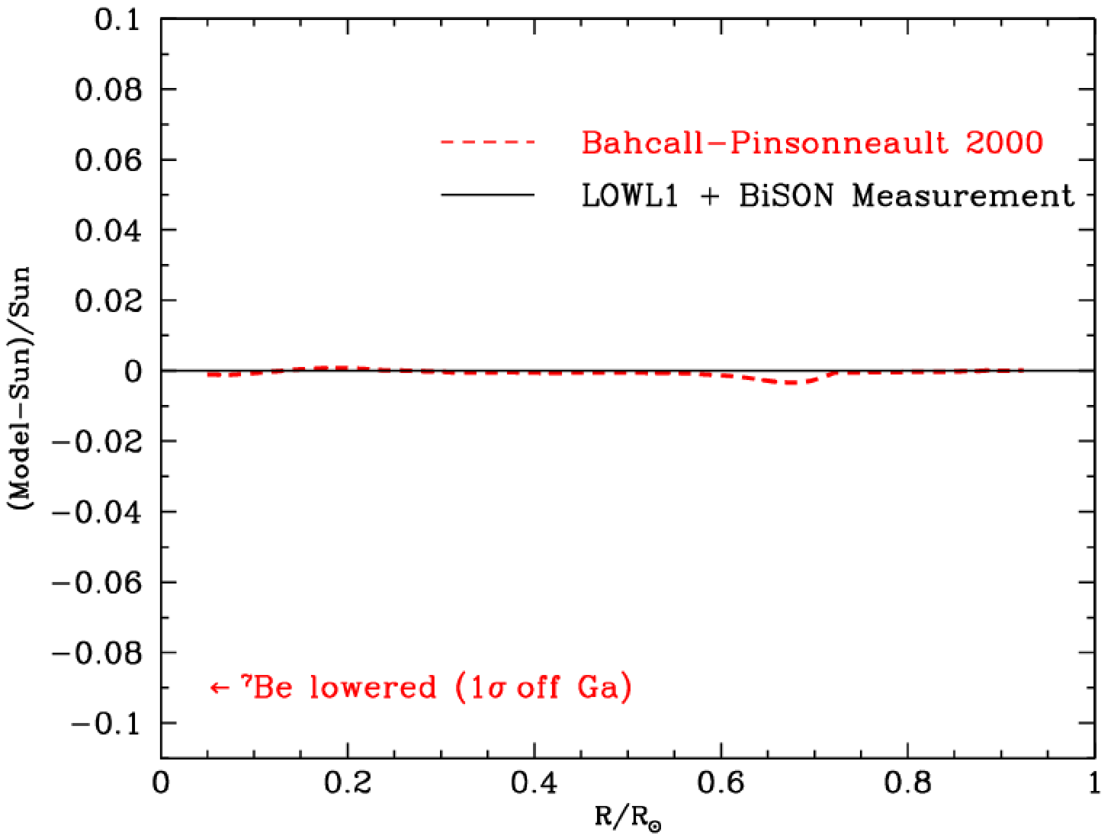
Problemi correnti nella fisica dei neutrini

AA2011/12

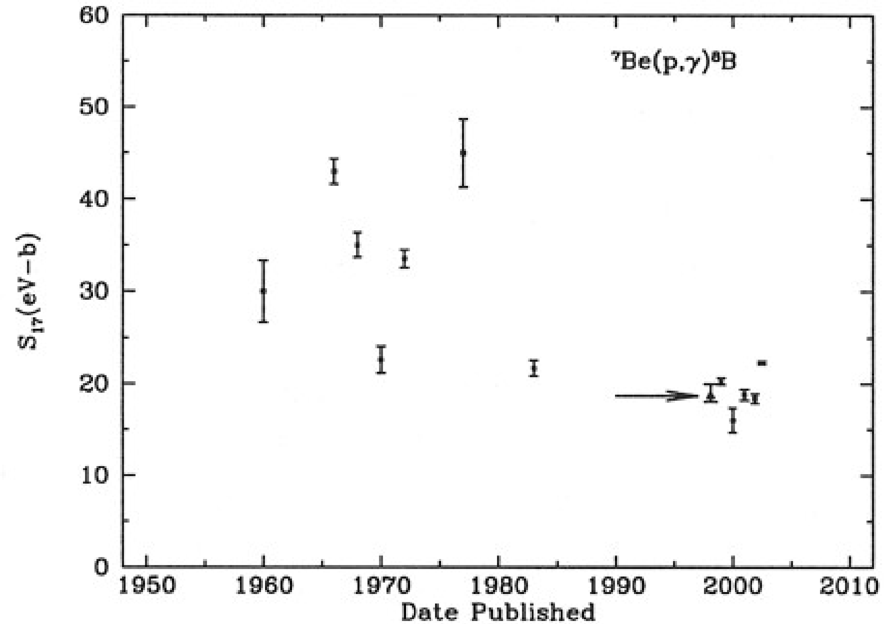
Sommario IV

- La conferma dell’ipotesi di oscillazione: SNO
- Oscillazioni si, ma dove?
- Un’altra conferma: KamLAND
- Verso delle misure di precisione: Borexino

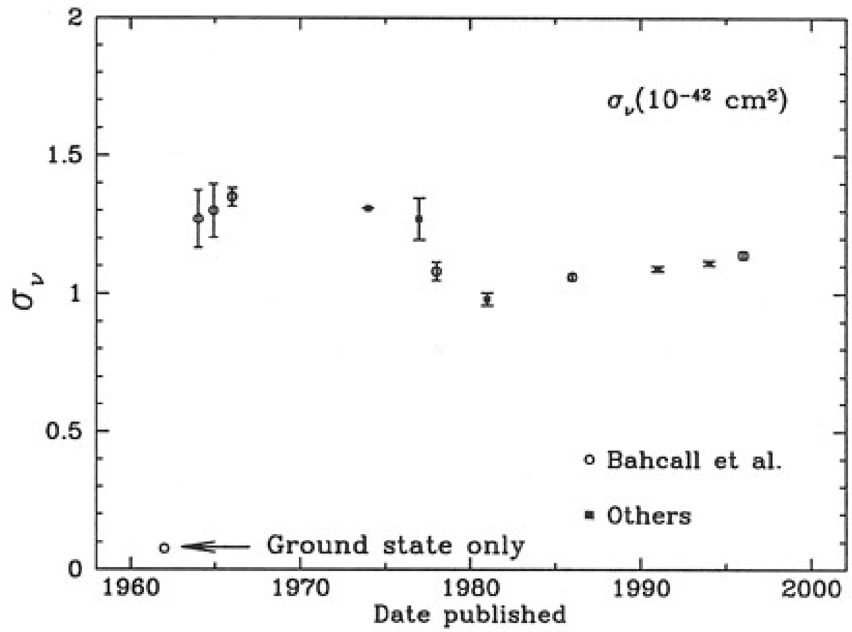
1997-2003



Bahcall–Pinsonneault 2000
LOWL1 + BiSON Measurement
← ^{7}Be lowered (1σ off Ga)
(Model−Sun)/Sun
$R/R_\odot$
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
−0.08
−0.1
0
0.2
0.4
0.6
0.8
1



$^7Be(p,\gamma)^8B$
S_{17}(eV−b)
Date Published

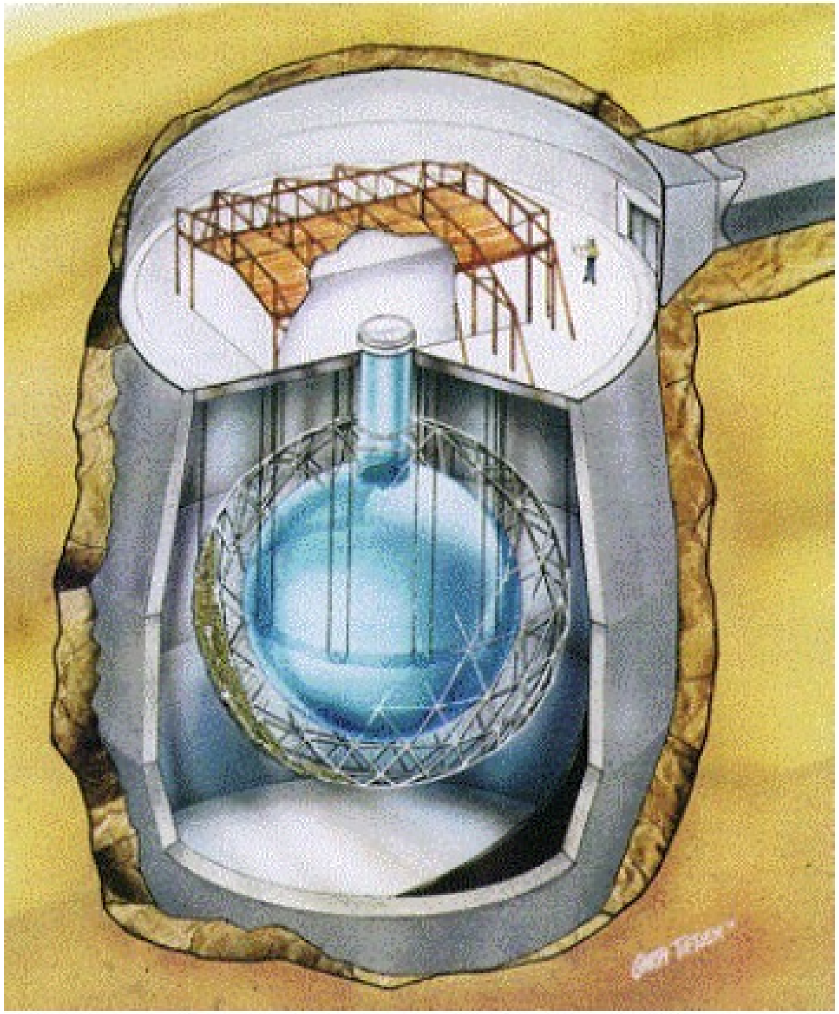


$\sigma_\nu(10^{-42}\ cm^2)$
σ_ν
Bahcall et al.
Others
Ground state only
Date published

SNO

Evidenza definitiva di oscillazione dei neutrini solari

(Sudbury Neutrino Observatory, Sudbury, Ontario, Canada)



SNO: rivelatore di luce Čerenkov emessa in 1000 tonn. di **acqua pesante** D_2O ultra-pura contenuta in una sfera acrilica (diam. 12 m), circondata da 7800 tonn. di acqua H_2O ultra-pura.

Raccolta di luce: 9456 fotomoltiplicatori, diam. 20 cm, su una superficie sferica di raggio 9.5 m

Profondità: 2070 m (6010 m H_2O eq.) in una miniera di nikel

Soglia di rivelazione energia elettroni: 5 MeV

Ricostruzione del punto di interazione dalla misura dei tempi relativi dei segnali dei fotomoltiplicatori

Rivelazione dei neutrini solari nell'esperimento SNO:

(ES) Diffusione elastica neutrino – elettrone : $\nu + e^- \rightarrow \nu + e^-$

Direzionale, $\sigma(\nu_e) \approx 6\,\sigma(\nu_\mu) \approx 6\,\sigma(\nu_\tau)$ (come in Super-K)

(CC) $\nu_e + d \rightarrow e^- + p + p$

Direzionalità debole: distribuzione angolare elettroni $\propto 1 - \frac{1}{3}\cos(\theta_{sun})$

Misura dell'energia del ν_e (perchè la maggior parte dell'energia del ν_e è trasferita all'elettrone)

(NC) $\nu + d \rightarrow \nu + p + n$

Sezione d'urto identica per i tre tipi di neutrino

Misura del flusso solare totale da $B^8 \rightarrow Be^8 + e^+ + \nu$ in presenza di oscillazioni

RIVELAZIONE DI $\nu + d \rightarrow \nu + p + n$

Rivelazione di fotoni ($\rightarrow e^+e^-$) da cattura del neutrone dopo rallentamento

- Prima fase (Novembre 1999 – Maggio 2001):

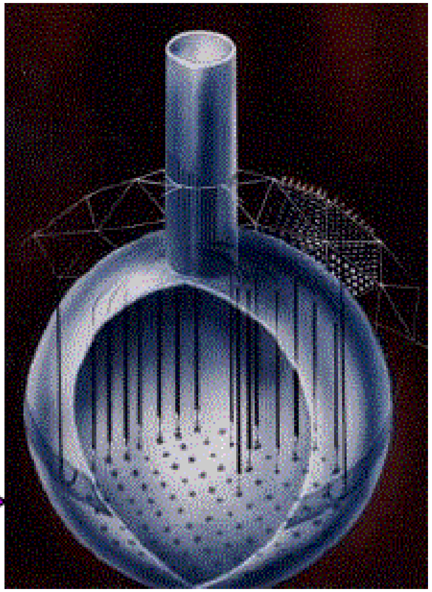
  $n + d \rightarrow H^3 + \gamma$ ($E_\gamma = 6.25$ MeV)

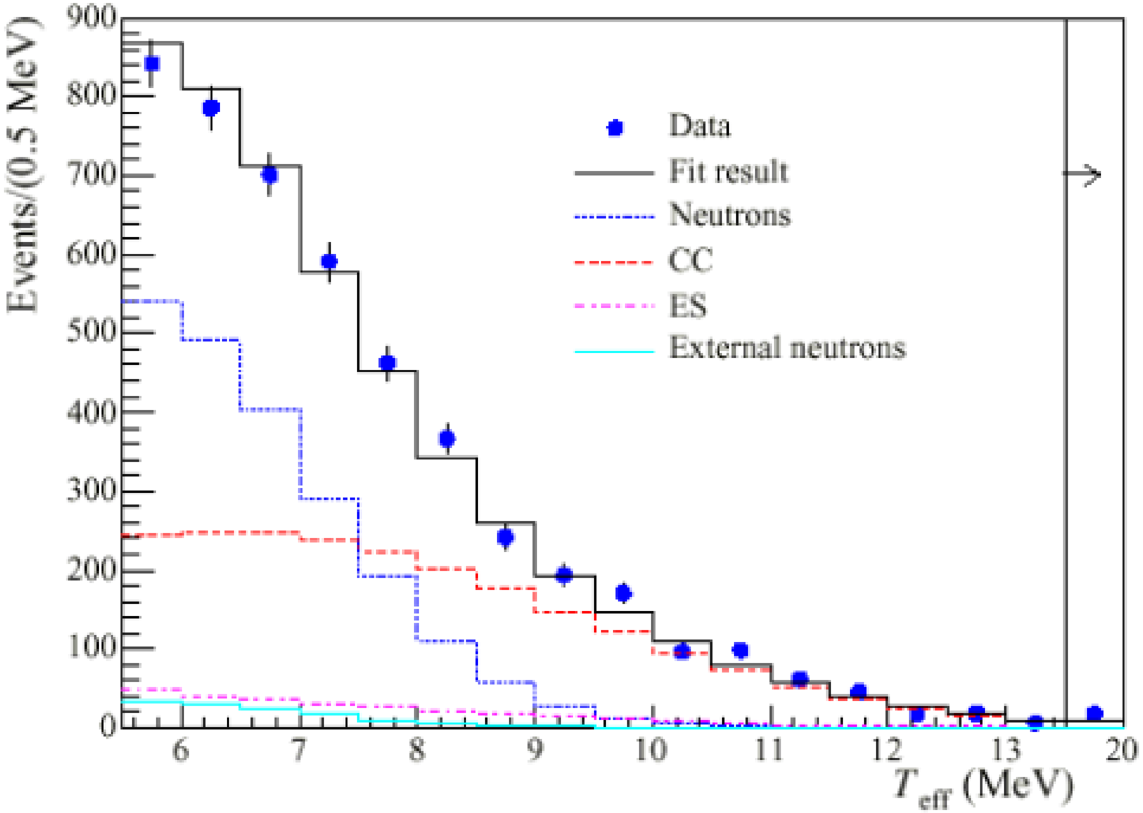
- Seconda fase : aggiunta di *NaCl* ultra-puro (2 tonn.)

  $n + Cl^{35} \rightarrow Cl^{36} +$ raggi γ ($\Sigma E_\gamma \approx 8.6$ MeV)

- In seguito: inserimento di contatori proporzionali a He^3

  $n + He^3 \rightarrow p + H^3$ (segnale mono-energetico)





Distribuzione energia depositata (ampiezza segnale)

Estrazione delle tre componenti mediante metodo di massima verosimiglianza

Numero di eventi:

CC: 2176 ± 78

ES: 279 ± 26

NC: 2010 ± 85

Fondo (neutroni esterni): 128 ± 42

Flussi di neutrini solari, misurati separatamente dai tre segnali:

$$\Phi_{CC} = (\ 1.72 \pm\ 0.05 \pm\ 0.11\ )\ \mathrm{x}\ 10^{6}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

<u>Nota</u>: $\Phi_{CC} \equiv \Phi(\nu_e)$

$$\Phi_{ES} = \ (\ 2.34 \pm\ 0.23 \ {}^{+\,0.15}_{-\,0.14}\ )\ \mathrm{x}\ 10^{6}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

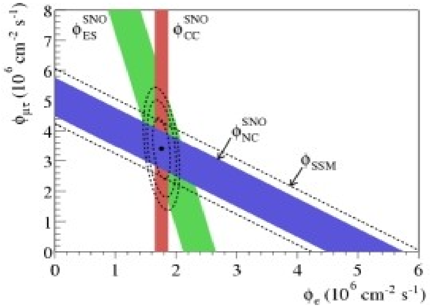
Calcolato nell'ipotesi che tutti i neutrini incidenti sono ν_e

$$\Phi_{NC} = (\ 4.81 \pm 0.19 \ {}^{+\,0.28}_{-\,0.27}\ )\ \mathrm{x}\ 10^{6}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$$

(stat) (sist)

$$\Phi_{SSM}(\nu) = 5.05\ {}^{+1.01}_{-0.81}\ \mathrm{x}\ 10^{6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

$$\frac{\Phi_{CC}}{\Phi_{NC}} = 0.358\ \pm 0.021\ {}^{+\,0.028}_{-\,0.029}$$



- **Flusso <u>totale</u> di neutrini solari in accordo con predizioni SSM**
  (misura della temperatura del nucleo solare con precisione ~ 0.5%)

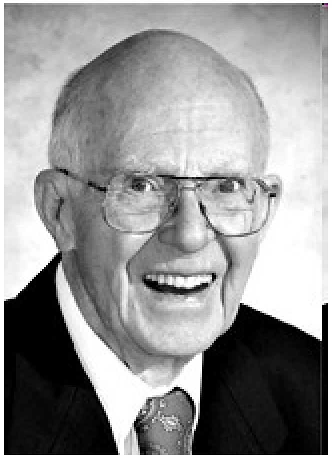
- **Composizione neutrini solari all'arrivo sulla Terra:**
  ~ 36% ν_e ; ~ 64% $\nu_\mu + \nu_\tau$ (rapporto ν_μ / ν_τ ignoto)

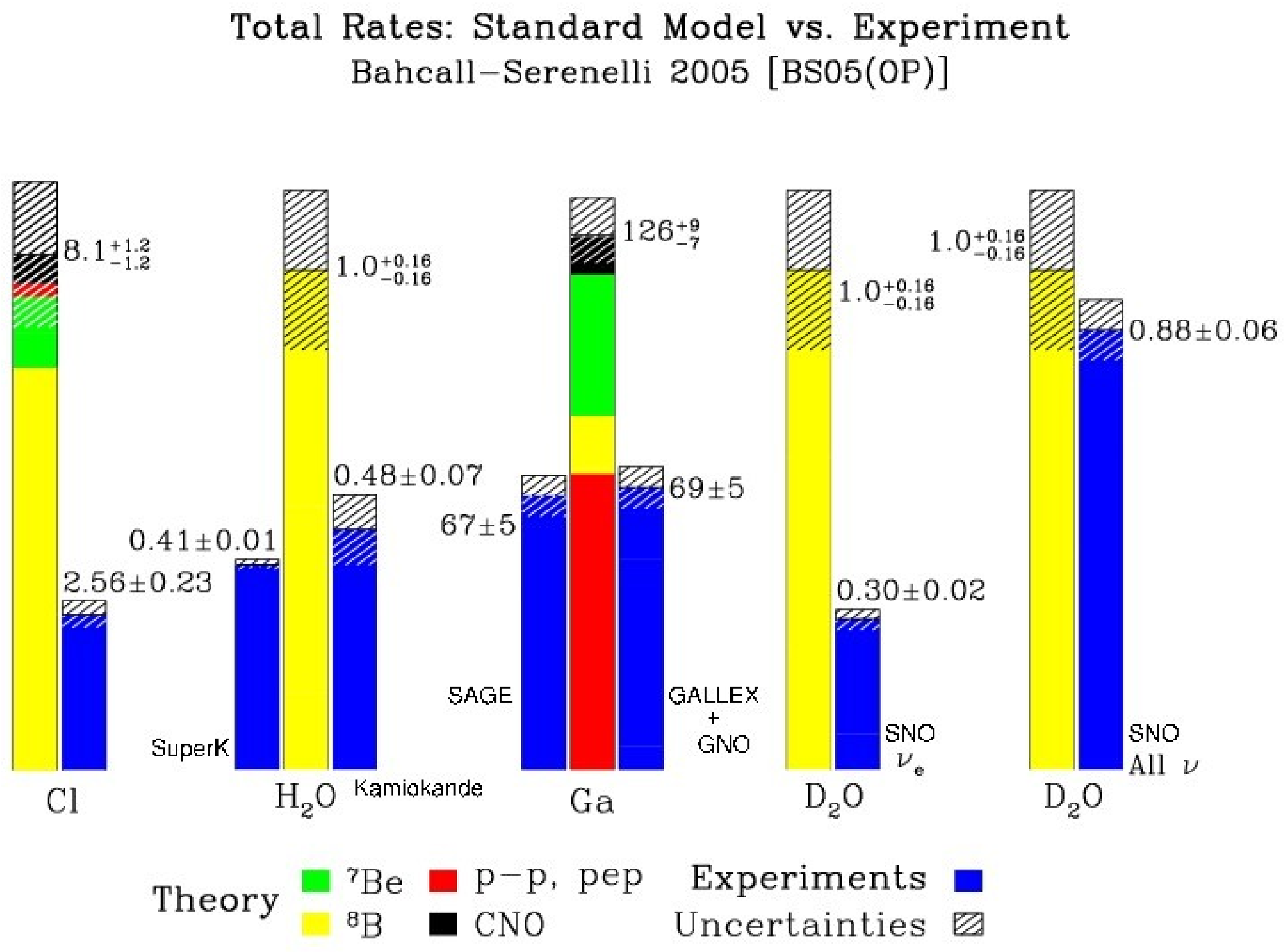
EVIDENZA DEFINITIVA DI OSCILLAZIONE DEI NEUTRINI SOLARI

"The Nobel Prize in Physics 2002"

http://nobelprize.org/nobel_prizes/physics/laureates/2002/

Davis (Homestake)
Koshiba (Kamiokande)
Giacconi (astronomia X)




Total Rates: Standard Model vs. Experiment
Bahcall–Serenelli 2005 [BS05(OP)]
$8.1^{+1.2}_{-1.2}$
2.56 ± 0.23
$1.0^{+0.16}_{-0.16}$
0.41 ± 0.01
0.48 ± 0.07
SuperK
Kamiokande
126^{+9}_{-7}
67 ± 5
69 ± 5
SAGE
GALLEX
+
GNO
$1.0^{+0.16}_{-0.16}$
0.30 ± 0.02
SNO
ν_e
$1.0^{+0.16}_{-0.16}$
0.88 ± 0.06
SNO
All ν
Cl
H_2O
Ga
D_2O
D_2O
Theory
^7Be
^8B
p–p, pep
CNO
Experiments
Uncertainties

Scomparsa di ν_e solari: interpretazione

Ipotesi: mixing di due neutrini

Oscillazioni nel vuoto

Spettro ν_e rivelato sulla Terra $\Phi(\nu_e) = \mathcal{P}_{ee}\,\Phi_0(\nu_e)$

($\Phi_0(\nu_e) \equiv$ spettro ν_e alla produzione)

Probabilità di rivelare ν_e:

$$\mathcal{P}_{ee} = 1 - \sin^2(2\theta)\sin^2\!\left(1.267\,\Delta m^2 \frac{L}{E}\right) \qquad \begin{pmatrix} L\ [\mathrm{m}] \\ E\ [\mathrm{MeV}] \\ \Delta m^2\ [\mathrm{eV}^2] \end{pmatrix}$$

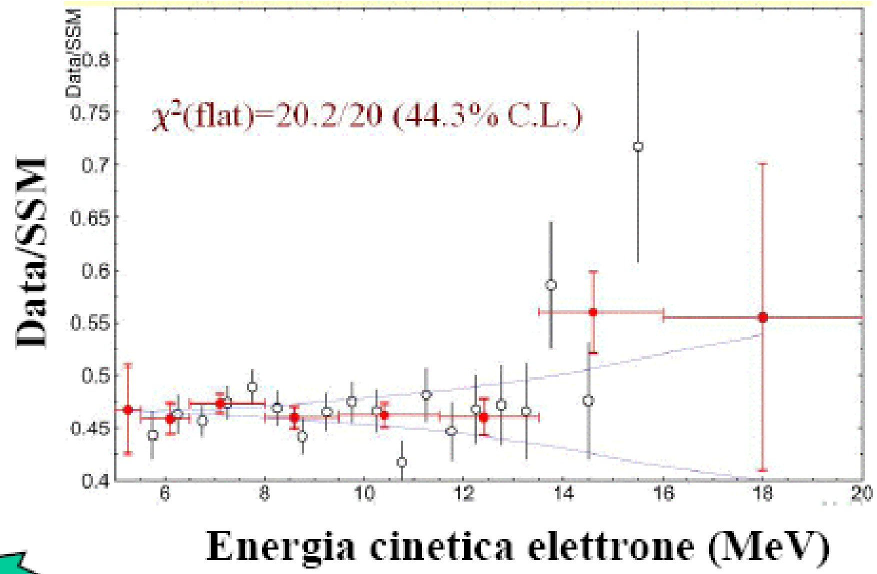
L = 1.496 x 10^{11} m (distanza media Sole – Terra con 3.3% di variazione annuale dovuta all'eccentricità dell'orbita terrestre)

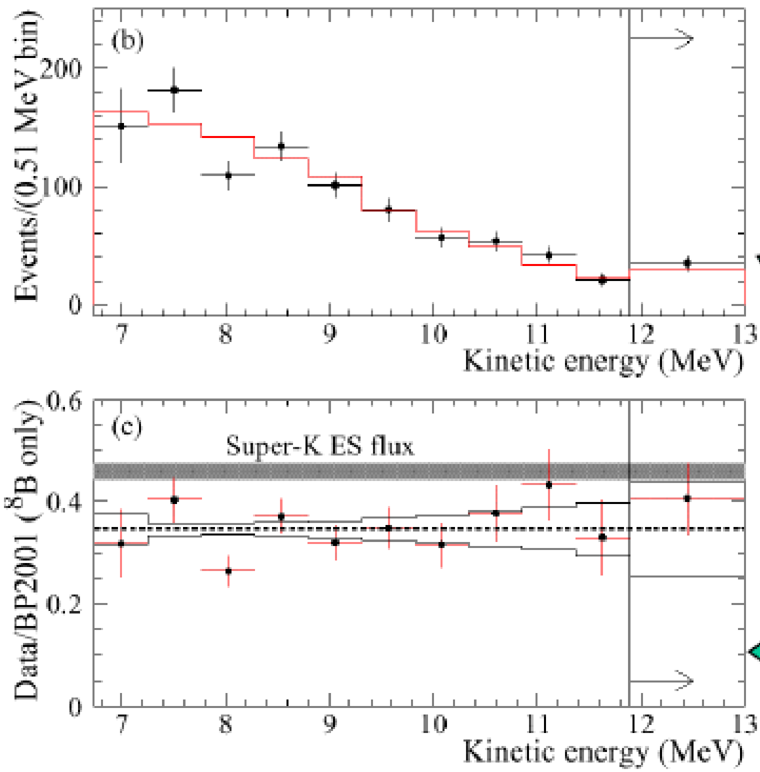
Effetti previsti per Super-K, SNO:

- **Distorsioni dello spettro ≥ 20%** (dipendenza di $\mathcal{P}_{ee}$ da E)
- **Modulazione stagionale ≥ 10%** (dipendenza di $\mathcal{P}_{ee}$ da L)

Distorsioni dello spettro

Super-K 2002



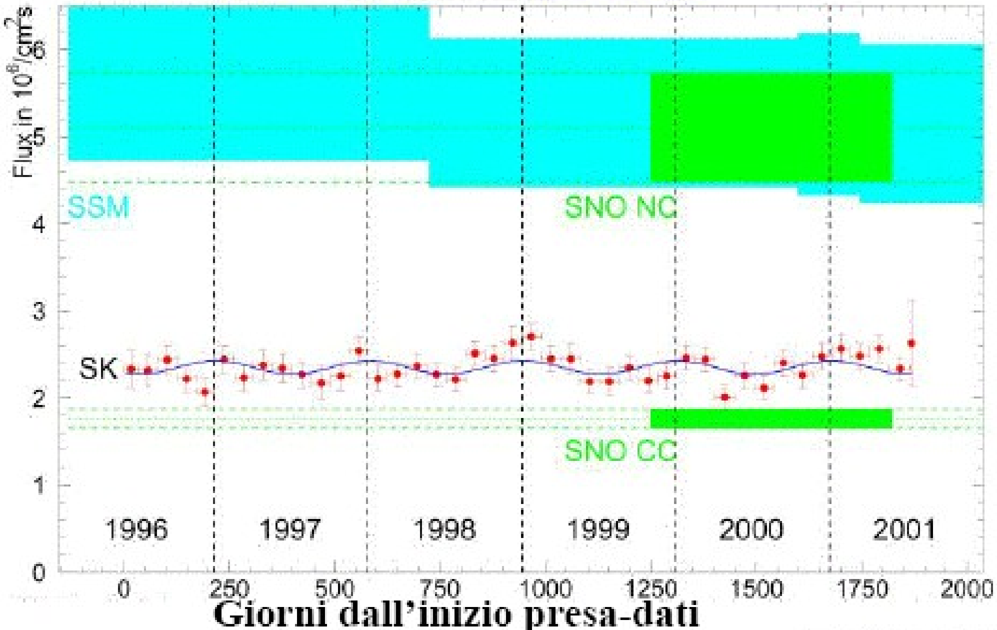


SNO: $\nu_e + d \rightarrow e^- + p + p$
distribuzione energia elettrone

SNO: dati / predizione SSM

deficit ν_e indipendente dall'energia entro gli errori di misura (assenza di distorsioni dello spettro)

Modulazione stagionale

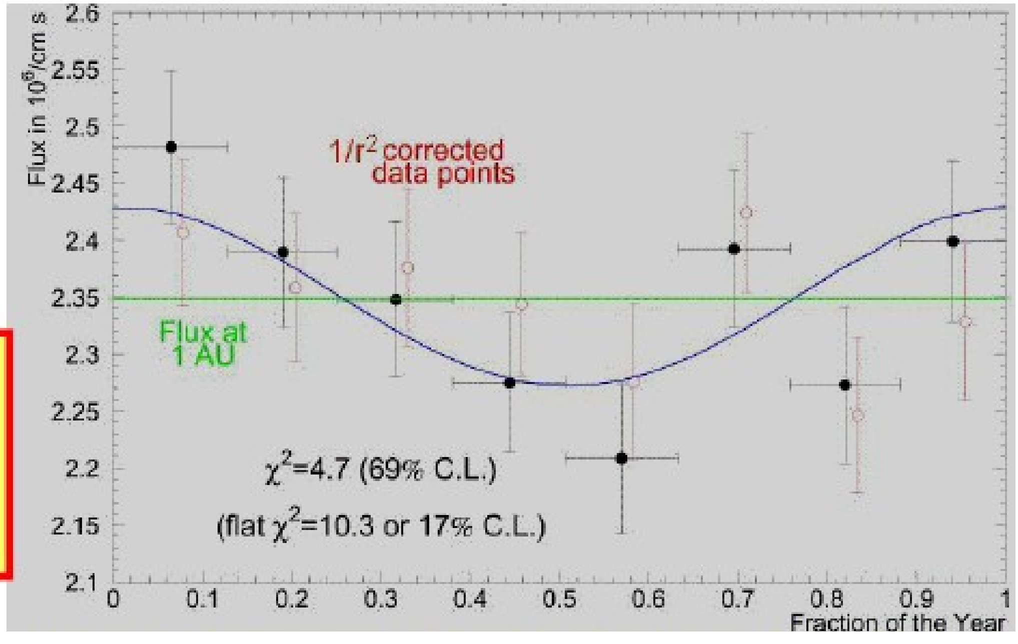


Variazione annuale della distanza Sole - Terra: 3.3% $\Rightarrow$ modulazione stagionale del flusso di neutrini solari

Modulazione stagionale in assenza di oscillazioni dovuta alla variazione dell'angolo solido: ~ 6.6%

Effetto osservato compatibile con la sola variazione di angolo solido

Le oscillazioni di neutrini nel vuoto non descrivono il deficit di ν_e solari osservato



OSCILLAZIONI DI NEUTRINI NELLA MATERIA

Rifrazione dei neutrini nella materia (L. Wolfenstein, 1978)

Indice di rifrazione : $$\boxed{n = 1 + \varepsilon = 1 + \frac{2\pi}{p^2} N f(0)}$$

p: impulso del neutrino
N: densità dei centri di diffusione
$f(0)$: ampiezza di diffusione a $\theta = 0°$

Nel vuoto: $$E = \sqrt{p^2 + m^2}$$

Onda piana nella materia: $\Psi = e^{i(n\mathbf{p}\cdot\mathbf{r} - Et)}$

$$\Rightarrow \quad E' = \sqrt{(np)^2 + m^2} \approx E + \frac{p^2}{E}\varepsilon \qquad (|\varepsilon| << 1)$$

Conservazione dell'energia:

$$E = E' + V$$

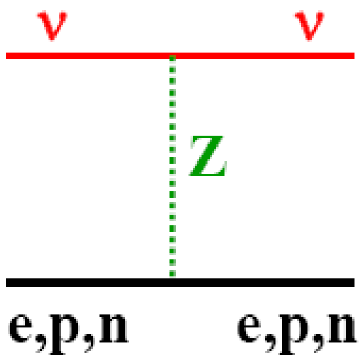
V $\equiv$ energia potenziale del neutrino nella materia

$$\Rightarrow \quad \boxed{V = -\frac{p^2}{E}\varepsilon = -\frac{2\pi}{E} N f(0)}$$

$V < 0$: potenziale attrattivo ($n > 1$)
$V > 0$: potenziale repulsivo ($n < 1$)

Energia potenziale del neutrino nella materia

1. Contributo da scambio Z (identico per i tre tipi di neutrino)



$$V_Z(p) = -V_Z(e) = \frac{\sqrt{2}}{2} G_F N_p (1 - 4\sin^2\theta_w)$$

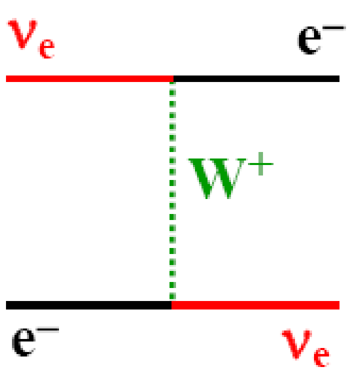
$$V_Z(n) = -\frac{\sqrt{2}}{2} G_F N_n$$

G_F: costante di Fermi

N_p (N_n): densità protoni (neutroni)

θ_w: angolo di mixing debole

2. Contributo da scambio W (soltanto per ν_e!)



$$\boxed{V_W[eV] = \sqrt{2} G_F N_e \approx 7.63 \times 10^{-14} \frac{Z}{A} \rho}$$

N_e: densità elettroni

ρ: densità di materia [g/cm^3]

NOTA: $V(\nu) = -V(\bar{\nu})$

Esempio: mixing $\nu_e - \nu_\mu$ in un mezzo di densità costante

(risultati identici per mixing $\nu_e - \nu_\tau$)

Nella base del "flavour": $\nu = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$

Equazione che descrive l'evoluzione dello stato: $\underbrace{H}_{\text{matrice 2x2}} \nu = i \dfrac{\partial \nu}{\partial t}$

$$H = (E + V_Z)\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \frac{1}{2E}\begin{vmatrix} M_{ee}{}^2 & M_{e\mu}{}^2 \\ M_{\mu e}{}^2 & M_{\mu\mu}{}^2 \end{vmatrix} + V_W \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$$

(Ricordare: $\sqrt{p^2 + M^2} \approx p + \dfrac{M^2}{2p} \approx E + \dfrac{M^2}{2E}$ per $M << p$)

$$M_{ee}{}^2 = \frac{1}{2}(\mu^2 - \Delta m^2 \cos 2\theta)$$

$$M_{e\mu}{}^2 = M_{\mu e}{}^2 = \frac{1}{2}\Delta m^2 \sin 2\theta$$

$$M_{\mu\mu}{}^2 = \frac{1}{2}(\mu^2 + \Delta m^2 \cos 2\theta)$$

$$\mu^2 = m_1{}^2 + m_2{}^2$$

$$\Delta m^2 = m_2{}^2 - m_1{}^2$$

NOTA: m_1, m_2, θ definiti nel vuoto

$$H = (E + V_Z)\underbrace{\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}}_{\text{termine diagonale: nessun mixing}} + \underbrace{\frac{1}{2E}\begin{vmatrix} M_{ee}{}^2 + 2EV_W & M_{e\mu}{}^2 \\ M_{\mu e}{}^2 & M_{\mu\mu}{}^2 \end{vmatrix}}_{\text{termine responsabile del mixing } \nu_e - \nu_\mu}$$

ρ = costante $\longrightarrow$ *H* indipendente dal tempo

Diagonalizzazione di *H* $\Rightarrow$ autovalori e autostati

Autovalori nella materia

$$\boxed{M^2 = \frac{1}{2}(\mu^2 + \xi) \pm \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - \xi)^2 + (\Delta m^2)^2 \sin^2 2\theta}}$$

$$\xi \equiv 2EV_W \approx 1.526 \times 10^{-7} \frac{Z}{A} \rho E \quad [\mathrm{eV}^2]$$

(ρ in g/cm^3, E in MeV)

Angolo di mixing nella materia

$$\boxed{\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - \xi}}$$

$\xi = \Delta m^2 \cos 2\theta \equiv \xi_{res} \Rightarrow$ mixing massimo ($\theta_m = 45°$) anche nel caso di angolo di mixing nel vuoto molto piccolo: "risonanza MSW"

(scoperta da Mikheyev e Smirnov nel 1985)

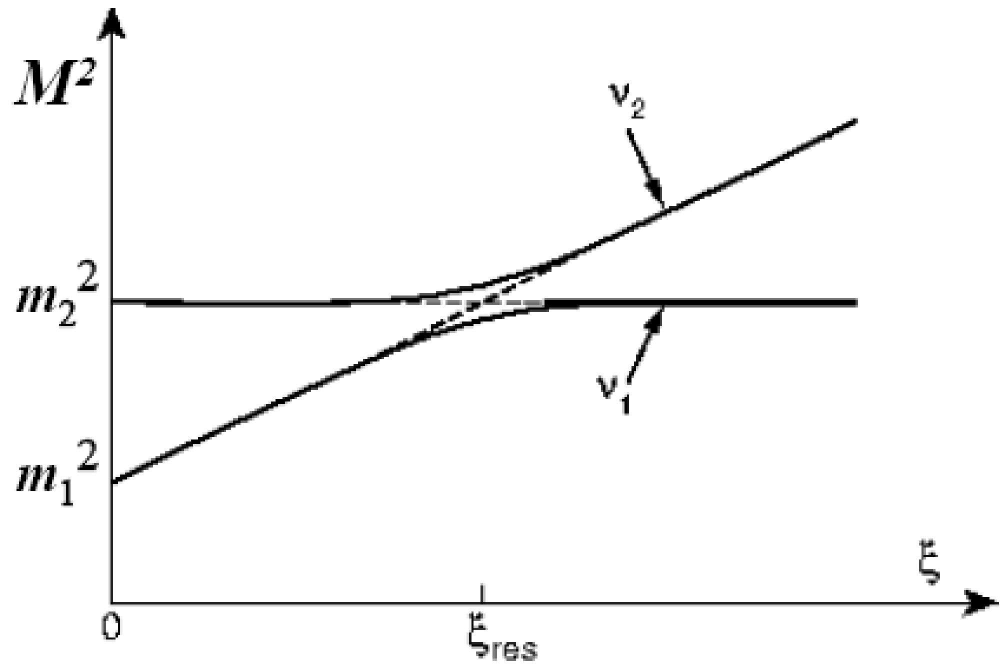
Nota 1: la risonanza MSW può esistere solo se $\theta < 45°$ (altrimenti $\cos 2\theta < 0$)

Nota 2: Per $\bar{\nu}_e$ $\xi < 0 \Rightarrow$ assenza di risonanza MSW se $\theta < 45°$

Autovalori della massa in funzione di ξ

$$\xi \equiv 2EV_W \approx 1.526\times10^{-7}\frac{Z}{A}\rho E$$

$$\boxed{\xi_{res} = \Delta m^2 \cos 2\theta}$$



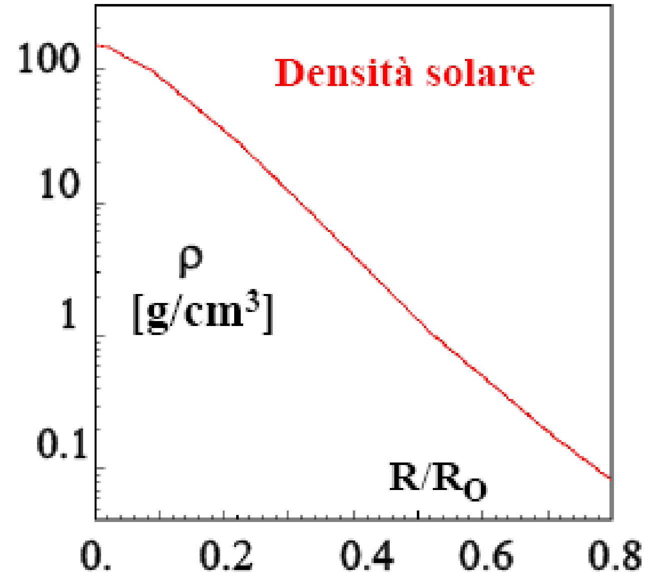
Lunghezza di oscillazione nella materia:

$$\lambda_m = \lambda \frac{\Delta m^2}{\sqrt{(\Delta m^2 \cos 2\theta - \xi)^2 + (\Delta m^2)^2 \sin^2 2\theta}}$$

($\lambda \equiv$ lunghezza di oscillazione nel vuoto)

Per $\xi = \xi_{res}$: $$\lambda_m = \frac{\lambda}{\sin 2\theta}$$

Effetto di materia sulle oscillazioni dei neutrini solari



Neutrini solari: prodotti in un mezzo di alta densità (il nucleo del Sole).

Densità variabile lungo il percorso nel Sole: $\rho = \rho(t)$

Formalismo delle oscillazioni nella materia

Evoluzione temporale: $H\nu = i\, \partial\nu / \partial t$

H (matrice 2 x 2) dipende dal tempo attraverso $\rho(t)$

→ **non esistono autostati di H**

Risoluzione numerica dell'equazione di evoluzione:

$$\nu(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$ (puro ν_e alla produzione)

$$\nu(\delta) = \nu(0) + \left(\frac{\partial \nu}{\partial t}\right)_{t=0} \delta = \nu(0) - iH(0)\nu(0)\delta$$ (δ = intervallo temporale molto piccolo)

....

$$\nu(t+\delta) = \nu(t) + \left(\frac{\partial \nu}{\partial t}\right)_{t} \delta = \nu(t) - iH(t)\nu(t)\delta$$

....

(fino all'uscita del neutrino dal Sole)

Soluzione generica:

$$\nu(t) = a_1(t)\nu_1 + a_2(t)\nu_2 \qquad (|a_1|^2 + |a_2|^2 = 1)$$

ν_1, ν_2 : autostati "locali" dell'Hamiltoniana indipendente dal tempo (ρ fissato)

Alla produzione ($t = 0$, nel nucleo del Sole):

$$\boxed{\nu_e = \cos\theta_m^0 \nu_1(0) + \sin\theta_m^0 \nu_2(0)}$$

dove: $\theta_m^0 = \theta_m(0)$; $\nu_1(0)$, $\nu_2(0)$ autostati di H per $\rho=\rho(0)$

Ipotesi: θ (angolo di mixing nel vuoto) $< 45°$: $\cos\theta > \sin\theta$ nel vuoto

Se $\xi > \xi_{res}$ alla produzione, $\theta_m(0) > 45°$

$$\Rightarrow \quad \boxed{a_1(0) = \cos\theta_m^0 < a_2(0) = \sin\theta_m^0}$$

(componente ν_2 del ν_e > componente ν_1)

$$\xi > \xi_{res} \quad \rightarrow \quad E[MeV] > \frac{\xi_{res}}{2V_W} \approx \frac{6.6\times 10^6 \Delta m^2 \cos 2\theta}{(Z/A)\rho} \qquad \begin{pmatrix} \Delta m^2 \ [\mathrm{eV}^2] \\ \rho \ \ [\mathrm{g/cm}^3] \end{pmatrix}$$

Soluzioni "adiabatiche"

(variazione di ρ trascurabile su una lunghezza di oscillazione)

$$a_1(t) \approx a_1(0) \;\; ; \;\; a_2(t) \approx a_2(0)$$

a ogni istante *t*

All'uscita dal Sole ($t = t_E$):

$$\nu(t_E) = a_1(0)\nu_1(t_E) + a_2(0)\nu_2(t_E)$$

$\nu_1(t_E)$, $\nu_2(t_E)$: autostati di massa nel vuoto

Se $a_1(0) < a_2(0)$: $\left|\langle \nu_\mu | \nu(t_E) \rangle\right| > \left|\langle \nu_e | \nu(t_E) \rangle\right|$

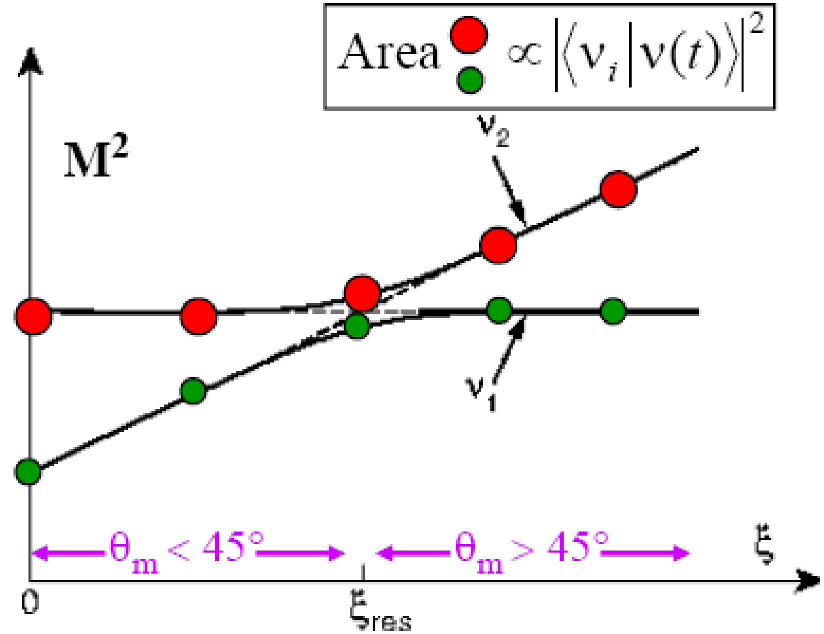
perchè nel vuoto ($\theta < 45°$)

$$\left|\langle \nu_\mu | \nu_2 \rangle\right| > \left|\langle \nu_e | \nu_2 \rangle\right|$$

DEFICIT DI ν_e ALL'USCITA DAL SOLE

$$\xi \equiv 2EV_W \approx 1.526 \times 10^{-7} \frac{Z}{A} \rho E$$

$$\xi_{res} \equiv \Delta m^2 \cos(2\theta)$$



For simplicity, we shall consider only oscillations between electron and muon neutrinos. The neutrino mass states $|\nu_1\rangle$ and $|\nu_2\rangle$ are assumed to have distinct masses m_1 and m_2, respectively. We define the neutrino flavor states $|\nu_e\rangle$ and $|\nu_\mu\rangle$ in terms of two mass states:

$$|\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle \quad \text{, and} \tag{1a}$$

$$|\nu_\mu\rangle = -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle \quad . \tag{1b}$$

We further assume that an electron neutrino is born at time $t = 0$. That neutrino will evolve in time as a superposition of states with time-dependent coefficients. The neutrino can be described by either mass states or flavor states:

$$|\nu(t)\rangle = a_1(t)\,|\nu_1\rangle + a_2(t)\,|\nu_2\rangle = a_e(t)\,|\nu_e\rangle + a_\mu(t)\,|\nu_\mu\rangle \quad , \tag{2}$$

where

$$a_e(t) = a_1(t)\cos\theta + a_2(t)\sin\theta \quad \text{, and} \tag{3a}$$

$$a_\mu(t) = -a_1(t)\sin\theta + a_2(t)\cos\theta \quad . \tag{3b}$$

In general, the time development of the neutrino states described in Equation (2) has a phase that depends on both the momentum and the energy of the neutrino. For example, an electron neutrino evolves as

$$|\nu_e(t)\rangle = \cos\theta\, e^{ip\cdot x - iE_1 t}|\nu_1\rangle + \sin\theta\, e^{ip\cdot x - iE_2 t}|\nu_2\rangle \quad . \tag{5}$$

We work in units in which $\hbar = c = 1$. Let us first consider the evolution of $|\nu(t)\rangle$ as a superposition of mass eigenstates during an infinitesimal time Δt. We assume a common momentum for each mass state, so that only the difference between the energies of the mass states (due to the difference in the neutrino masses) governs the time development of the state. With $p >> m_k$, we can approximate the energy as

$$E_k = \sqrt{p^2 + m_k^2} \approx p + m_k^2/2p = p + M_k \ , \tag{6}$$

where $M_k = m_k^2/2p$ ($k = 1, 2$). The neutrino evolves in time Δt as

$$\begin{aligned} |\nu(t+\Delta t)\rangle &= a_1(t+\Delta t)e^{-iE_1\Delta t}|\nu_1\rangle + a_2(t+\Delta t)e^{-iE_2\Delta t}|\nu_2\rangle \\ &\approx a_1(t+\Delta t)\, e^{-iM_1\Delta t}|\nu_1\rangle + a_2(t+\Delta t)e^{-iM_2\Delta t}|\nu_2\rangle \quad . \end{aligned} \tag{7}$$

We have dropped the overall phase factor of $\exp(-ip\Delta t)$ in Equation (7) because it has no bearing on the final result. With the help of Equations (4a) and (4b), we can write Equation (7) in the flavor basis:

$$|\nu(t+\Delta t)\rangle = [a_1(t+\Delta t)e^{-iM_1\,\Delta t}\cos\,\theta + a_2(t+\Delta t)e^{-iM_2\,\Delta t}\sin\,\theta]\,|\nu_e\rangle$$
$$+ [-a_1(t+\Delta t)e^{-iM_1\,\Delta t}\sin\,\theta + a_2(t+\Delta t)e^{-iM_2\,\Delta t}\cos\,\theta]\,|\nu_\mu\rangle \quad . \tag{8}$$

We next consider the neutrino as a superposition of flavor states:

$$|\nu(t)\rangle = a_e(t)\,|\nu_e\rangle + a_\mu(t)\,|\nu_\mu\rangle \quad . \tag{9}$$

Because only electron neutrinos interact via charged currents, the two flavor states have different forward-scattering amplitudes, and each sees a different effective refractive index in matter. We assume that the change in the probability amplitudes $a_e(t)$ and $a_\mu(t)$ during an infinitesimal time Δt can be expressed as a simple phase shift that is proportional to the refractive index:

$$a_e(t+\Delta t) \approx a_e(t)\,\exp\,[ip\,(n_{\mathrm{nc}} + n_{\mathrm{cc}} - 1)\Delta \mathrm{t}] = a_e(t)\exp\,[i(\xi + \eta)\Delta \mathrm{t} \quad , \text{ and} \tag{10a}$$

$$a_\mu(t+\Delta t) \approx a_\mu(t)\,\exp\,[ip\,(n_{\mathrm{nc}} - 1)\Delta \mathrm{t}] = a_\mu(t)\exp\,[i\xi\,\Delta \mathrm{t}] \quad , \tag{10b}$$

where $\xi = (2\pi N_e\,/p)f_{\mathrm{nc}}$ and $\eta = \sqrt{2}G_F\,N_e$. The latter relation is the matter oscillation term. We have also used $\Delta x \approx \Delta t$. The neutrino state, therefore, evolves as

$$|\nu(t+\Delta t)\rangle = a_e(t)e^{i(\xi+\eta)\Delta t}\,|\nu_e\rangle + a_\mu(t)e^{\,i\xi\,\Delta t}\,|\nu_\mu\rangle$$
$$= a_e(t)e^{i\eta\Delta t}\,|\nu_e\rangle + a_\mu(t)\,|\nu_\mu\rangle \quad , \tag{11}$$

where again we have dropped the overall phase factor of $\exp(i\xi\Delta t)$ because it does not affect the final result. Equations (8) and (11) are expressions for $|\nu(t+\Delta t)\rangle$. Equating the coefficients of $|\nu_e\rangle$ and $|\nu_\mu\rangle$ results in a set of coupled equations:

$$a_1(t+\Delta t)e^{-iM_1\,\Delta t}\cos\theta + a_2(t+\Delta t)e^{-iM_2\,\Delta t}\sin\theta = a_e(t)e^{i(\xi+\eta)\Delta t}\ , \tag{12a}$$

$$-a_1(t+\Delta t)e^{-iM_1\,\Delta t}\sin\theta + a_2(t+\Delta t)e^{-iM_2\,\Delta t}\cos\theta = a_\mu(t)e^{i\xi\,\Delta t}\ . \tag{12b}$$

Both sides of Equations (12a) and (12b) are expanded to first order in Δt,

$$[a_1(t) + \dot{a}_1(t)\Delta t - ia_1(t)M_1\Delta t]\cos\theta + [a_2(t) + \dot{a}_2(t)\Delta t - ia_2(t)M_2\Delta t]\sin\theta = a_e(t)(1+i\eta\Delta t) \tag{13a}$$

$$-[a_1(t) + \dot{a}_1(t)\Delta t - ia_1(t)M_1\Delta t]\sin\theta + [a_2(t) + \dot{a}_2(t)\Delta t - ia_2(t)M_2\Delta t]\cos\theta = a_\mu(t)\ , \tag{13b}$$

where a dot indicates the time derivative. Equations (4c) and (4d) are used to express $a_1(t), \dot{a}_1(t)$, $a_2(t)$, and $\dot{a}_2(t)$ in terms of $a_e(t)$, $\dot{a}_e(t)$, $a_\mu(t)$, and $\dot{a}_\mu(t)$. Following more algebraic operations,

$$-i\dot{a}_e(t) = [M_1\cos^2\theta + M_2\sin^2\theta + \eta]a_e(t) + (M_2 - M_1)\cos\theta\sin\theta\, a_\mu(t)\ , \tag{14a}$$

$$-\dot{a}_\mu(t) = (M_2 - M_1)\cos\theta\sin\theta\, a_e(t) + [M_1\sin^2\theta + M_2\cos^2\theta]a_\mu(t)\ . \tag{14b}$$

These expressions can be cast in a Schrödinger-like equation for a column matrix A consisting of the probability amplitudes $a_e(t)$ and $a_\mu(t)$:

$$-i\frac{dA}{dt}=HA\,, \tag{15}$$

$$\text{where } H=\begin{pmatrix} M_1\cos^2\theta+M_2\sin^2\theta+\eta & (M_2-M_1)\cos\theta\sin\theta \\ (M_2-M_1)\cos\theta\sin\theta & M_1\sin^2\theta+M_2\cos^2\theta \end{pmatrix} \text{ and } A=\begin{pmatrix} a_e(t) \\ a_\mu(t) \end{pmatrix}$$

The eigenvalues of the matrix H are given by

$$\chi_{1,2}=\frac{\eta+M_1+M_2}{2}\mp\frac{\sqrt{\eta^2+(M_2-M_1)^2-2\eta(M_2-M_1)\cos 2\theta}}{2}\,. \tag{16}$$

Equation (15) can then be solved:

$$A(t)=\left[\frac{\chi_2 e^{i\chi_1 t}-\chi_1 e^{i\chi_2 t}}{\chi_2-\chi_1}I+\frac{e^{i\chi_2 t}-e^{i\chi_1 t}}{\chi_2-\chi_1}H\right]A(0)\,, \tag{17}$$

where I is the identity matrix. At time $t=0$, the beam consists only of electron neutrinos. Thus, $a_e(0)=1$, and $a_\mu(0)=0$ so that

$$a_e(t)=\frac{\chi_2 e^{i\chi_1 t}-\chi_1 e^{i\chi_2 t}}{\chi_2-\chi_1}+\frac{e^{i\chi_2 t}-e^{i\chi_1 t}}{\chi_2-\chi_1}(M_1\cos^2\theta+M_2\sin^2\theta+\eta)\,, \tag{18a}$$

$$a_\mu(t)=\frac{e^{i\chi_2 t}-e^{i\chi_1 t}}{\chi_2-\chi_1}(M_2-M_1)\cos\theta\sin\theta\,. \tag{18b}$$

The probability of detecting a muon neutrino after a time t is given by

$$P_{\mathrm{MSW}}(\nu_e \to \nu_\mu) = \left|\langle \nu_\mu | \nu(t) \rangle\right|^2 = \left|a_e(t)\langle \nu_\mu | \nu_e \rangle + a_\mu(t)\langle \nu_\mu | \nu_\mu \rangle\right|^2 = \left|a_\mu(t)\right|^2 \qquad (19)$$

so that

$$P_{\mathrm{MSW}}(\nu_e \to \nu_\mu) = \frac{(M_2 - M_1)^2 \cos^2\theta \sin^2\theta}{(\chi_2 - \chi_1)^2} 2(1 - \cos(\chi_2 - \chi_1)t)$$

$$= \frac{(M_2 - M_1)^2 \sin^2 2\theta}{(\chi_2 - \chi_1)^2} \sin^2 \frac{(\chi_2 - \chi_1)}{2} t \ . \qquad (20)$$

By substituting in the expressions for χ_1, χ_2, M_1, M_2, we have

$$(\chi_2 - \chi_1)^2 = (M_2 - M_1)^2 \left[\sin^2 2\theta + \left(\frac{\eta}{M_2 - M_1} - \cos 2\theta \right)^2 \right] \qquad (21a)$$

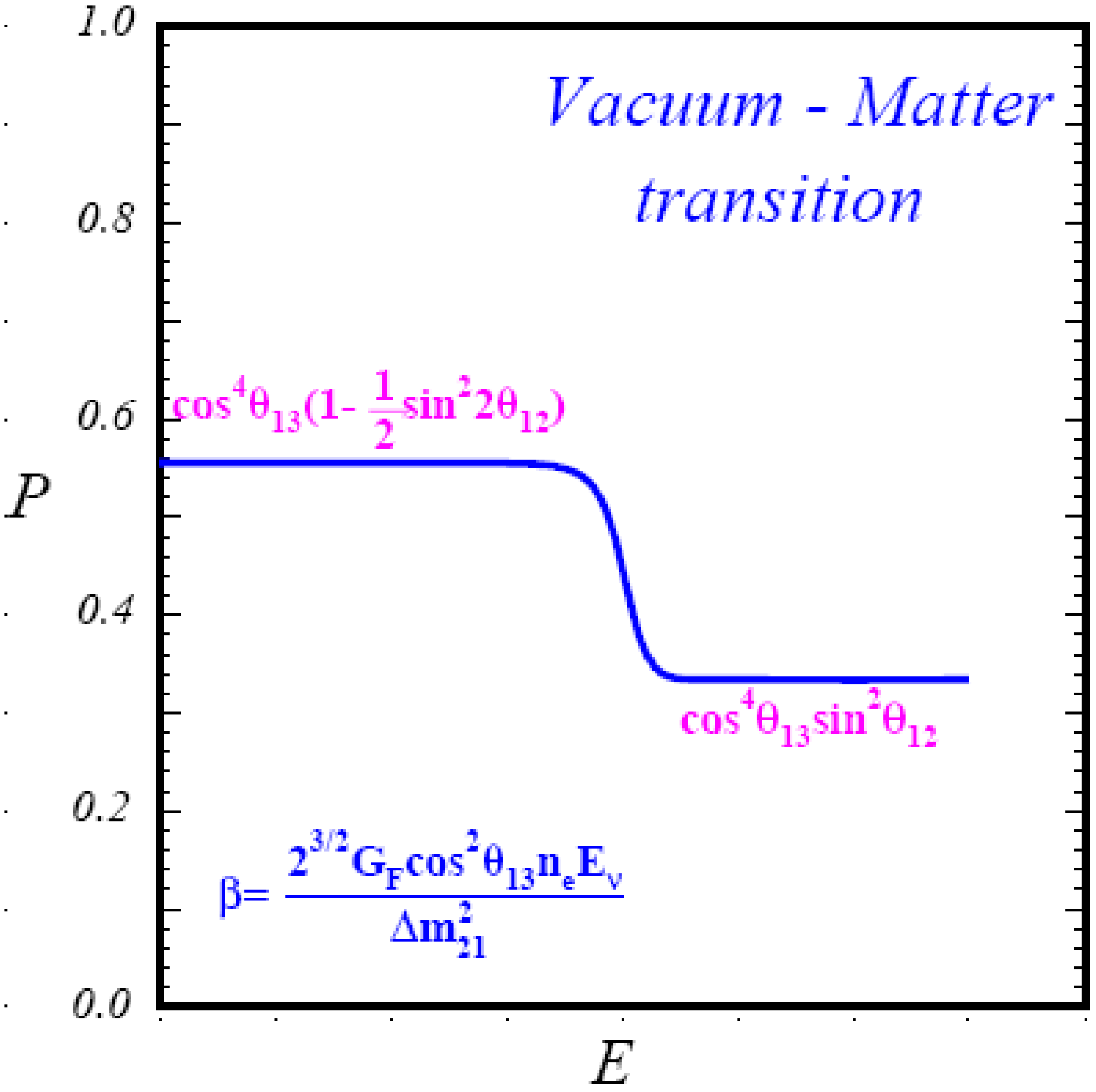
$$(M_2 - M_1) = \frac{\Delta m^2}{2p} \approx \frac{\Delta m^2}{2E_\nu} \ . \qquad (21b)$$

Recalling that $x = t$, and $\eta = \sqrt{2}G_F N_e$, we arrive at the MSW probability for an electron neutrino to oscillate into a muon neutrino:

$$P_{\mathrm{MSW}}(\nu_e \to \nu_\mu) = \frac{\sin^2 2\theta}{W^2} \sin^2\left(\frac{\pi x W}{\lambda}\right) ,$$

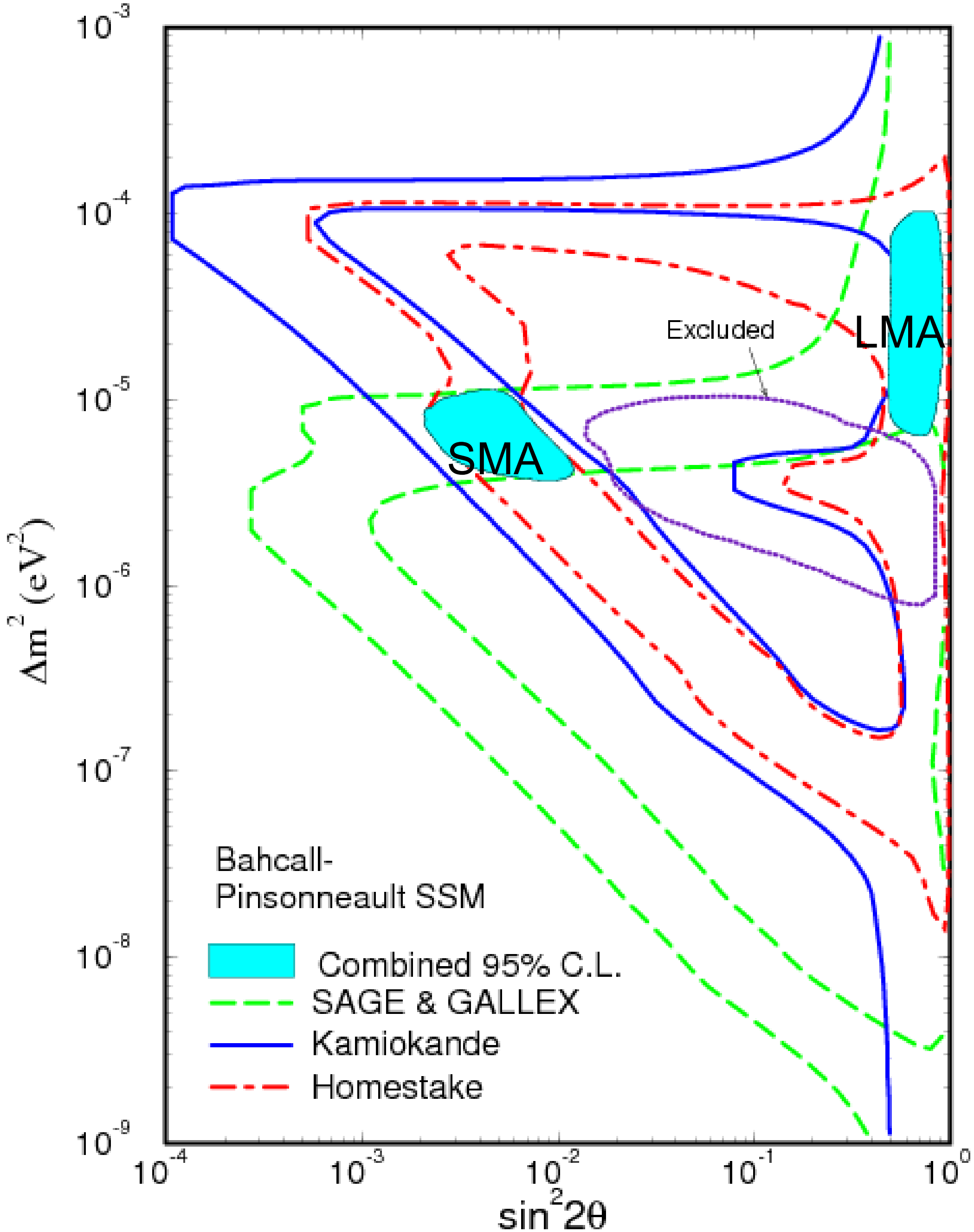
where λ is the *in vacuo* oscillation length,

$$W^2 = \sin^2 2\theta + \left(\sqrt{2}G_F N_e \frac{2E_\nu}{\Delta m^2} - \cos 2\theta\right)^2 \ , \qquad (23)$$



Per i neutrini solari la transizione dovrebbe avvenire intorno a 2MeV

1994 MSW



Oscillazioni nel vuoto
$\Delta m^2 < 10^{-10}$

"Best fit" ai dati di SNO

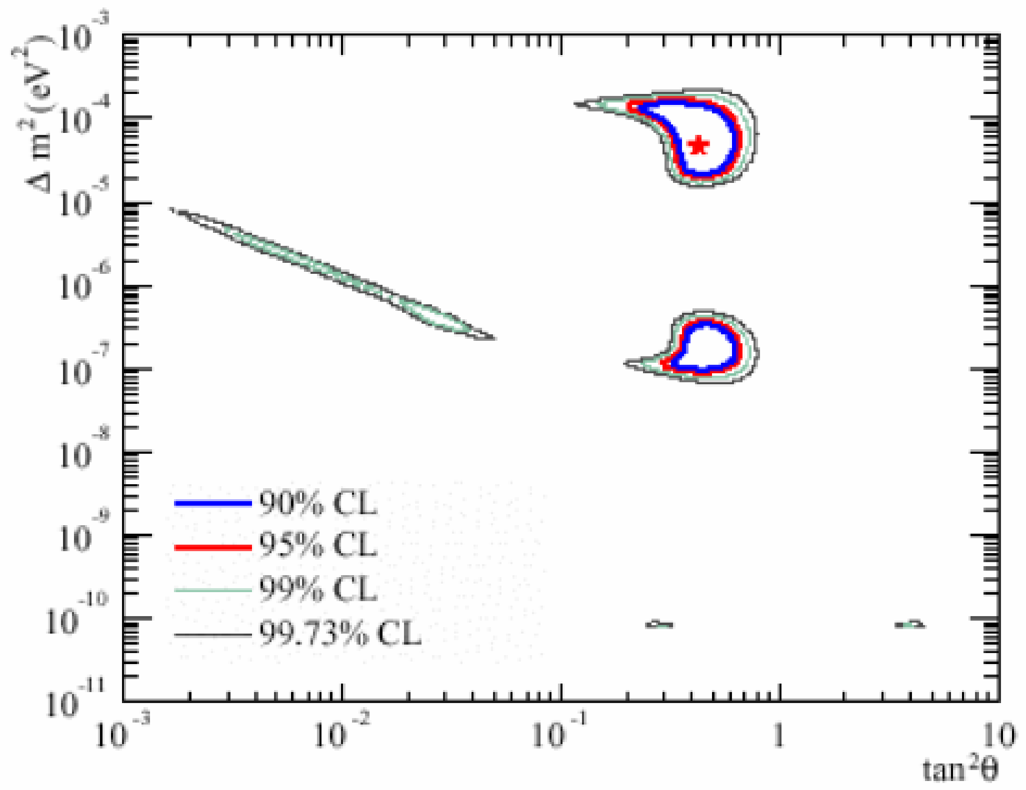
★ **Best fit:**

$$\Delta m^2 = 5.0 \times 10^{-5} \text{ eV}^2$$

$$\tan^2 \theta_{12} = 0.45$$

$$\chi^2 / N_{dof} = 68.9 / 69$$

Livelli di confidenza per "fits" con 2 parametri liberi	
CL	$\Delta\chi^2 = \chi^2 - \chi^2_{min}$
68.27%	2.30
90%	4.61
95%	5.99
99%	9.21
99.73%	11.83



<u>NOTA</u>: $\tan^2\theta$ preferita a $\sin^2 2\theta$ perchè $\sin^2 2\theta$ è simmetrico rispetto a $\theta = 45°$

$$\sin 2(45^0 - \theta) = \sin(90^0 - 2\theta) = \sin(90^0 + 2\theta) = \sin 2(45^0 + \theta)$$

e le soluzioni MSW esistono solo se $\theta < 45°$

"Best fit" globale
ai risultati di tutti gli esperimenti sui neutrini solari

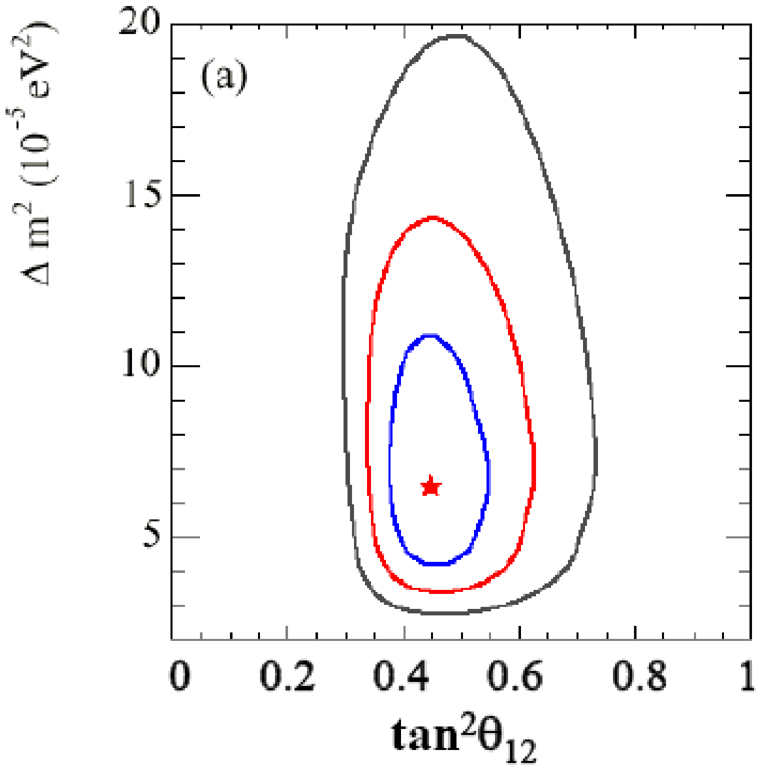
Soluzione LMA-MSW

★ **Best fit:**

$$\Delta m^2 = (\,6.5\,^{+4.4}_{-2.3}\,)\ \mathrm{x}\ 10^{-5}\ \mathrm{eV}^2$$

$$\tan^2\theta_{12} = 0.45\,^{+0.09}_{-0.08}$$

$$\chi^2 / N_{dof} = 113.1 / 116$$



2003: KamLAND

- Prima (?) evidenza di oscillazioni con neutrini da sorgenti artificiali

<u>Reattori nucleari</u>: sorgenti intense, isotrope di $\bar{\nu}_e$ da decadimento β dei frammenti di fissione.

Spettro in energia ($E \leq 10$ MeV, $\langle E \rangle \approx 3$ MeV) determinato sperimentalmente.

Frequenza di produzione $\bar{\nu}_e$: $1.9 \times 10^{20}\ P_{th}\ \ s^{-1}$

(P_{th}: potenza termica del reattore in GW)

Incertezza sul flusso $\bar{\nu}_e$: ± 2.7 %

<u>Rivelazione</u>:

$\bar{\nu}_e + p \rightarrow e^+ + n$ (sui protoni liberi di scintillatore liquido contenente idrogeno)

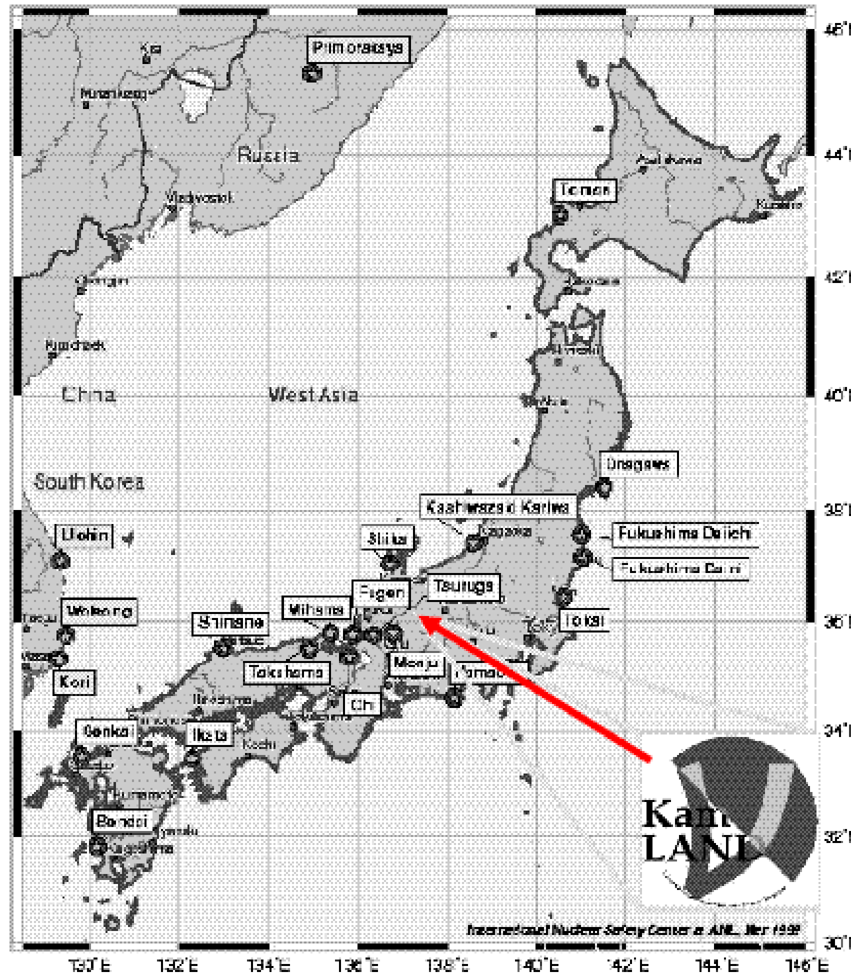
$e^+ e^- \rightarrow 2\gamma$
segnale iniziale
$E = E_\nu - 0.77$ MeV

"termalizzazione" da collisioni multiple ($\langle t \rangle \approx 180\ \mu s$), seguita da cattura
$n + p \rightarrow d + \gamma$ ($E_\gamma = 2.2$ MeV)
segnale ritardato

KamLAND (KAMioka Liquid scintillator Anti-Neutrino Detector)

Sorgente $\bar{\nu}_e$: reattori nucleari in Giappone

Potenza termica totale 70 GW
>79% del flusso $\bar{\nu}_e$ prodotto da 26 reattori, 138 < L < 214 km
Media pesata delle distanze:
$<L>$: 180 km (peso = flusso $\bar{\nu}_e$)



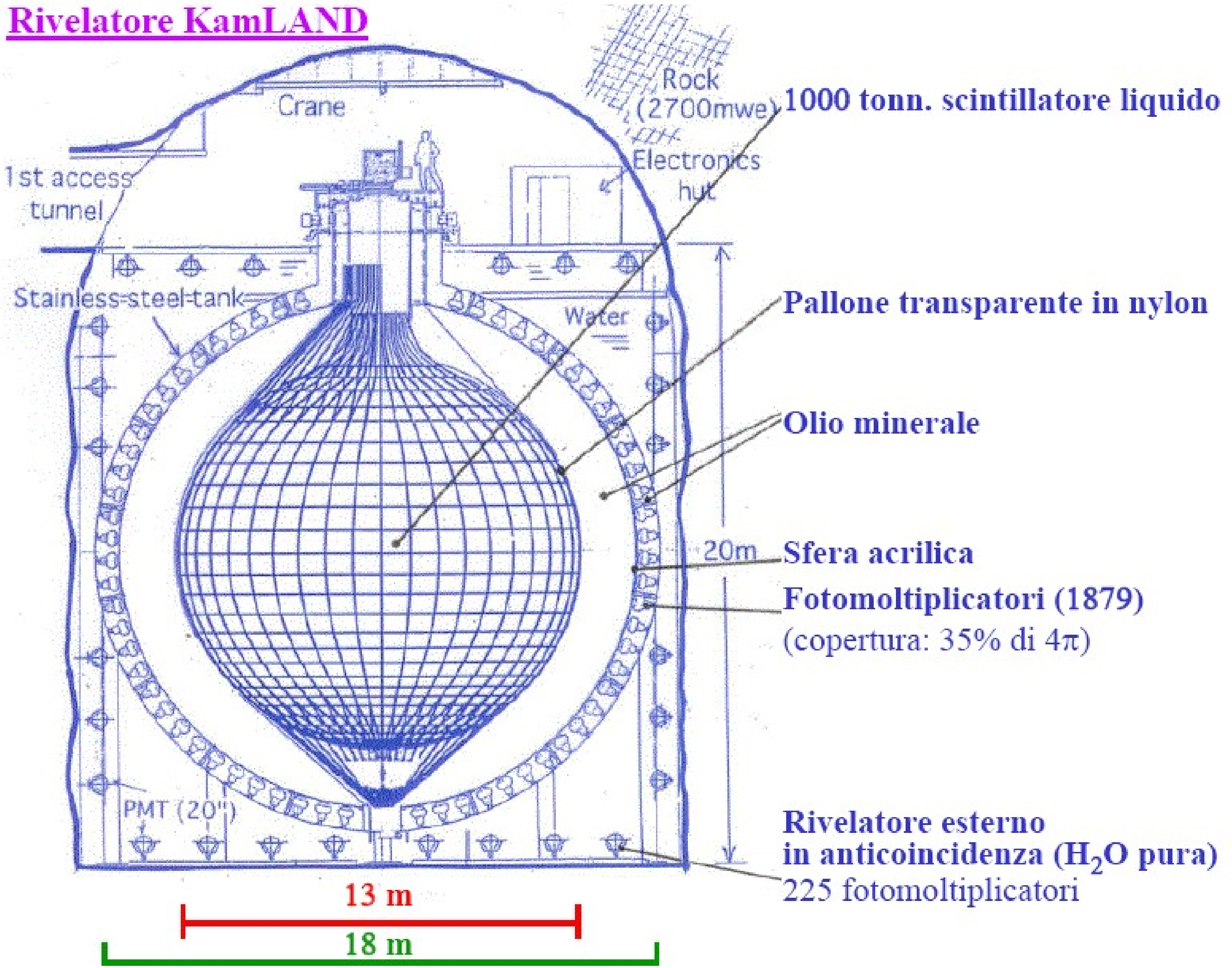
Flusso $\bar{\nu}_e$ predetto ≈ 1.3×10^6 cm^{-2} s^{-1}
(tutti i reattori a potenza massima, assenza di oscillazioni)

Lunghezza d'oscillazione media per $\Delta m^2 = 6.5 \times 10^{-5}$ eV^2 :

$$< \lambda_{osc} > \approx 120 \text{ km}$$

probabilità di scomparsa uguali per ν_e e $\bar{\nu}_e$?

Rivelatore KamLAND

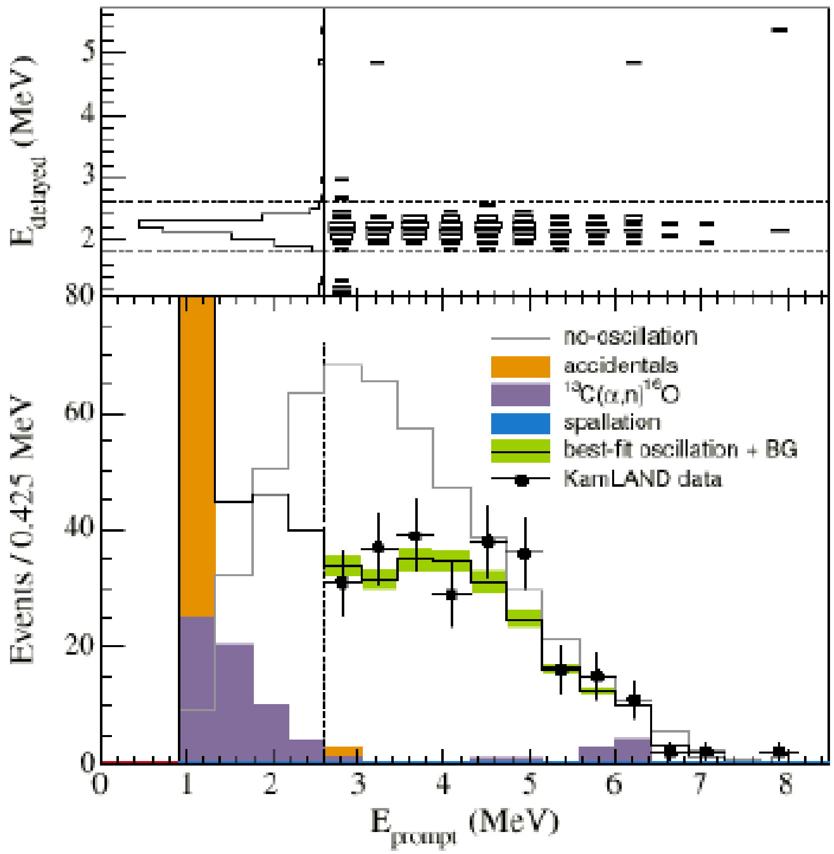


KamLAND: analisi dati Marzo 2002 – Gennaio 2004

Selezione eventi:

Segnale iniziale: $2.6 < E < 8.5$ MeV, distanza dal centro < 5.5 m

Segnale ritardato: $0.5 < \Delta t < 660$ μs, $\Delta R < 1.6$ m rispetto al segnale iniziale



258 eventi compatibili con $\bar{\nu}_e + p \rightarrow e^+ + n$ (segnale iniziale)

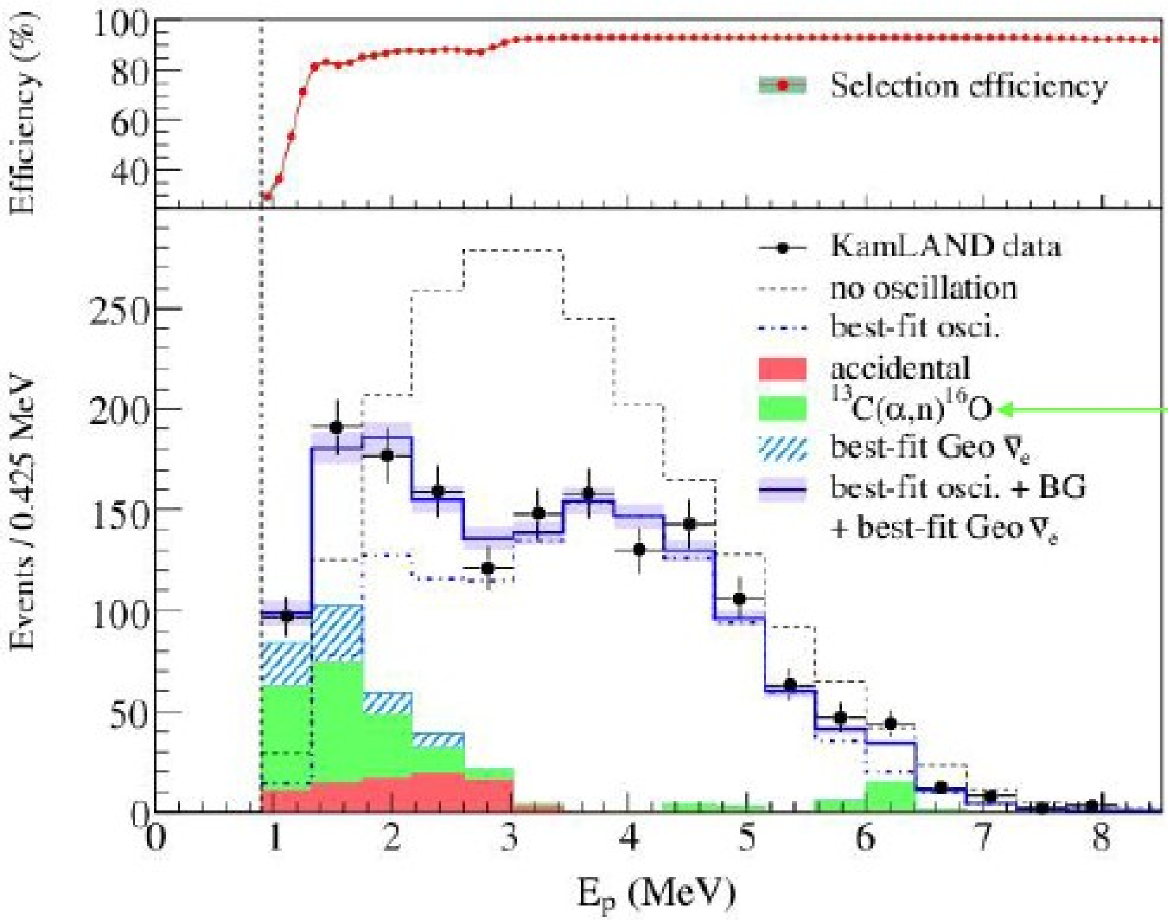
+ segnale ritardato **$n + p \rightarrow d + \gamma$** ($E_\gamma = 2.2$ MeV)

Fondo 17.8 ± 7.3 eventi

Numero eventi aspettati in assenza di oscillazioni: 365.2 ± 23.7

KamLAND: risultati recenti

(febbraio 2008)



Contaminazione dello scintillatore da radioattività α

Numero di eventi aspettati in assenza di oscillazioni: 2179 ± 89 (sist.)

Eventi di fondo: 276.1 ± 23.5

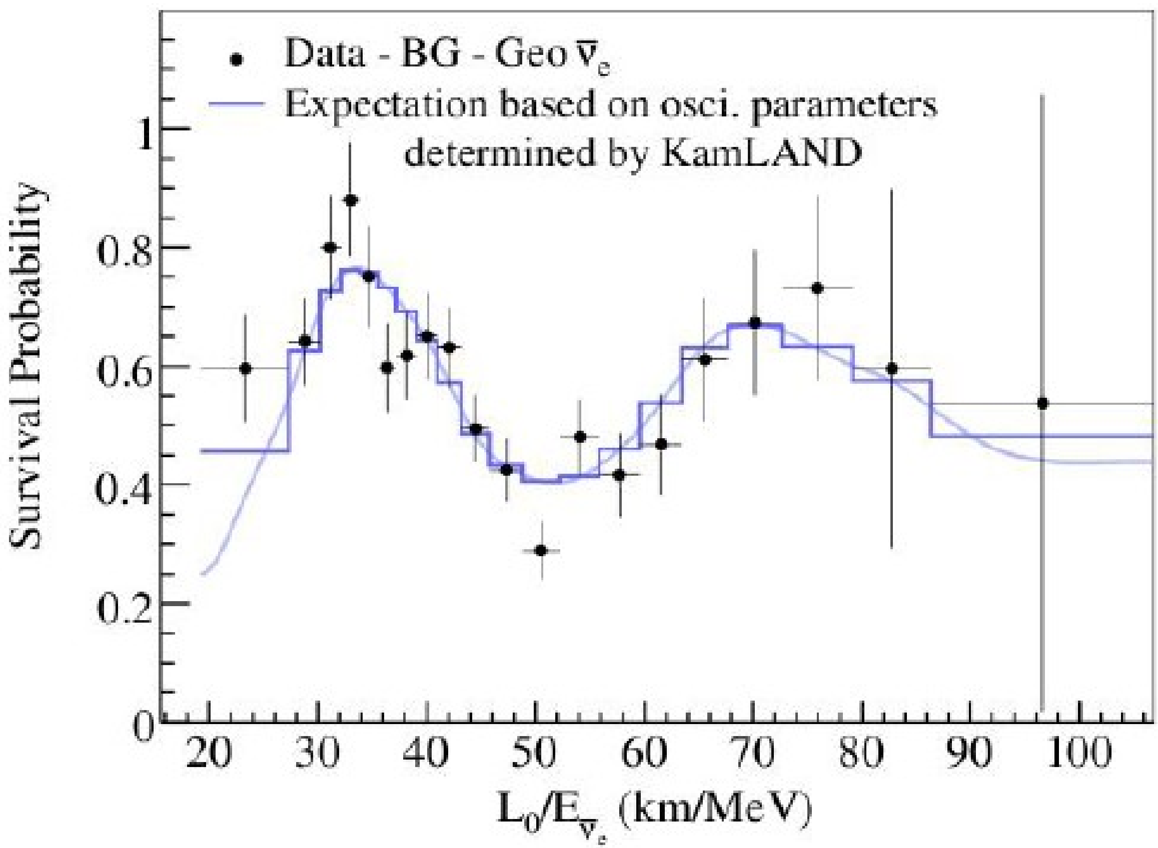
Numero di eventi osservati : 1609

KamLAND: probabilità di scomparsa di $\bar{\nu}_e$

$$\mathscr{P}_{ee} = 1 - \sin^2(2\theta)\sin^2(1.267\,\Delta m^2 \frac{L_0}{E})$$

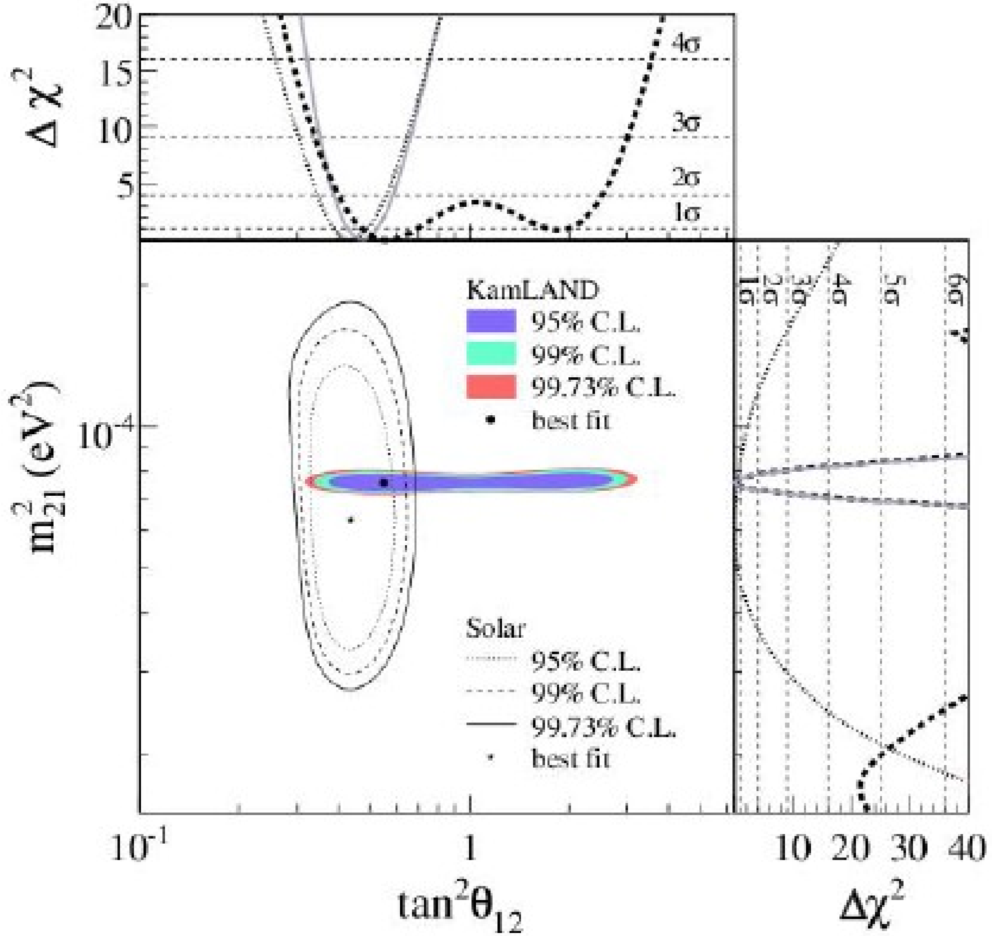
Best fit

$$\boxed{\begin{gathered}\Delta m^2 = (7.58^{+0.14}_{-0.13} \pm 0.15)\times 10^{-5}\ \mathrm{eV}^2 \\ \tan^2\theta = 0.56^{+0.10}_{-0.07}(\mathrm{stat})^{+0.10}_{-0.06}(\mathrm{sist})\end{gathered}}$$



L_0 = 180 km
distanza media
sorgente - rivelatore

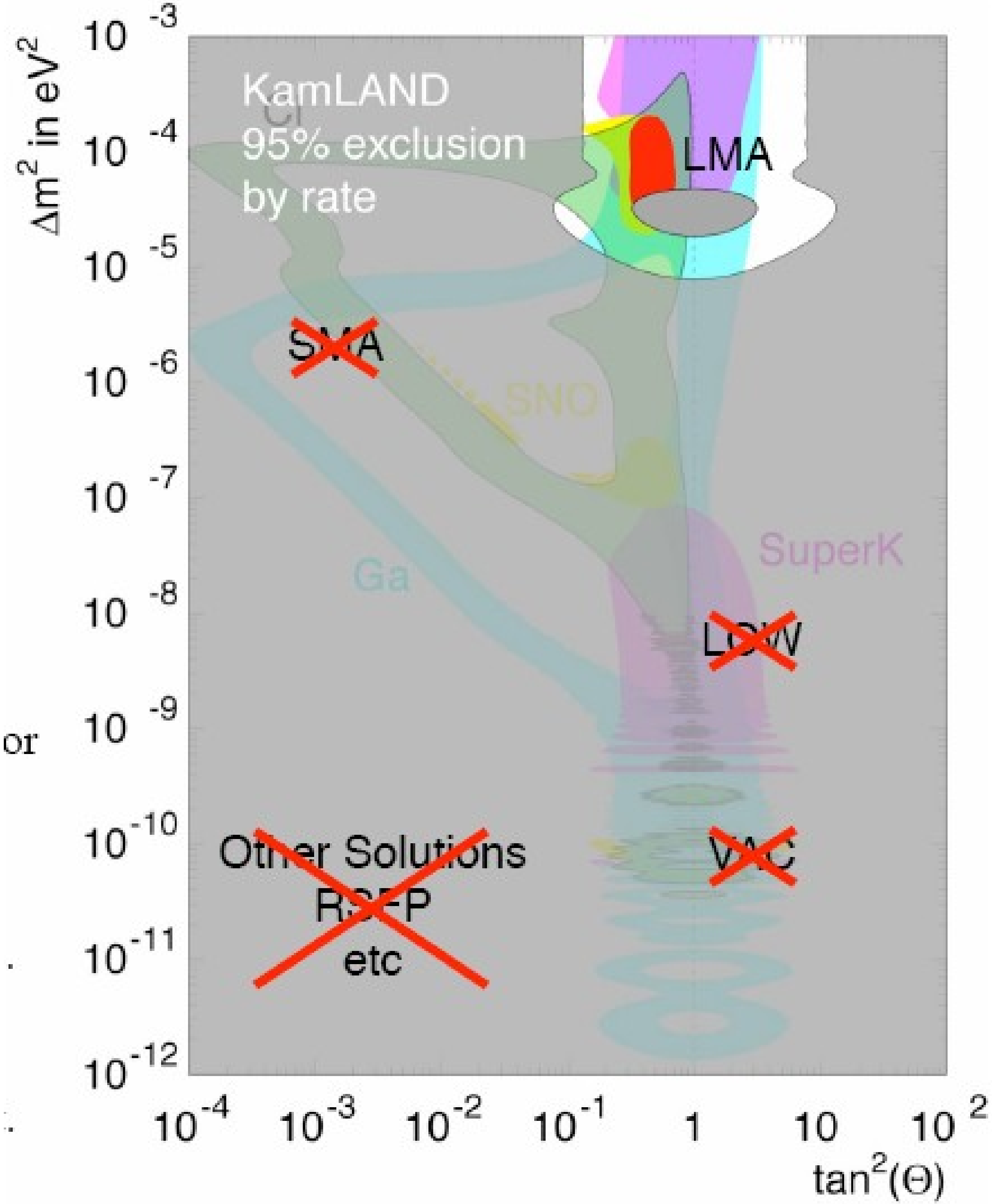
Best fit a tutti i dati sui neutrini solari + KamLAND



Best fit combinato: $\Delta m^2 = (7.59 \pm 0.21) \times 10^{-5}\ \text{eV}^2$

$$\tan^2\theta = 0.47^{+0.06}_{-0.05} \Rightarrow \theta = 34.4^{\circ}\,{}^{+1.6^{\circ}}_{-1.5^{\circ}}$$

or


KamLAND
95% exclusion
by rate
Cl
LMA
SMA
SNO
Ga
SuperK
LOW
VAC
Other Solutions
RSFP
etc
Δm^2 in eV^2
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10
10^{2}
$\tan^2(\Theta)$

Borexino

- Proposto nel 1991 @ LNGS
- Primi risultati pubblicati nel 2008

How do we detect ν /anti-ν in BX ??

Borexino is an ultrapure organic scintillator detector made by 278 tons of PC+PPO

ν_x are detected throught their scattering off electrons:

$\nu_x + e^- \longrightarrow \nu_x + e^-$ σ_{CC}=9.2 10^{-45} E_ν(MeV) cm^2 σ_{CC}~6 σ_{NC}

anti-ν_e are detected throught the inverse beta decay on protons:

$\nu_e + p \longrightarrow n + e^+$

- ✓ E_{thr} = 1.8 MeV
- ✓ E_{e+}=E_ν-0.78 MeV
- ✓ Delayed coincidence : τ_n ~ 256 μs in PC

Particle detection via the emitted scintillation light:

- ✓ *Very low energy threshold (40 keV);*
- ✓ *Good energy and spatial resolution (L.Y.~ 500 p.e./MeV)* ..but…
- ✓ *No directional information*
- ✓ *Background rejection critical: the ν_x induced events can't be distinguished from the other β events due to natural radioctivity*

A ultrapure detector is mandatory….

The BOREXINO detector

- PMT total collected charge -> light yield (p.e) -> event energy
- Photon arrival times on each PMT -> event position

ENERGY RESOLUTION

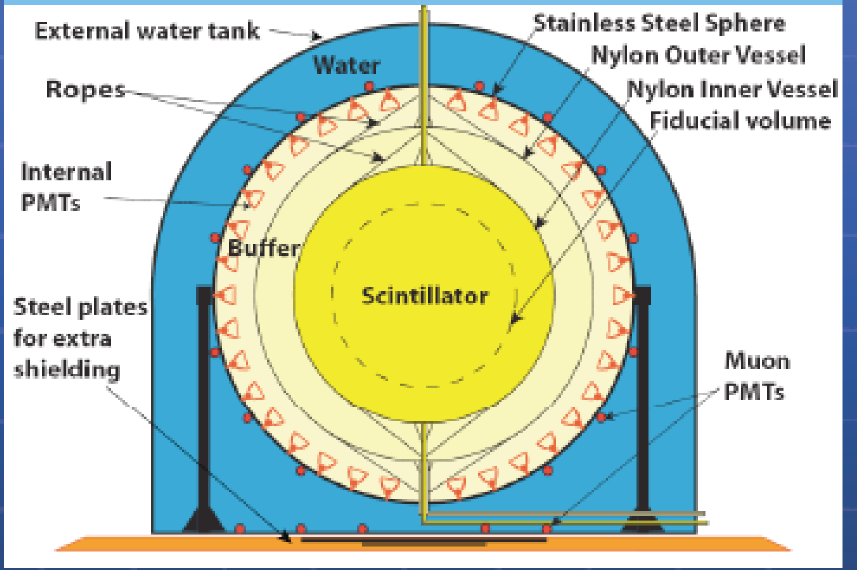
10% @ 200 keV
8% @ 400 keV
5% @ 1 MeV

SPATIAL RESOLUTION

35 cm @ 200 keV
16 cm @ 500 keV

Extreme radiopurity of scintillator = 15 years of work !!!

The dectector is now calibrated!!!

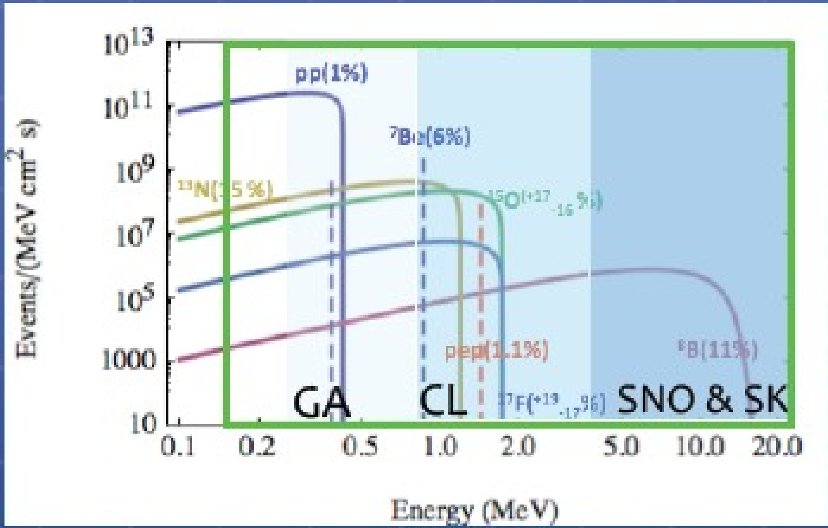


- **External backgrounds:** underground lab., principle of progressive shieldings
- **Internal backgrounds:** accurate material selections and clean manipulations, liquid handling plants in situ (WE, nitrogen stripping, distillation)

Most important backgrounds:

$^{238}U \sim 2\ 10^{-17}$ g/g, $^{232}Th \sim 5\ 10^{-18}$ g/g, $^{210}Po \sim 10$ c/d/t, $^{210}Bi \sim 15$ c/d/100t, $^{85}Kr \sim 30$ c/d/100t

Neutrino astrophysics: probing our knowledge of the Sun



BOREXINO

Importance of single solar-ν spectrum component precise flux measurements:

✓Solve the high/Low metallicity solar model controversy

✓Confirm MSW-LMA or exploit possible traces of non-standard neutrino-matter interaction / presence of mass varying ν's

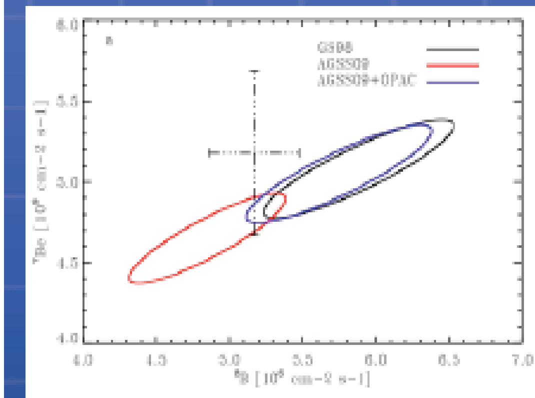
✓Fix the amount of solar energy produced via CNO cycle

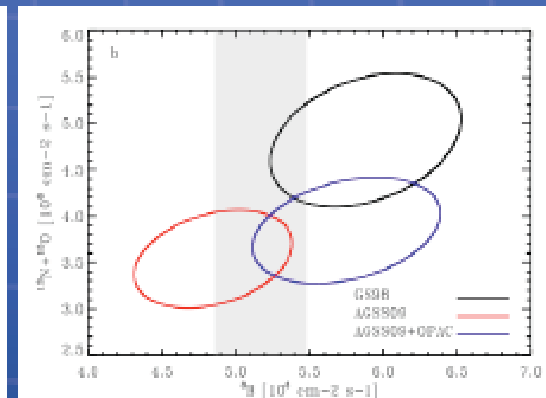
Sandra Zavatarelli, INFN Genova Italy

Neutrino astrophysics: probing our knowledge of the Sun

		GS98	AGS05
	pp	5.97×10^{10}	6.04×10^{10}
	pep	1.41×10^{8}	1.44×10^{8}
	hep	7.91×10^{3}	8.24×10^{3}
10%	^{7}Be	5.08×10^{9}	4.54×10^{9}
	^{8}B	5.88×10^{6}	4.66×10^{6}
40%	^{13}N	2.82×10^{8}	1.85×10^{8}
	^{15}O	2.09×10^{8}	1.29×10^{8}
	^{17}F	5.65×10^{6}	3.14×10^{6}

Serenelli arXiv:0910.3690



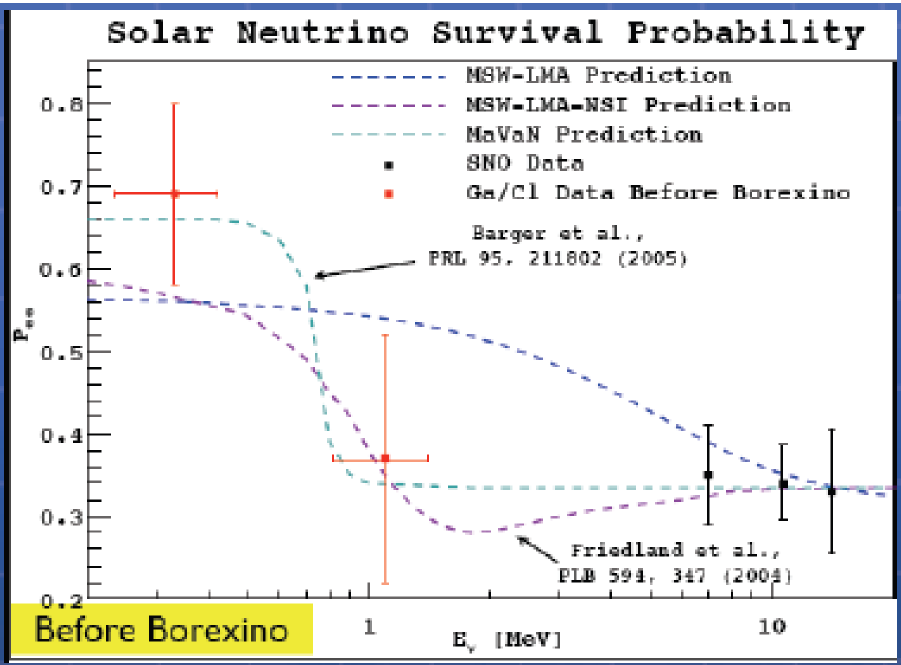


Flux: $cm^{-2}s^{-1}$ (BPS09)

- ✓Solve the high/Low metallicity solar model controversy
- ✓Confirm MSW-LMA or exploit possible traces of non-standard neutrino-matter interaction / presence of mass varying ν's
- ✓Fix the amount of solar energy produced via CNO cycle

Sandra Zavatarelli, INFN Genova Italy

Neutrino astrophysics: probing our knowledge of the Sun

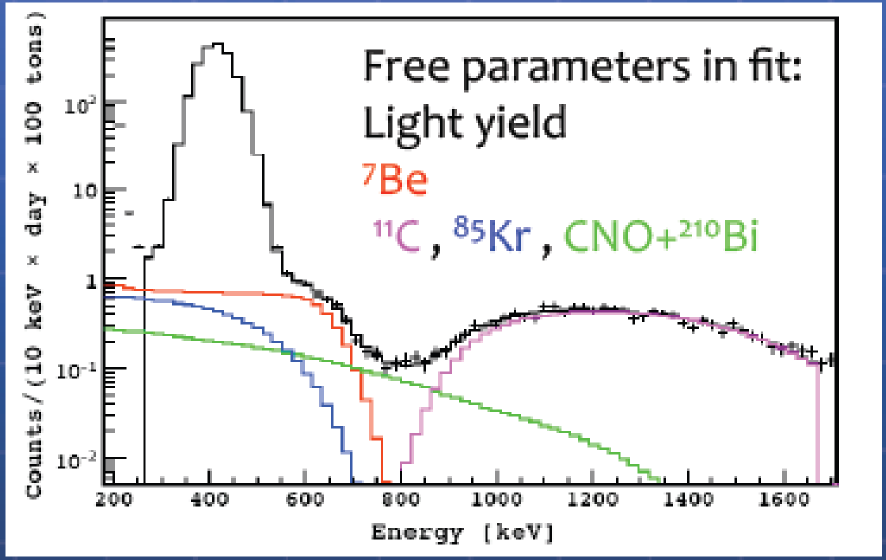


- ✓Solve the high/Low metallicity solar model controversy
- ✓Confirm MSW-LMA or exploit possible traces of non-standard neutrino-matter interaction / presence of mass varying ν's
- ✓Fix the amount of solar energy produced via CNO cycle

Neutrino astrophysics: the measure of the ^{7}Be solar neutrino flux

1st result (30 % precision) - Phys.Lett.B (2007): ^{7}Be Rate = $47 \pm 7_{stat} \pm 12_{syst}$ cpd/100t (47.4 days)

2nd result (10% precision)- PRL 101 (2008): **^{7}Be Rate = $49 \pm 3_{stat} \pm 4_{sys}$ cpd/100 tons (192 days)**



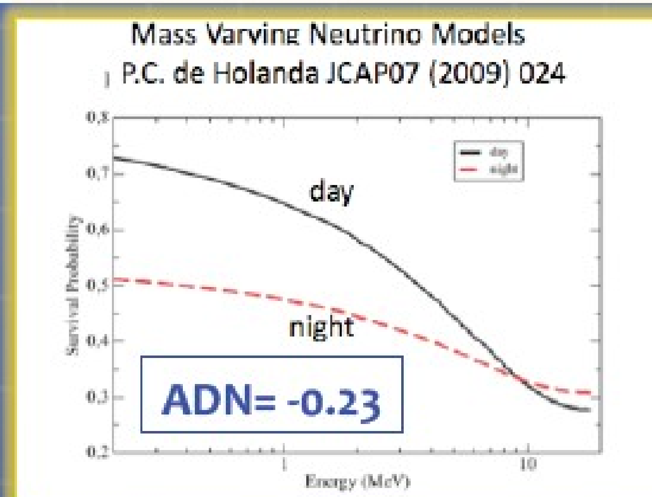
Expected rate cpy/100 t		
No oscillations	BPS07 (GS98)	BPS07 (AGS05)
75 ± 4	48 ± 4	44 ± 4

3rd result: now a 5% precision measurement and the seasonal variation study are possible!!!

- **Detector calibrated**
- **Monte Carlo fitting procedure implemented**
- **^{85}Kr content known at 16% level (delayed coincidence)**
- **3 years of statistics!!!**

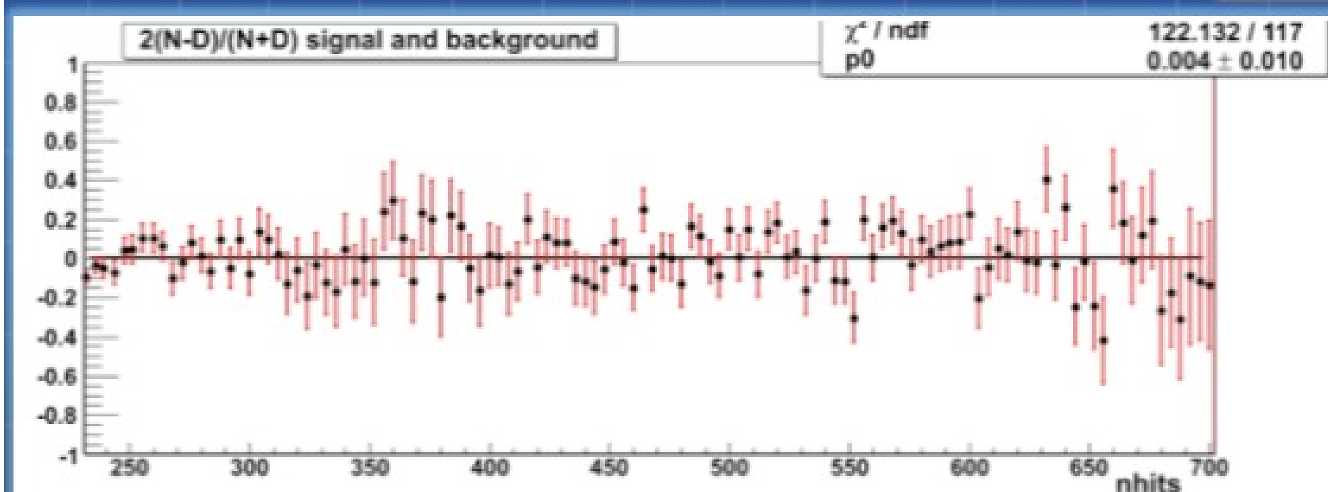
Neutrino astrophysics:
^{7}Be solar neutrino flux day/night asymmetry

- LMA solution to SNP -> no asymmetry
- MaVaN models -> possible asymmetry



$$ADN = \frac{N - D}{(N + D)/2}$$

Borexino result: *ADN*= 0.007 ± 0.073 (*stat*)



Day spectrum 387.5 d
Night spectrum 401.57 d
Stat. Error: 2.3 cpd/100t

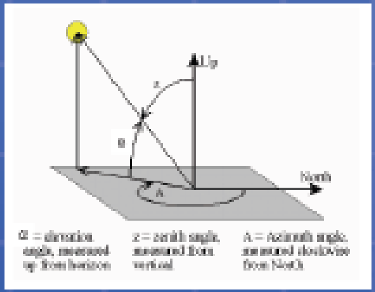
MaVaN model rejected at more than 3σ

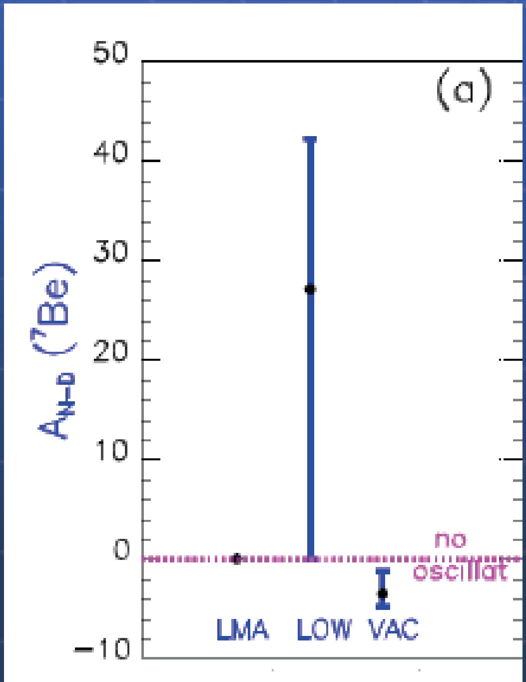
Sandra Zavatarelli, INFN Genova Italy

The D/N asymmetry in the ^{7}Be flux

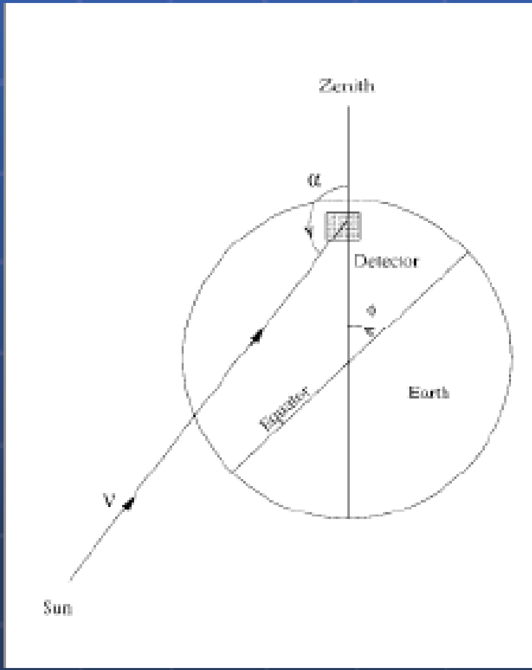
- MSW mechanism: ν interaction in the Earth could lead to a ν_e regeneration effect
- Solar ν flux higher in the night than in the day
- The amount of the effect depends
  - detector latitude
  - energy of the neutrinos

$$\theta \ \Delta m^2$$



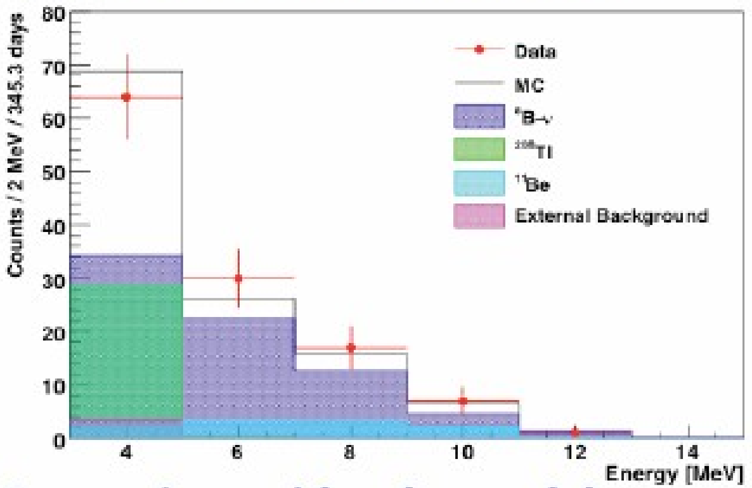


The absence of a day night effect for the ^{7}Be is a further confirmation of the LMA solution of the solar neutrino problem

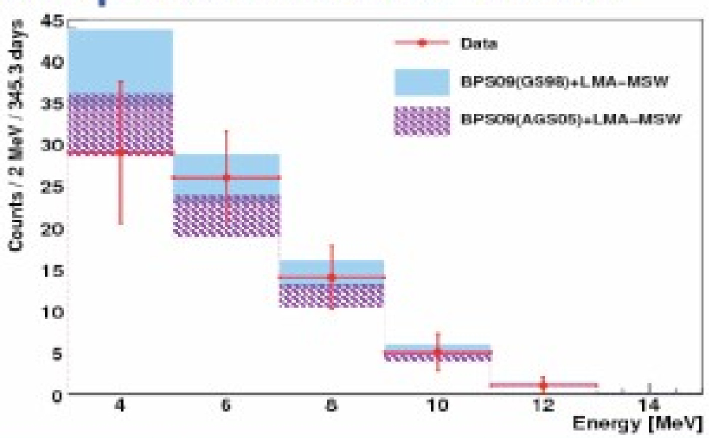


Neutrino astrophysics: the ^{8}B-ν final spectrum compared with models and other results

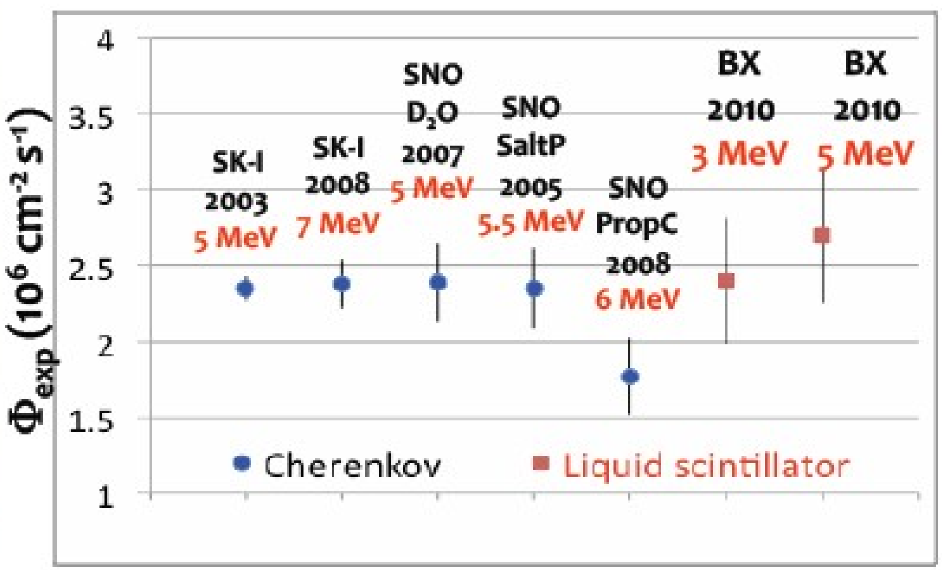
Final spectrum (exp.: 97 tons y)



Comparison with solar models



^{8}B solar ν flux measurements via elastic scattering



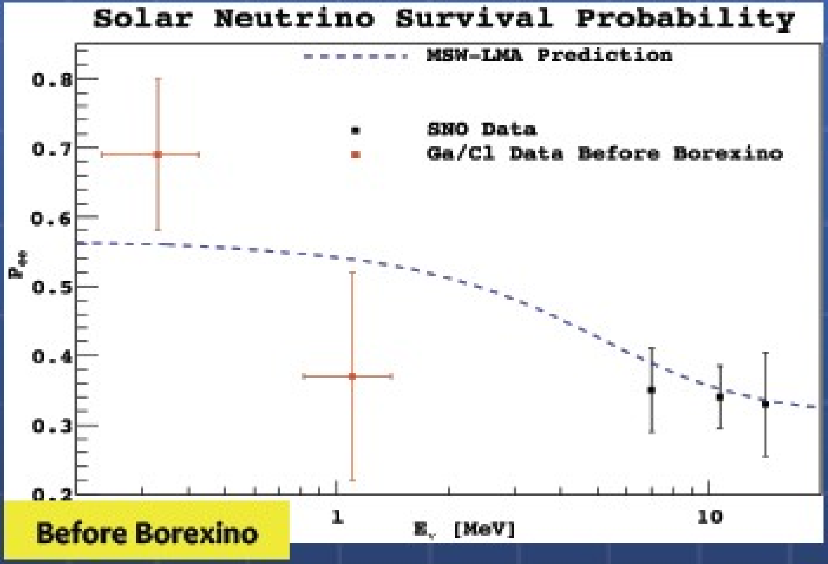
Threshold is defined @ 100% trigger efficiency

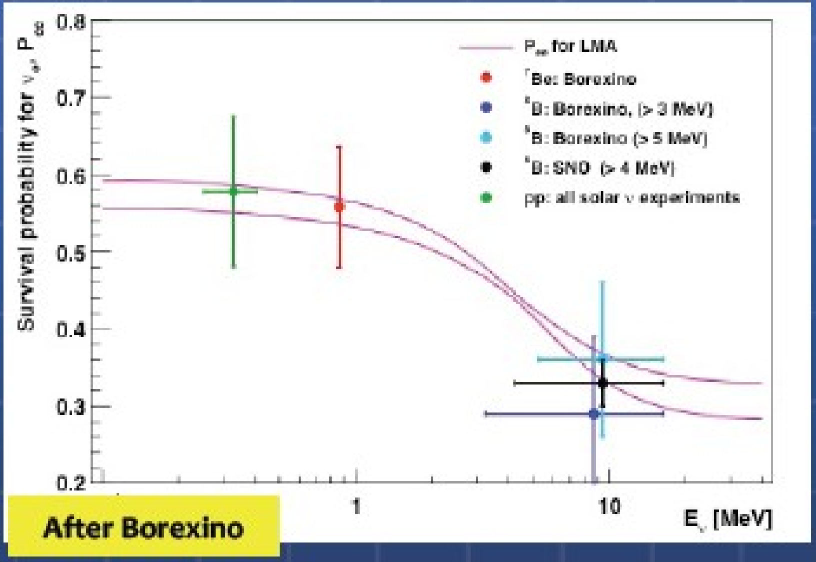
Borexino	3.0–16.3 MeV	5.0–16.3 MeV
Rate [cpd/100 t]	$0.22\pm0.04\pm0.01$	$0.13\pm0.02\pm0.01$
Φ^{ES}_{exp} [10^6 cm^{-2}s^{-1}]	$2.4\pm0.4\pm0.1$	$2.7\pm0.4\pm0.2$
$\Phi^{ES}_{exp}/\Phi^{ES}_{th}$	0.88 ± 0.19	1.08 ± 0.23

Neutrino astrophysics: testing the LMA solution to the solar neutrino problem

✓ **Borexino is the first experiment able to investigate simultaneously, in real time, the vacuum and matter regimes of oscillation**

Solar ν_e survival probability in vacuum-matter transition





^{7}Be ν: $P_{ee}=(0.56 \pm 0.10)$ ^{8}B ν: $\overline{P_{ee}}=(0.29 \pm 0.10)$ Distance = 1.9 σ

✓CNO, pep and pp ν-flux measurement: possible in case of positive result of running purifications