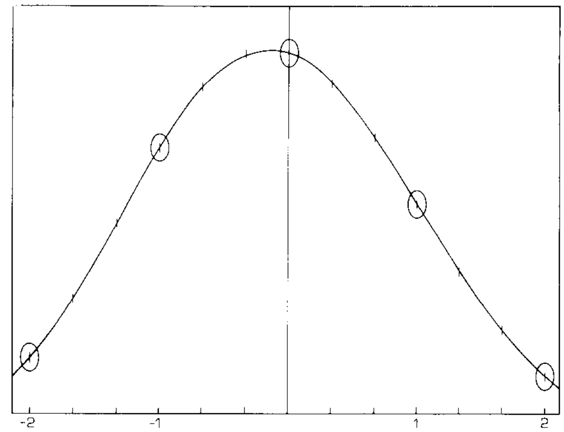

Fig. 4. Theoretical (full line) and experimental (heavy dots) intensity distribution in the image of two unresolved point sources. Encircled experimental points have been used for the correction procedure. Nyquist distance is taken as unit on the x-axis.

4. Experiment

The superresolving method described in section 2 has been tested through the experimental setup sketched in fig. 3. A laser beam, made incoherent by the passage through the rotating ground glass G, illuminates two parallel slits S_1 and S_2. The spatially incoherent secondary sources S_1 and S_2 are then imaged through the low-pass filter P. In fig. 4, theoretical and experimental intensity distributions are reported. As it is evident from fig. 4, the two sources are not resolved in the image. A set of measured intensity data, corresponding to points (encircled in fig. 4) spaced at the Nyquist rate $1/2\nu_M$, has then been used for the convolution with the correction function calculated in section 3. The resulting intensity distribution $(I_o)_{sup}(x)$ (see eq. (2.7)) is included between the upper and lower curve of fig. 5. The uncertainty is produced by the measurement errors on the reconstruction process and it was evaluated by repeating the experiment several times to obtain different noise realizations. In any case, the two image points are now clearly resolved, i.e. the superresolution effect has been obtained.

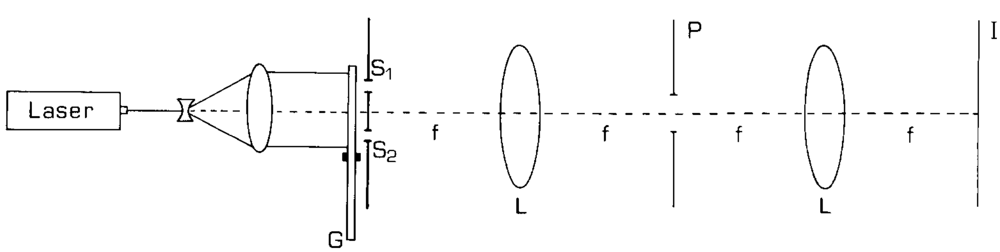

Fig. 3. Experimental setup used for testing the superresolving method. G: rotating ground glass; S_1, S_2: secondary spatially incoherent sources; L: converging lens of focal length f; P: low-pass filter; I: image plane.

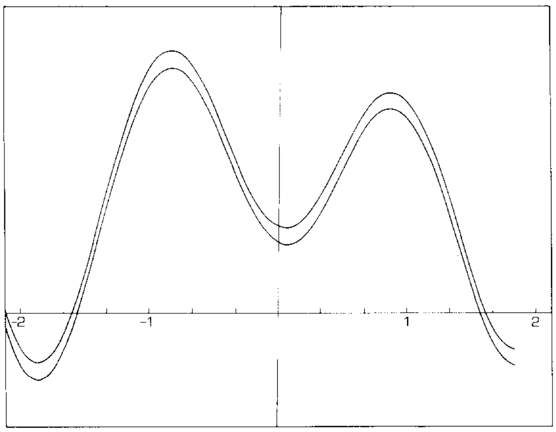

Fig. 5. Curves representing the superior and inferior limits for the superresolved image intensity distribution. Unit on the x-axis is taken as in fig. 4.

5. Conclusions

An image postprocessing method able to emulate a superresolving pupil in partially coherent light has been proposed not long ago [6]. Such a method requires the convolution between the cross-spectral density measured at the image plane and a suitable correction function. In this paper, we demonstrated that, in the limiting case of incoherent illumination, the method can be modified in such a way as to imply only the convolution between the actual image intensity distribution and a suitable correction function. Such a function has been explicitly evaluated and then used for processing an experimentally determined intensity distribution representing two unresolved image points. Due to the band-limited nature of the image, it has been possible to use for the convolution only a discrete set of measured data, corresponding to points spaced at the Nyquist rate. Furthermore, a low number of experimental data has been sufficient to obtain superresolution in the corrected image intensity distribution.

References

[1] W.T. Cathey, B.R. Frieden, W.T. Rhodes and C.K. Rushforth, J. Opt. Soc. Am. A1 (1984) 241.

[2] R.W. Gerchberg, Optica Acta 21 (1974) 709;
A. Papoulis, IEEE Trans. Circ. Syst. CAS-22 (1975) 735;
P. DeSantis and F. Gori, Optica Acta 22 (1975) 691;
C.K. Rushforth and R.L. Frost, J. Opt. Soc. Am. 70 (1980) 1539.

[3] C.K. Rushforth and R.W. Harris, J. Opt. Soc. Am. 58 (1968) 539;
B.R. Frieden, in: Progress in optics, Vol. 9, ed. E. Wolf (Pergamon, New York, 1971) p. 311.

[4] M. Bertero, C. DeMol and G. Viano, in: Inverse scattering problems in optics, ed. H.P. Baltes (Springer-Verlag, Berlin, 1980) p. 161.

[5] G. Toraldo di Francia, Nuovo Cimento Suppl. 9 (1952) 426.

[6] P. DeSantis, F. Gori, G. Guattari and C. Palma, Optics Comm. 60 (1986) 13.

[7] L. Mandel and E. Wolf, J. Opt. Soc. Am. 66 (1976) 529.

[8] J.W. Goodman, Introduction to Fourier optics (McGraw-Hill, New York, 1968).

[9] K. Dutta and J.W. Goodman, J. Opt. Soc. Am. 67 (1977) 796.