# Elementi di Fisica Teorica Contemporanea

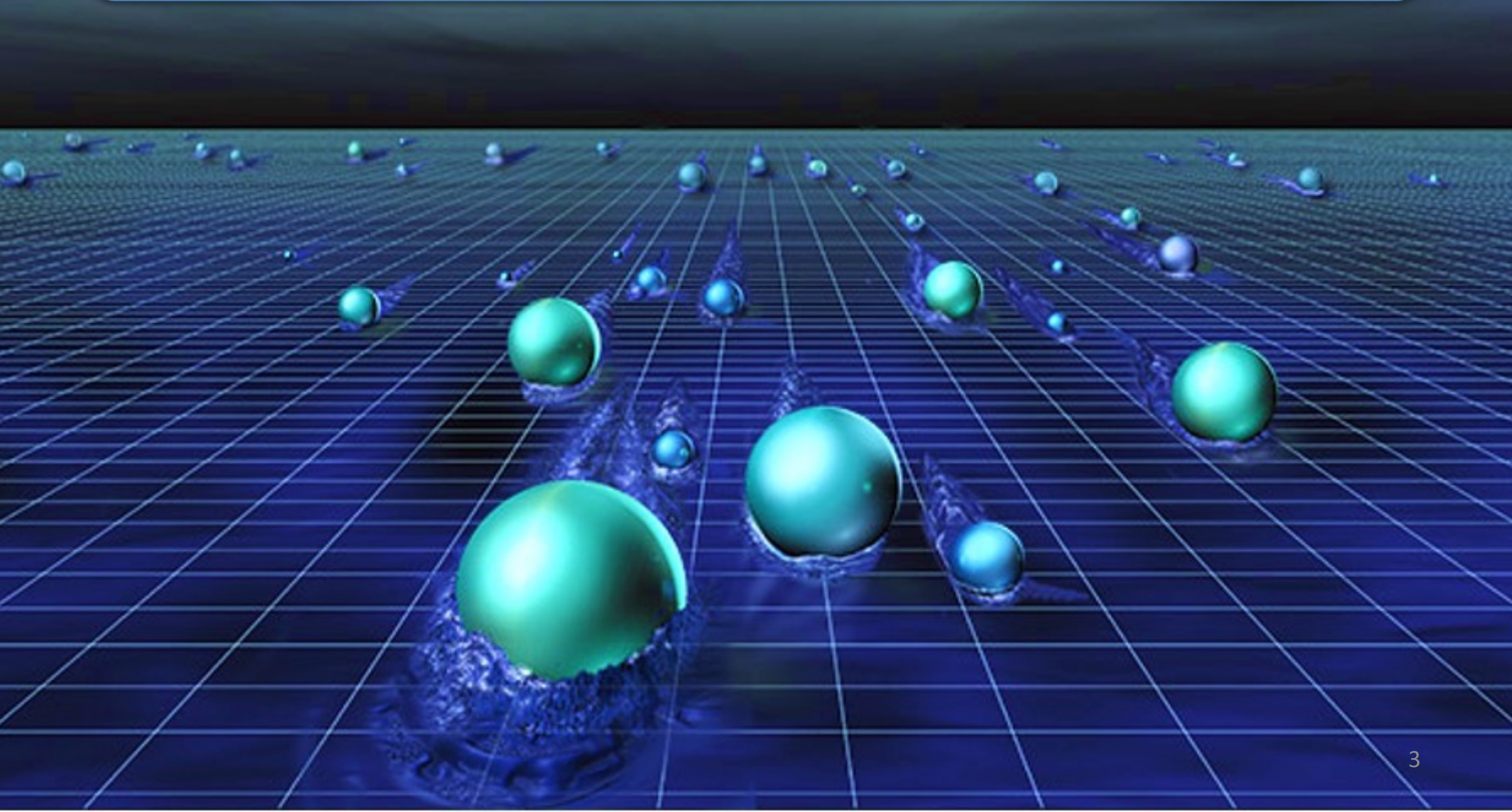
1. Teoria della Relatività
2. Meccanica quantistica
3. Particelle e campi (V. Lubicz / C. Tarantino)*
4. Gravità quantistica

(*) Testo consigliato:
Q. Ho-Kim, N. Kumar, C.S. Lam: *Invitation to Contemporary Physics*

# Particelle e campi
# Parte 3

## Sommario

# 1) La Teoria Quantistica dei Campi



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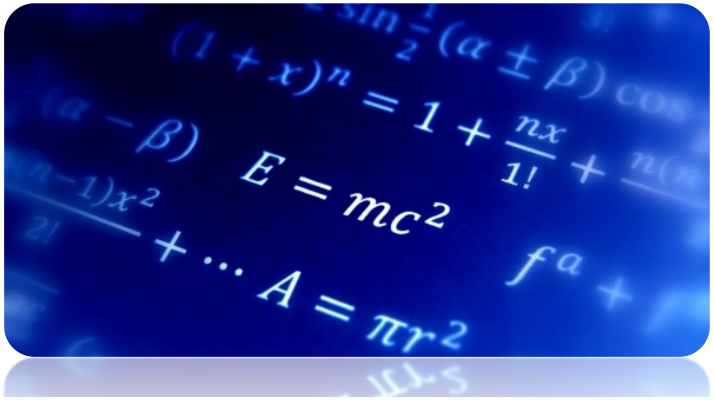
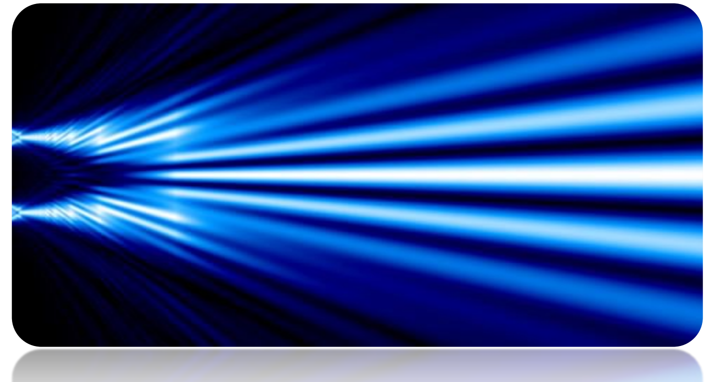



**Relativity**



**Non-relativistic Quantum Mechanics**

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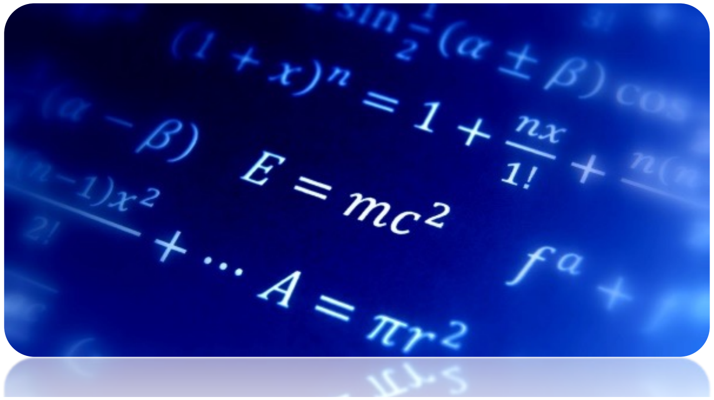


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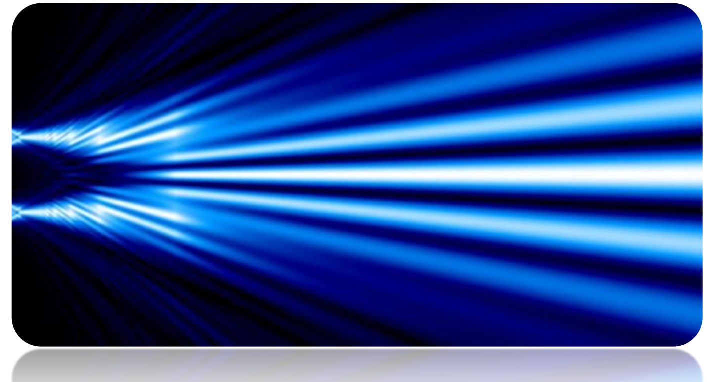

**Non-relativistic Quantum Mechanics**

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right)\psi$$

↕ **Schrodinger equation**

$$E = \frac{\vec{p}^{\,2}}{2m} + V(x)$$

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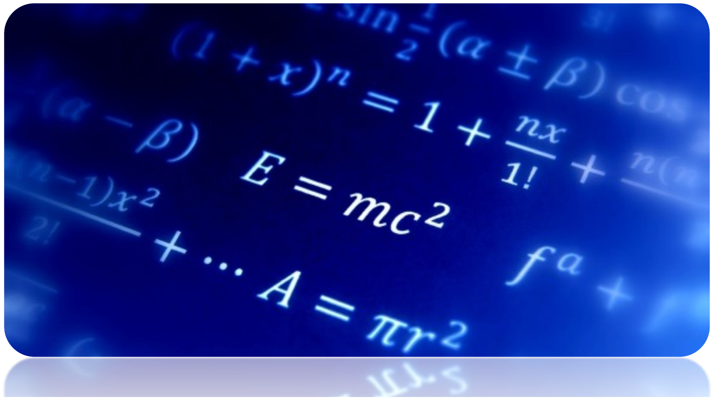
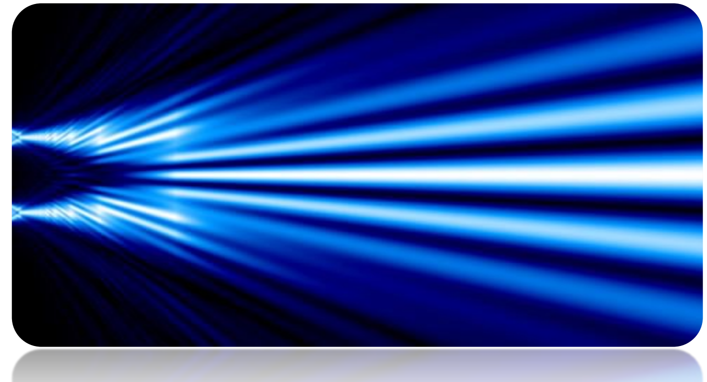



**Relativity**

$$E^2 = c^2p^2 + m^2c^4$$



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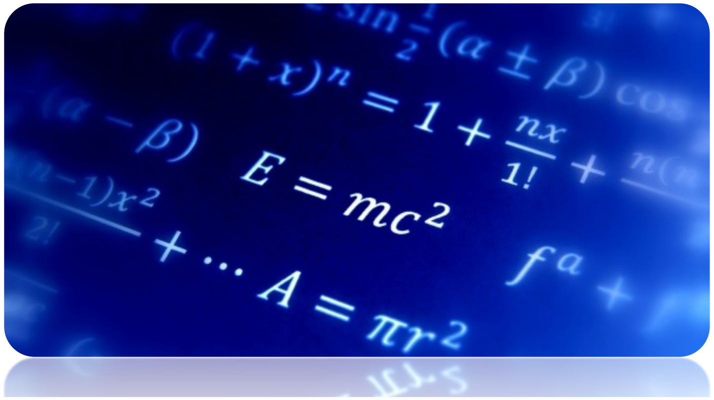


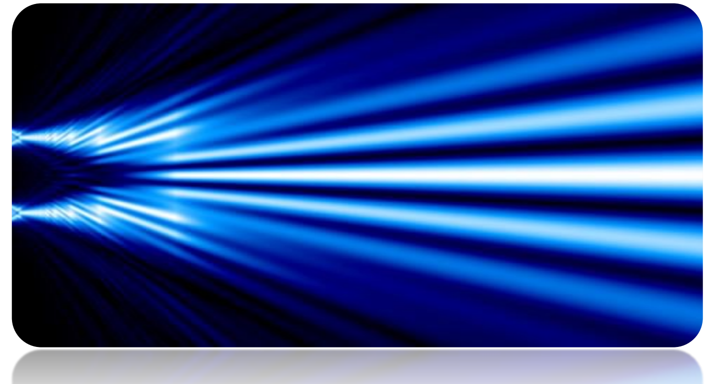

### Relativity

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = \left(\nabla^2 - \frac{m^2c^2}{\hbar^2}\right)\psi$$

↕ **Klein-Gordon equation**

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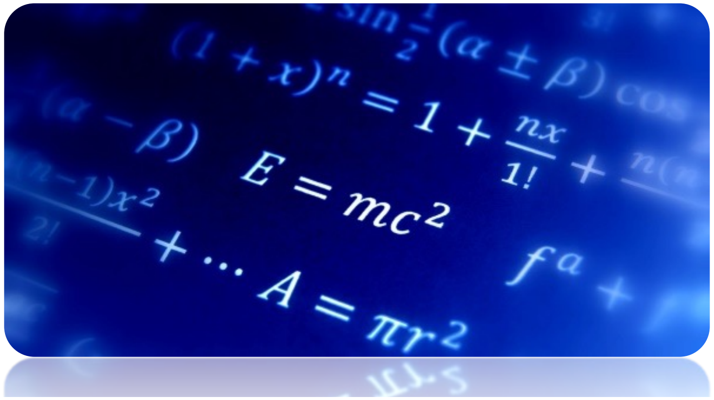


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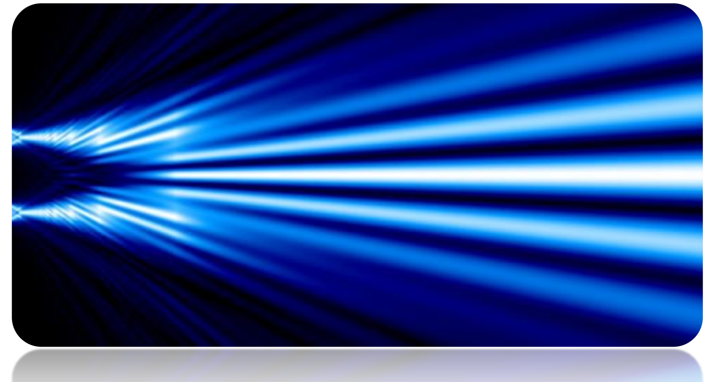

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$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right)\psi$$

**Schrodinger equation**

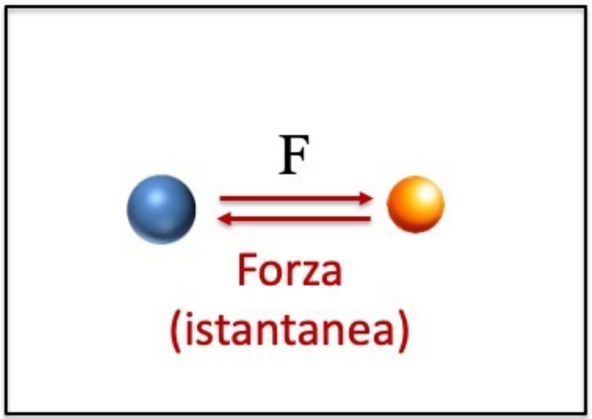
**Combining Quantum Mechanics and Relativity, however, is not just a change of "formula"**

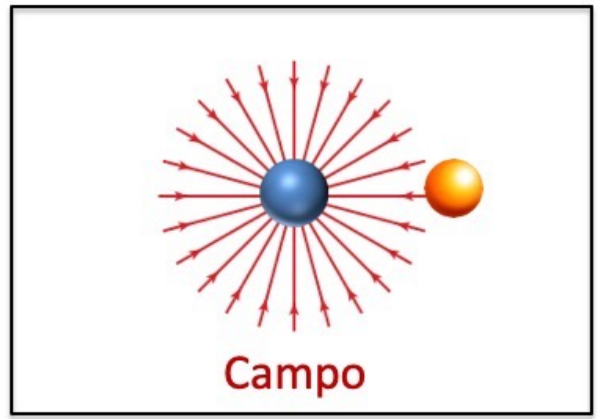
# We already discussed an important change with general relativity

## Gravitazione e relatività

## non forze ma campi...

- Nella gravità newtoniana, l'idea di campo rappresenta solo un concetto ausiliario



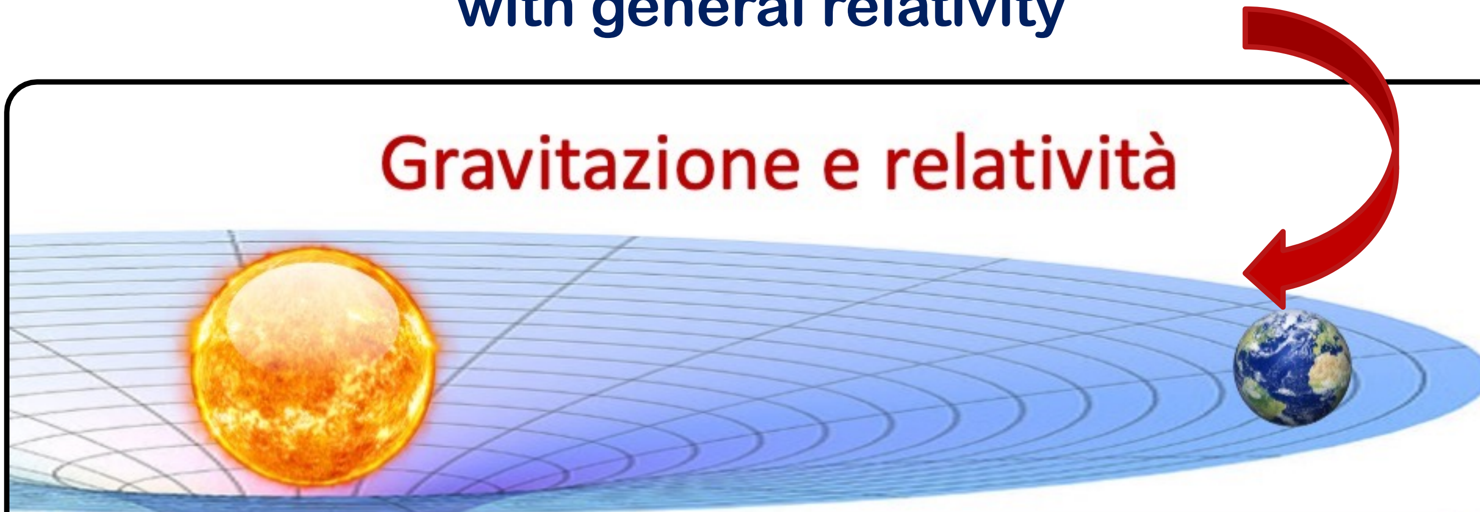




Ma le interazioni gravitazionali, come quelle elettromagnetiche, si propagano ad una velocità finita, la velocità della luce ➡ Il concetto di campo diventa essenziale ed acquista un significato fisico di per sé.

L'effetto di una massa, in un punto distante dello spazio, potrebbe continuare ad essere presente anche se, ad esempio, la massa nel frattempo fosse «scomparsa»

# We already discussed an important change with general relativity

## Gravitazione e relatività



**NEWTON:**

1) Ogni cosa attrae un'altra con forza inversamente proporzionale al quadrato della distanza

2) Un oggetto risponde ad una forza con accelerazione ad essa proporzionale

**EINSTEIN:**

**In the Einstein theory, the gravitational force is replaced by the gravitational field**

## We already discussed an important change with general relativity


**In a quantum relativistic theory, the fundamental interactions are described in terms of fields**

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**In a quantum relativistic theory, the fundamental interactions are described in terms of fields**

**Field theories deal with physical systems described by assigning the value of (one or more) fields in each spacetime point**

## We already discussed an important change with general relativity


**In a quantum relativistic theory, the fundamental interactions are described in terms of fields**

**A field, like the gravitational or the electromagnetic field, corresponds to a real physical property of spacetime in that point**

## The second important change is contained in this formula:

$$E = \sqrt{m^2 c^4 + c^2 \vec{p}^{\,2}}$$

- The mass is just a part of the total energy

Energy is conserved, while mass is not

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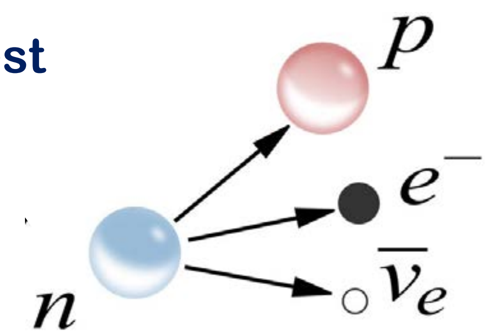
$$E = \sqrt{m^2 c^4 + c^2 \vec{p}^{\,2}}$$

- The mass is just a part of the total energy

Energy is conserved, while mass is not

- A particle with mass m has an energy $E = mc^2$ at rest

  If an energy $E = mc^2$ is available, a particle of mass m can be created (if no conservation law is violated)




The number of particles is not conserved

- The non-relativistic Schrödinger equation of Quantum Mechanics

$$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}-V(x)\right)\psi=0$$

was modified to describe relativistic systems, obtaining

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2+\frac{m^2c^2}{\hbar^2}\right)\psi=0$$

the Klein-Gordon equation
for scalar (spin 0) particles

$$\left(i\hbar\,\gamma^0\frac{1}{c}\frac{\partial}{\partial t}+i\hbar\,\vec{\gamma}\cdot\vec{\nabla}-mc\right)\psi=0$$

the Dirac equation
for spin=1/2 particles

These equations, however, are written in terms of the wave function, which describes single particle states.

**These equations cannot describe creation and annihilation of particles**

- **The problem is solved by promoting the theory to a**

## Relativistic Quantum Field Theory

**The wave function is replaced by a quantum field:**

**Klein-Gordon equation**
**for the scalar (spin 0) field**

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{m^2c^2}{\hbar^2}\right)\psi = 0$$

**Dirac equation**
**for the spin=1/2 field**

$$\left(i\hbar\,\gamma^0\frac{1}{c}\frac{\partial}{\partial t} + i\hbar\,\vec{\gamma}\cdot\vec{\nabla} - mc\right)\psi = 0$$

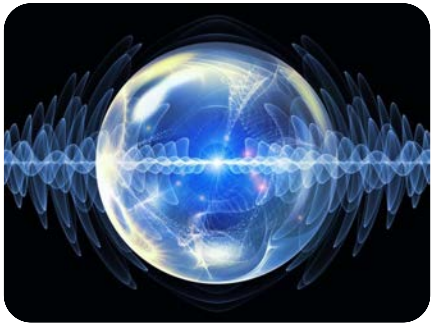
- **In each spacetime point, the quantum field defines the probability of particle creation and annihilation**

- **Particles turn out to be excitations of the field** **(like waves are excitations of the sea surface)**

- For each **elementary particle** there exist a **quantum field**

or we should better say

for each **field** there exist an **elementary particle**

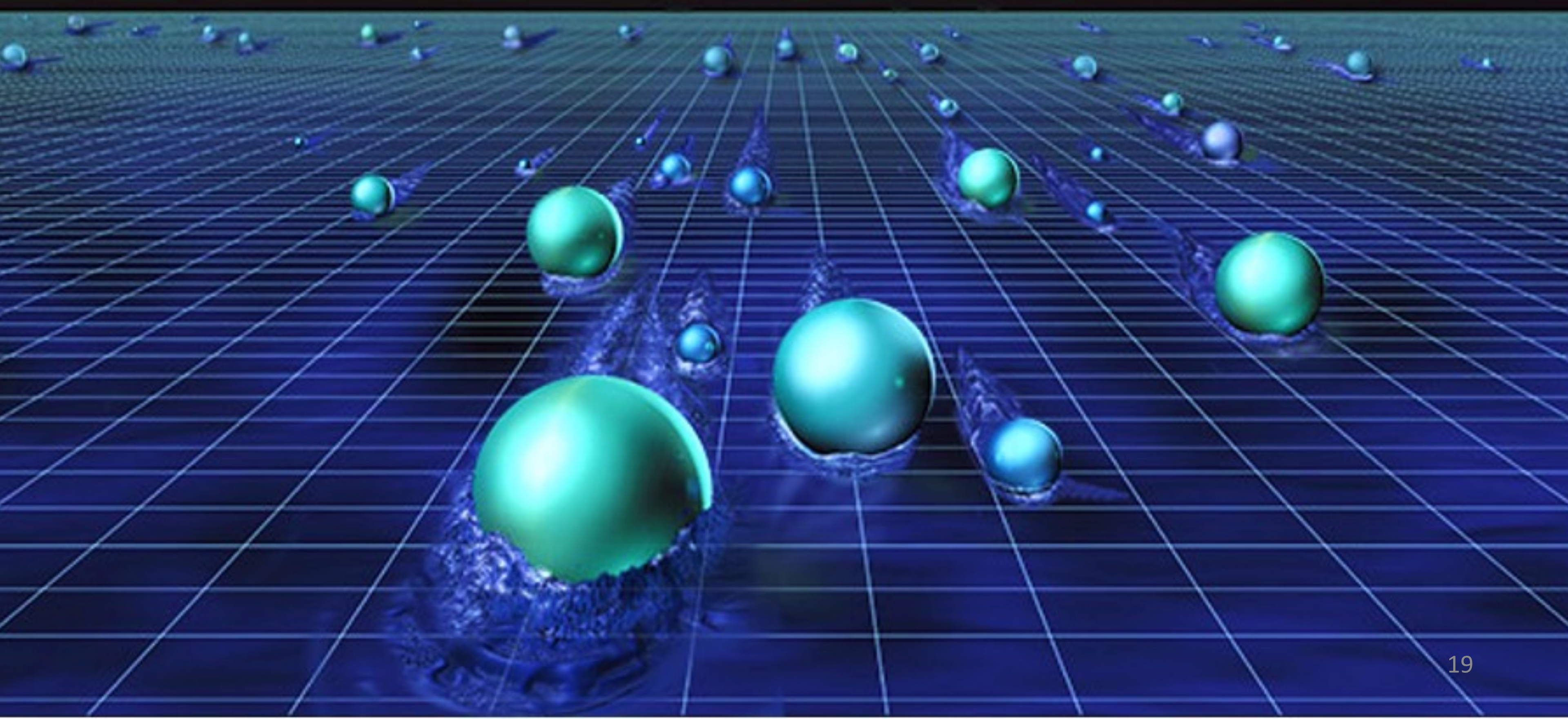
- We already know the particle (or the excitations) of the electromagnetic field: it's the photon

Then there is the electron field. It's a Dirac (spin 1/2) field.

There is the muon field and the neutrinos fields, the quark fields and the Higgs field and …. So:

**Which are the elementary particles and corresponding fields?**

# 2) I Costituenti Elementari della Materia



# Elementary Constituents of Matter

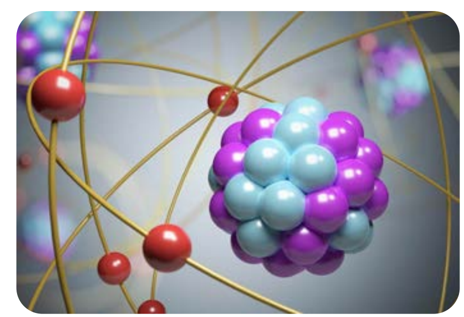
**Ambition of humankind:**
**Establish order and regularity in our complex surroundings**

- **Ancient cultures have identified elementary constituents in various elements**

  **air, fire, water, earth, …**


- **But the first "scientific" progress arrived in the 19th century, with the discover of the**

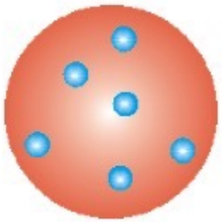
  **atom**



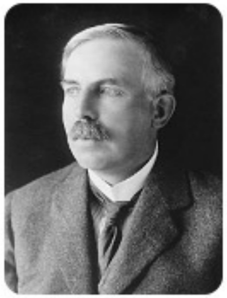
**We have already discussed the beginning of this story**

# LA STRUTTURA DELL'ATOMO

- Alla fine del 1800 l'ipotesi atomica (Dalton 1808) è largamente accettata. Ma l'atomo è considerato "indivisibile".
- 1897: Thomson scopre l'elettrone (carica negativa, massa ≪ massa atomica)

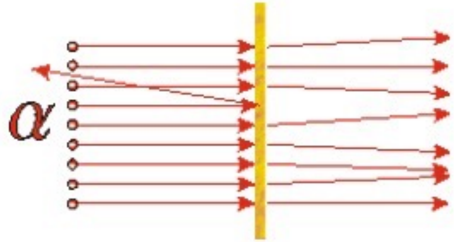


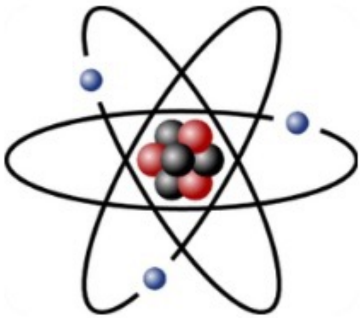
- 1909: Esperimento di Rutherford + Geiger e Marsden



Ernest Rutherford

Risultato

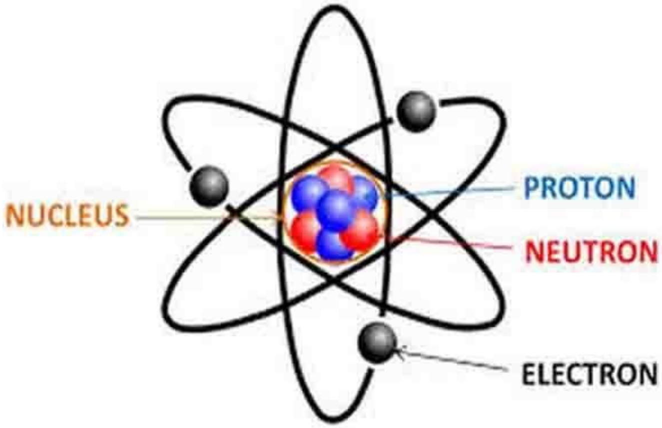


- 1932, James Chadwick: discovery of neutron

- Nuclei are made up of protons and neutrons

(collectively referred as nucleons)



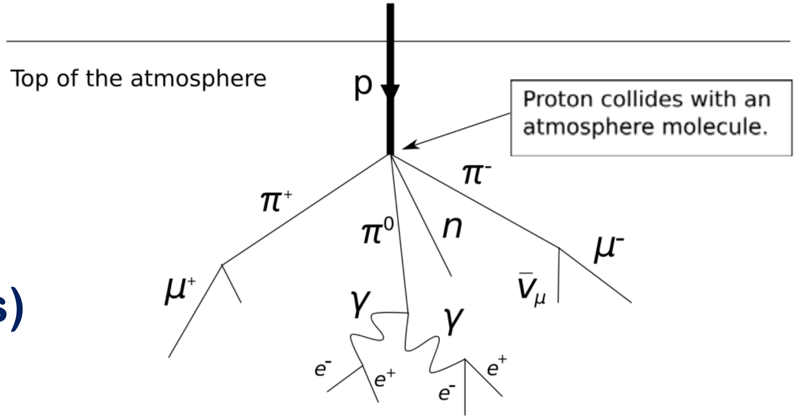

- Protons (charged) are as many as electrons (atoms are neutral)

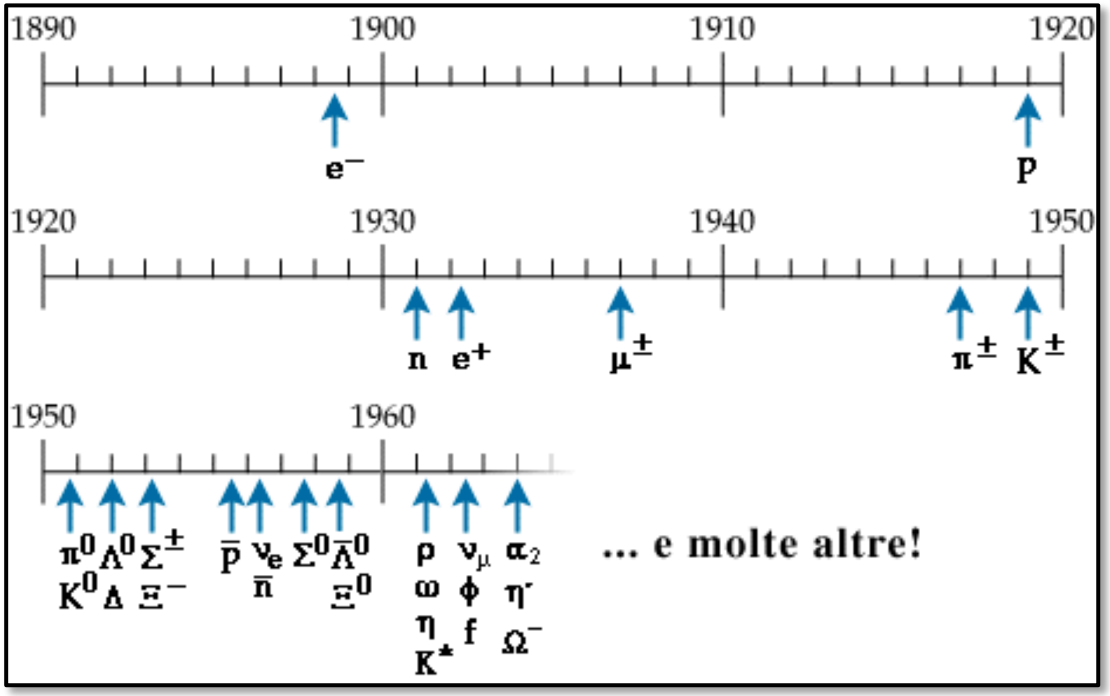
- Neutrons (neutral) can be of different number in different isotopes

  (a nucleus with too many neutrons becomes unstable by undergoing β-decay)

The elementary particles seemed to be no more atoms but protons, neutrons and electrons

- 1930's: new particles discovered in cosmic rays or in high energy accelerators (mainly hadrons, i.e. particles subject to nuclear forces, like nucleons)



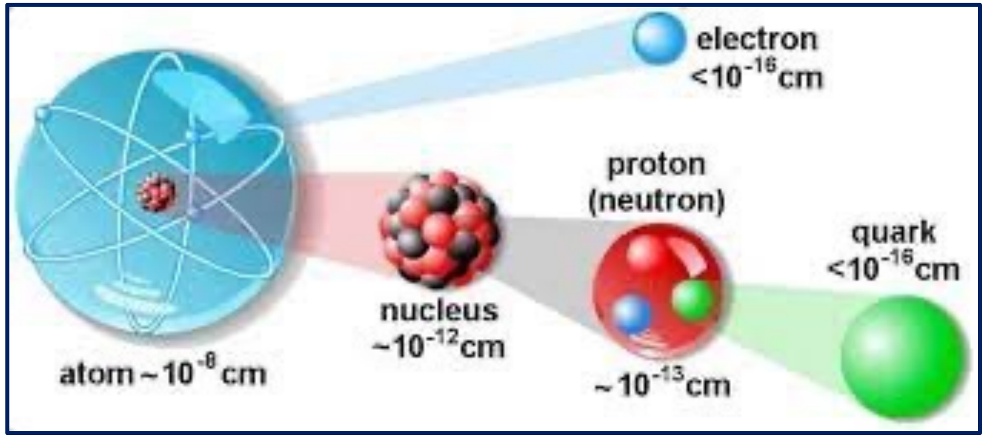




**There were many hadrons and they seemed to be unrelated**

- 1960's: Murray Gell-Mann and Yuval Ne'eman found some regularity (SU(3)-symmetry)

- 1964: Murray Gell-Mann and George Zweig discovered an inner hadron structure:

**QUARKS**




- Hadrons made up of 3 quarks are called baryons, those made up of a quark and an antiquark are called mesons

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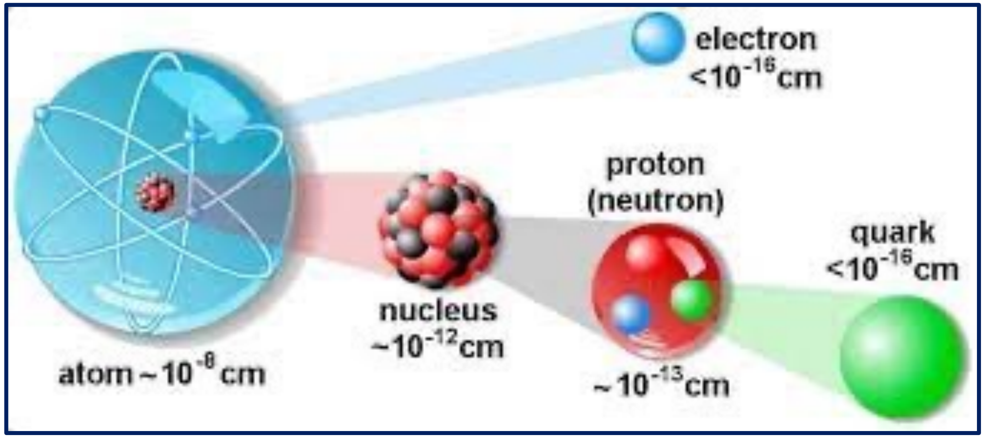
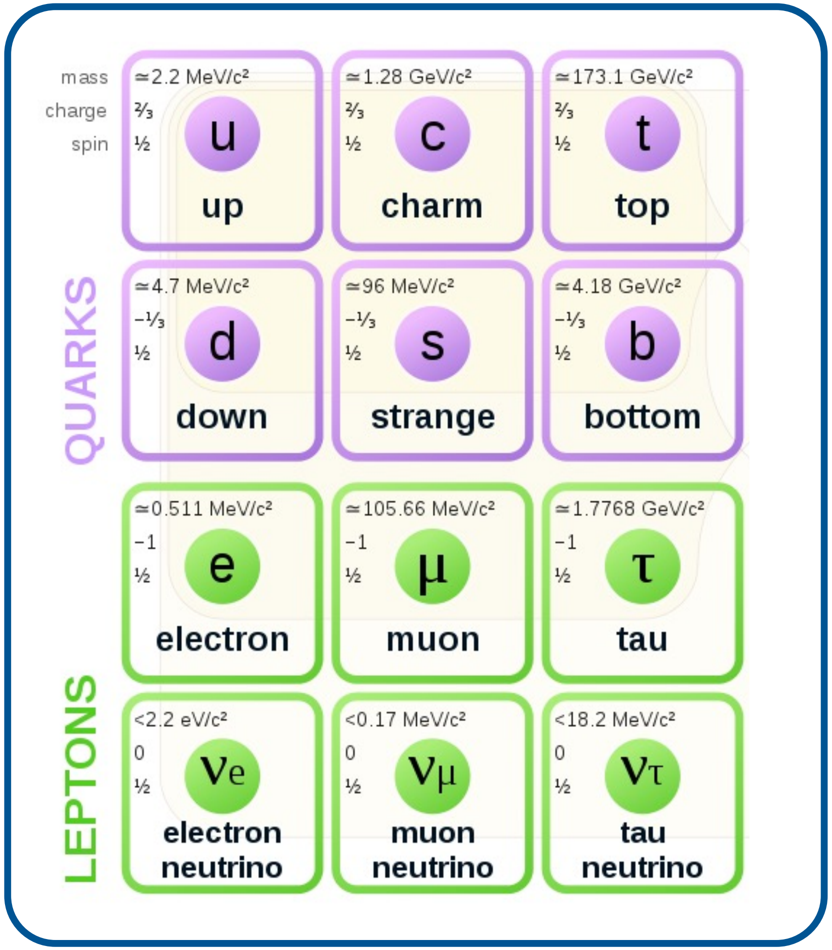



The elementary particles became:

• Quarks (composing nucleons and other hadrons)
• Leptons (unaffected by nuclear forces, like electrons)

# Elementary Constituents of Matter

## as we know them today




- Being elementary particles, they have **no internal structure**, i.e. they are not composed by more elementary constituents

**They are dimensionless pointlike particles**

We may well discover some internal structure in the future, and realize that some of these particles are not elementary

# Elementary Constituents of Matter

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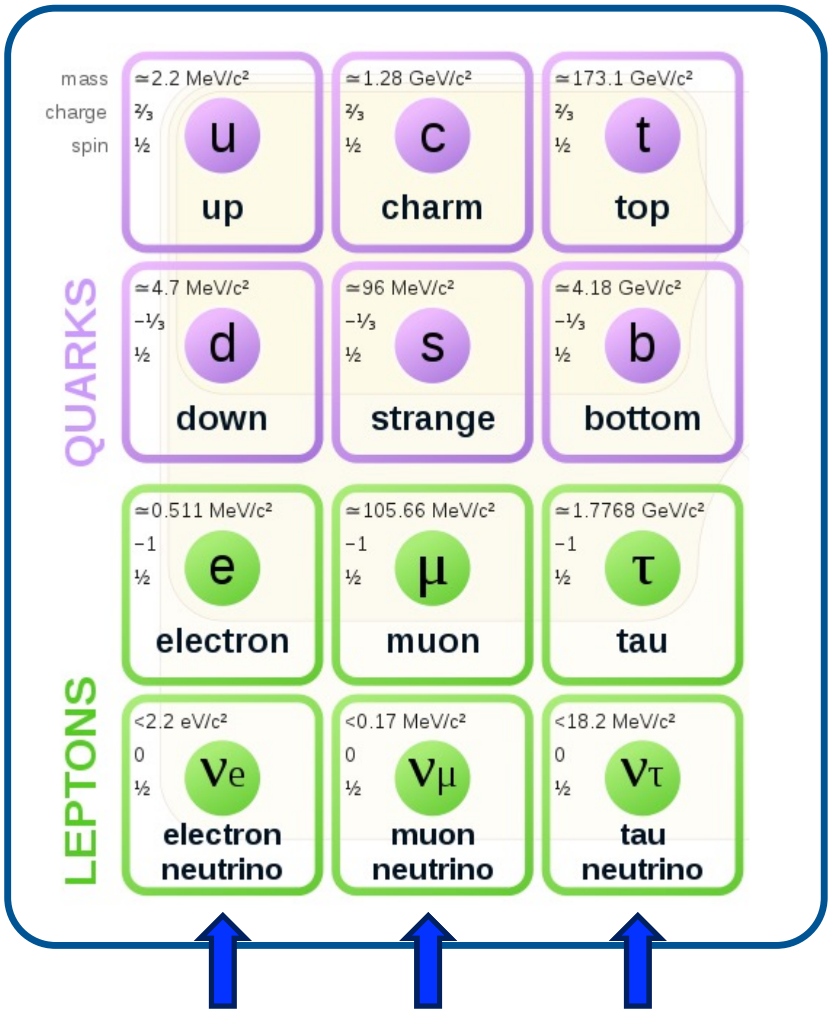


- Both quarks and leptons appear in

**3 generations (or families)**

- Constituents in different generations are almost clones of each other except for mass, that is larger for higher generations

- Constituents of the 2° and 3° generations are unstable.
  They cannot be found naturally but can be produced by high energy collisions

# Elementary Constituents of Matter

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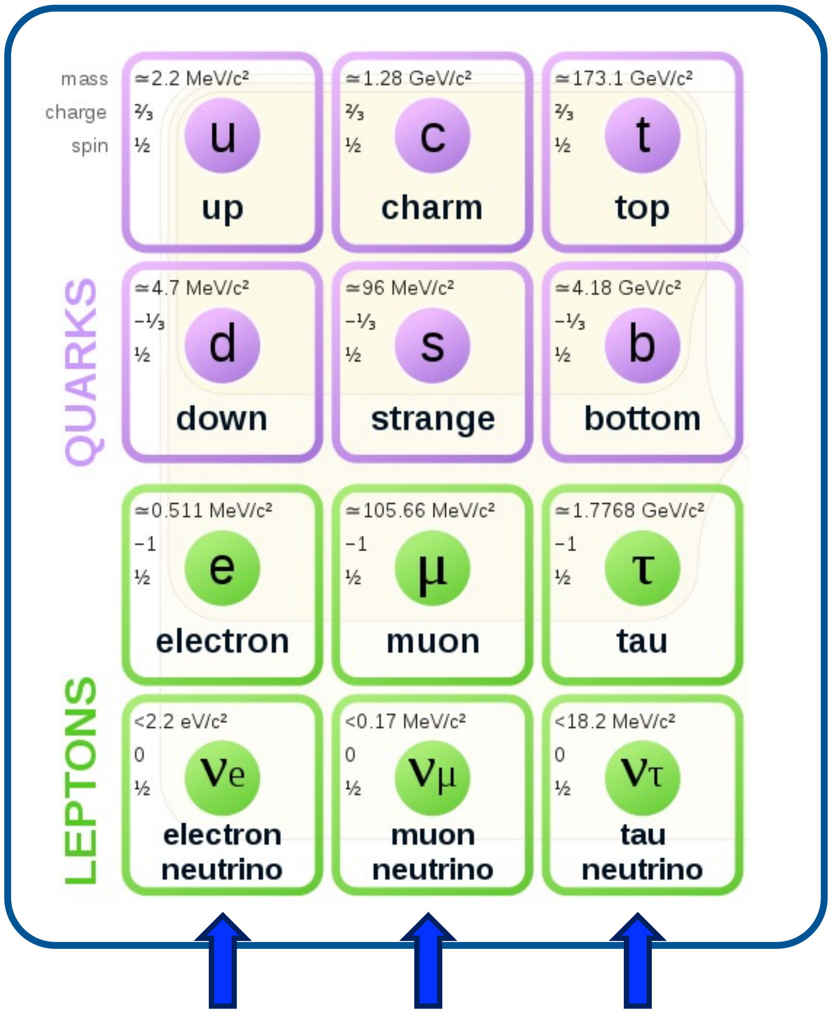


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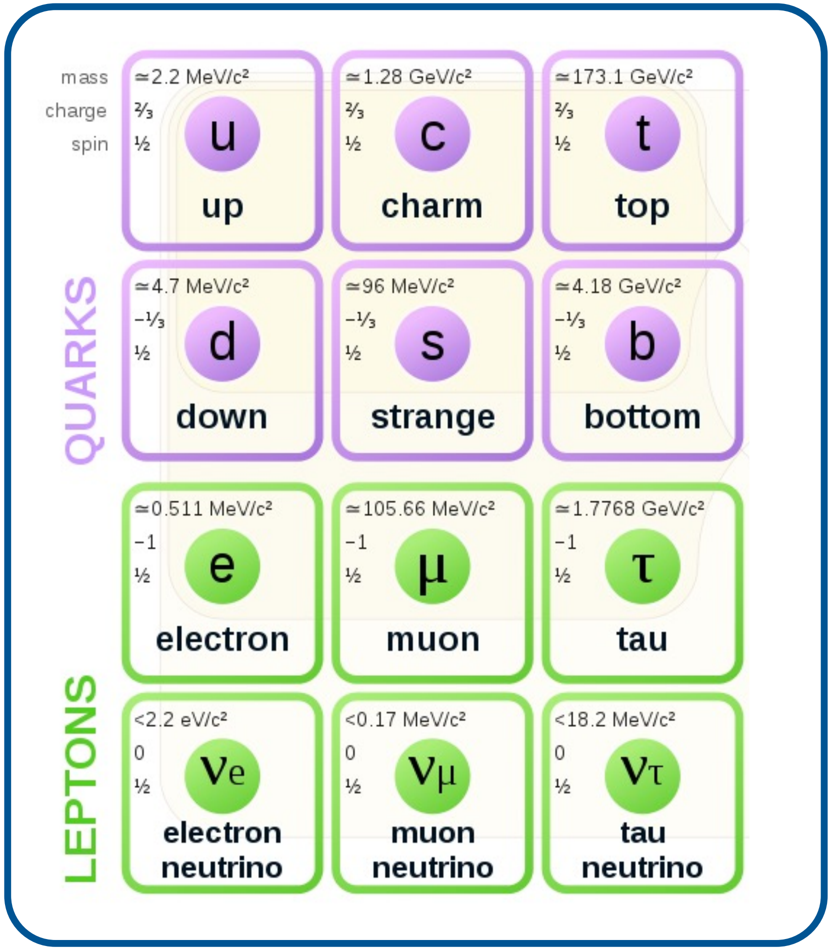
**We do not know why generations are 3!**

- When in 1936 the muon was discovered, Isidor Isaac Rabi said:


80 years later we still do not know!

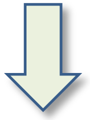
# Elementary Constituents of Matter

## as we know them today




- There are 2 quarks and 2 leptons in each of the 3 generations
- Matter constituents are fermions with spin

**s = 1/2**



They obey the Fermi-Dirac statistics and the Pauli exclusion principle

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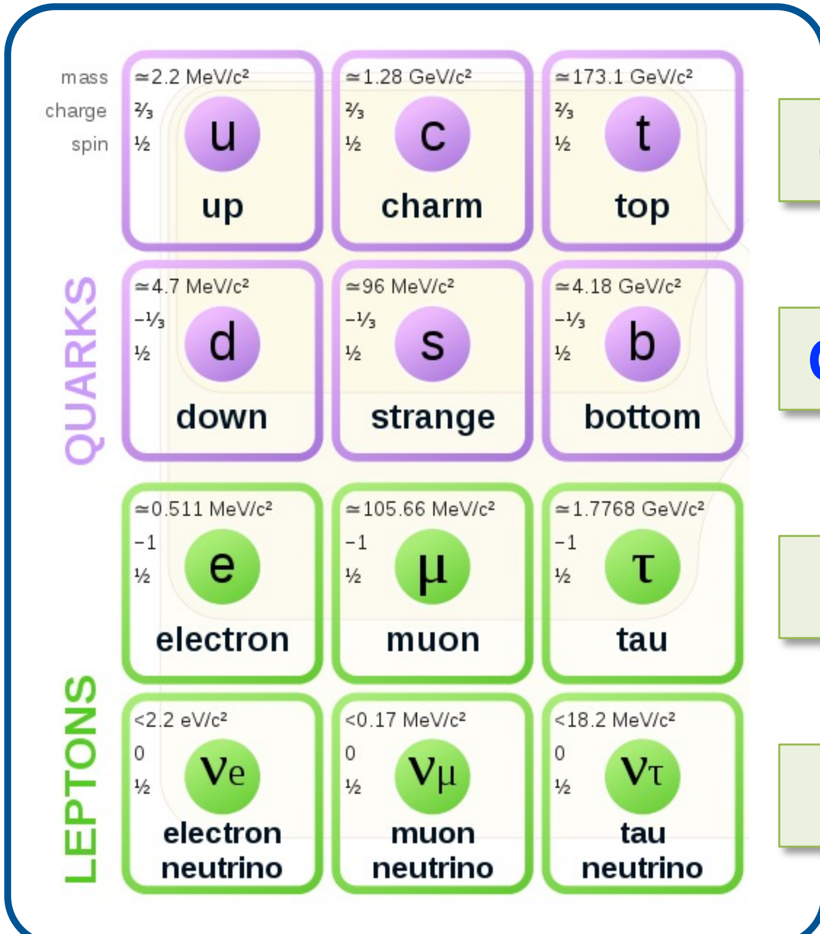



Q = 2/3

Q = -1/3

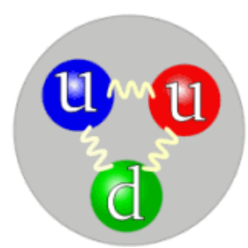
Q = -1

Q = 0

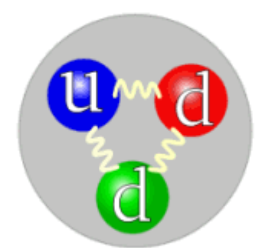
### Electric charges

- Leptons have integral units
- Quarks have multiples units 1/3

(quarks come in 3 colors)

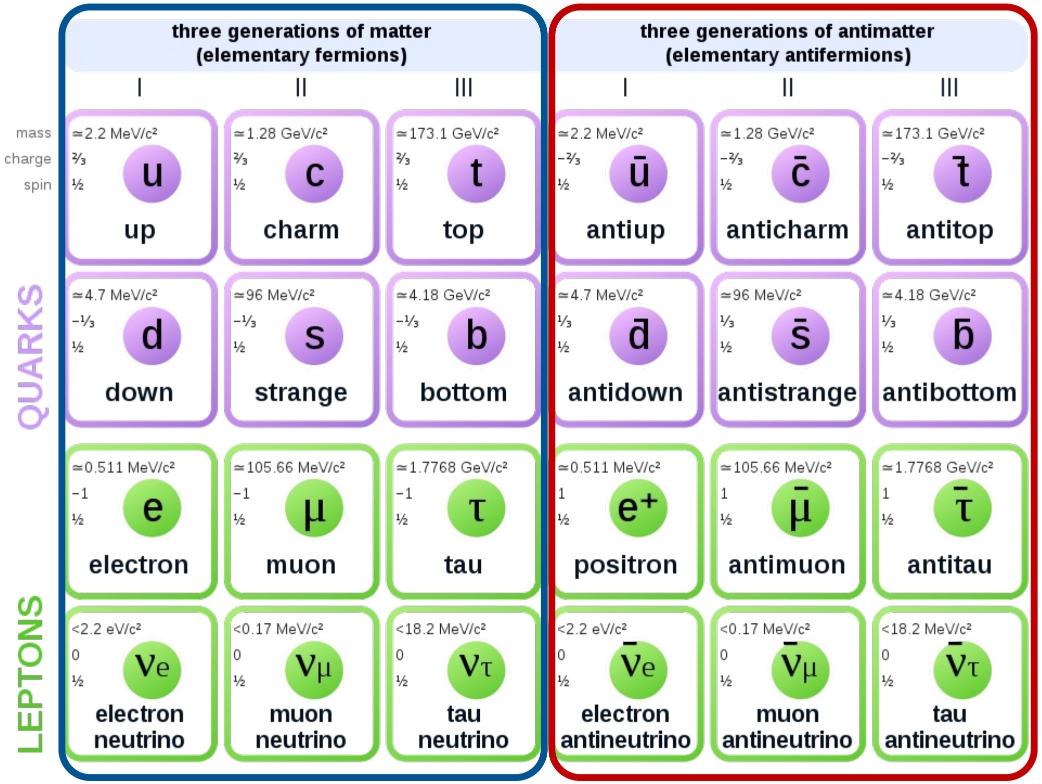



Proton




Neutron

# For each matter particle there exist an antiparticle



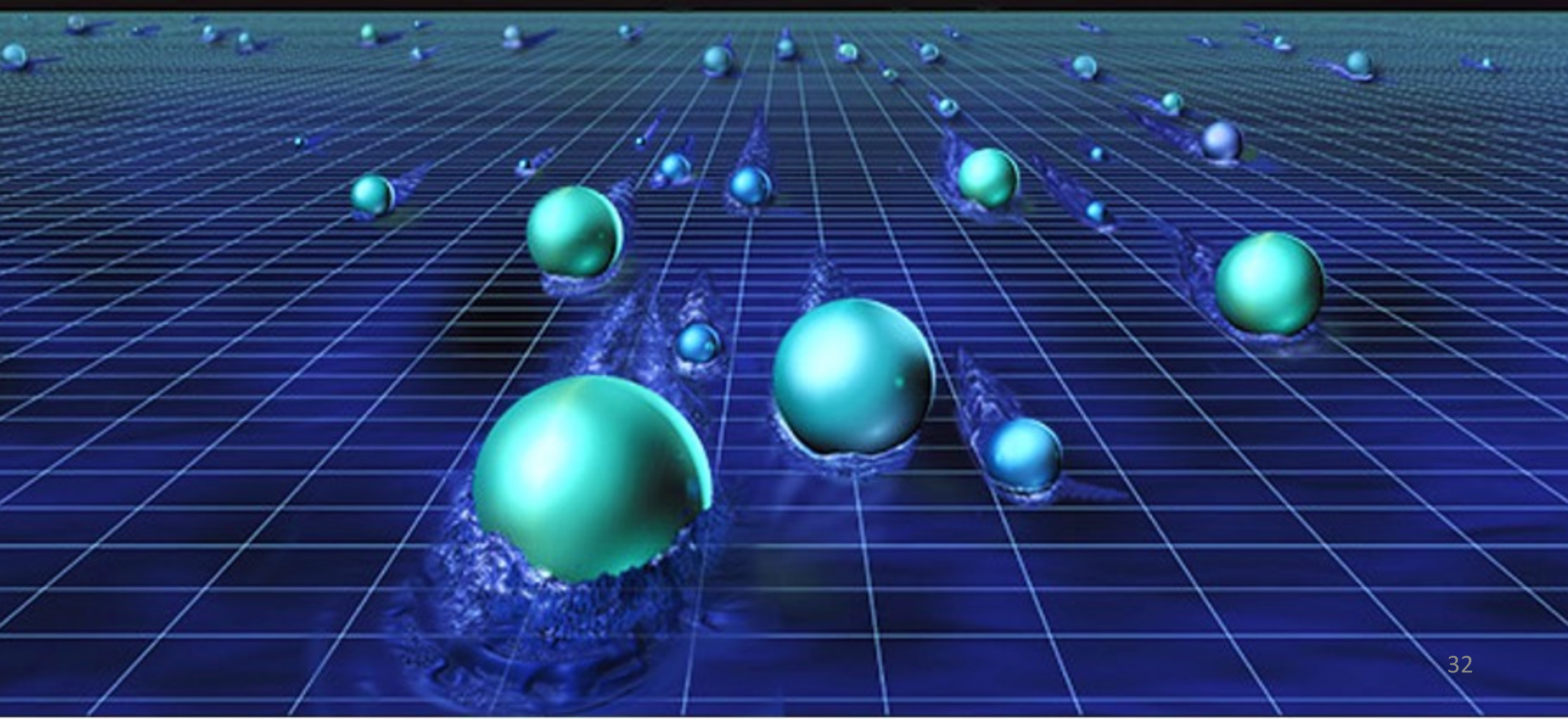

- Antiparticles have the same mass but opposite electric charge as the particle

  (and all the opposite additive quantum numbers)

- When they encounter particles of the same kind, they annihilate each other

The existence of antiparticles was predicted theoretically in 1929 by Paul Dirac.
A great success of relativistic quantum theory

# 3) Teoria delle Forze



# Fundamental Forces

- We observe a variety of forces in our world, both at a macroscopic and microscopic level

- What binds protons and neutrons together in a nucleus?
- And electrons and a nucleus into an atom?
- And atoms together into a molecule?
- And molecules together in ourselves?
- And us on the surface of this planet?
- …

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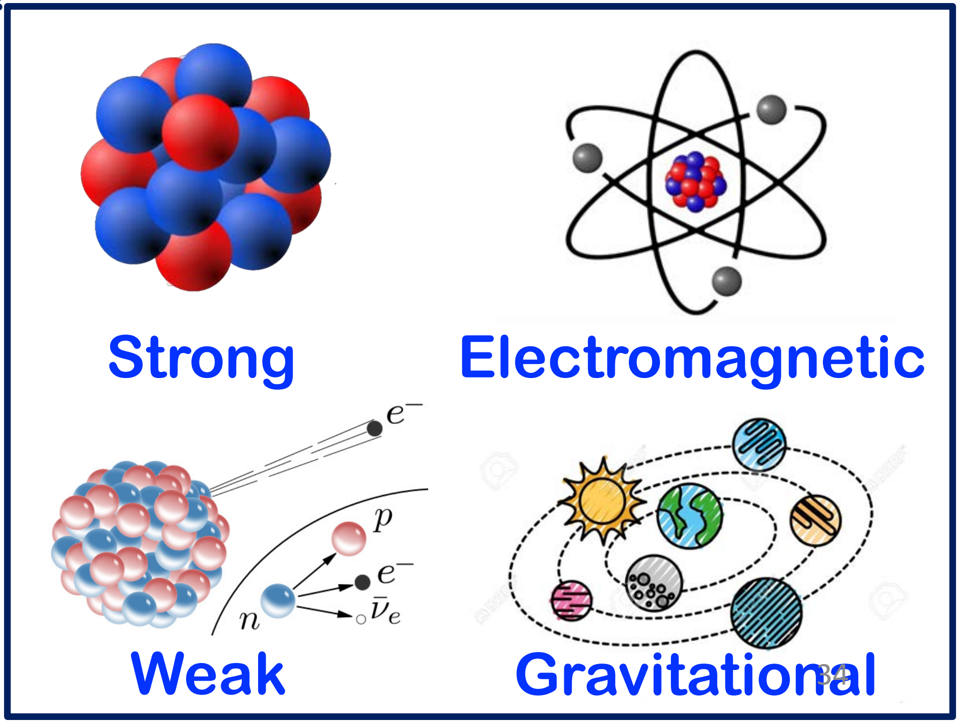
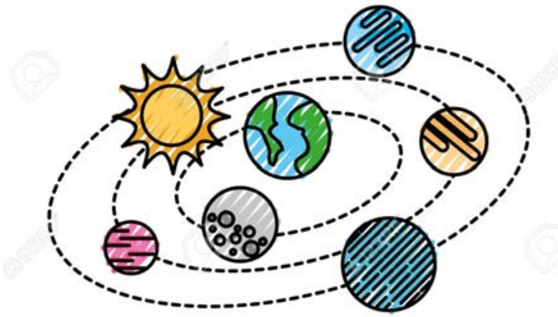
It turns out that all the complicated forces we experience are very complex manifestations of 4 fundamental forces

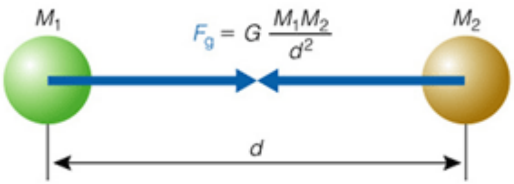
In order of decreasing strength



# Gravitational force

- The first fundamental force to be discovered and the weakest in strength
- Discovered by Newton in the 17° century while trying to explain the Kepler's laws
- Universal and attractive between any two masses and ruled by an inverse square law





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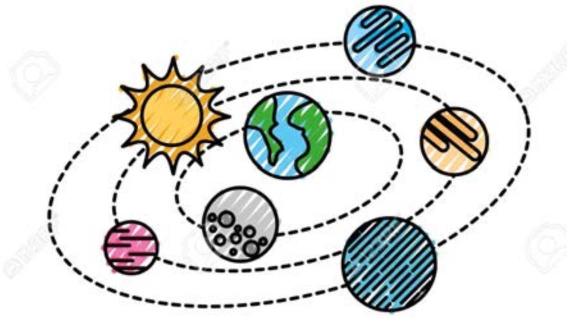
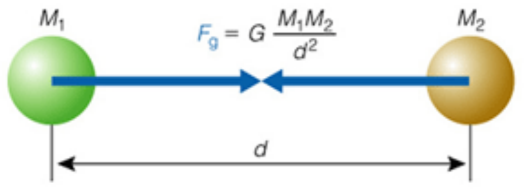
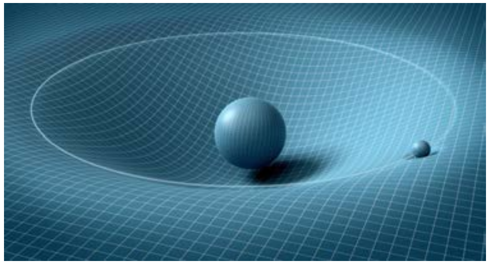
- Universal and attractive between any two masses and ruled by an inverse square law
- Reinterpreted by Einstein in 1915 in his theory of general relativity
- Gravity is interpreted as a revelation of the curvature of the spacetime
- General relativity predicts that light falls under gravity and predicts gravitational waves





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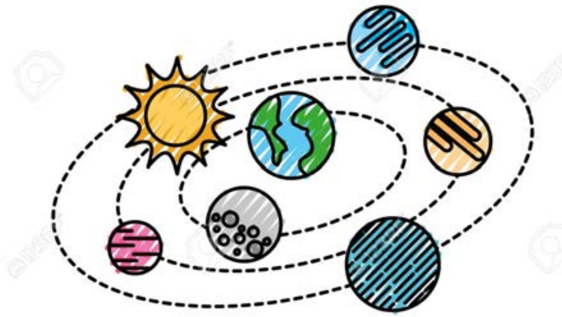

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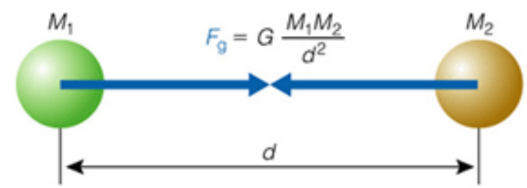
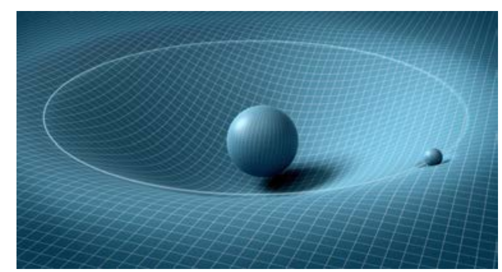
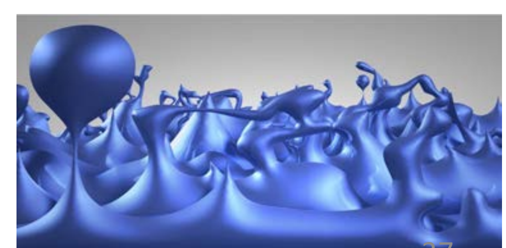



- Reinterpreted by Einstein in 1915 in his theory of general relativity
- Gravity is interpreted as a revelation of the curvature of the spacetime

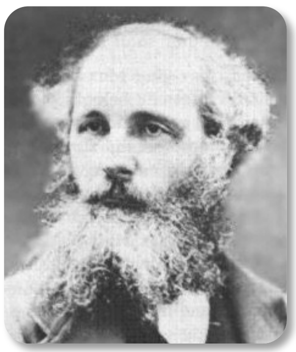


- General relativity predicts that light falls under gravity and predicts gravitational waves
- Quantum gravity is not understood!
  Quantum effects become relevant at the Planck scale ($L_P \sim 10^{-33}$ cm or $M_P \sim 10^{19}$ GeV/$c^2$)



# Electromagnetic force

- Electricity and magnetism were first thought to be unrelated until Maxwell's equations (1865)

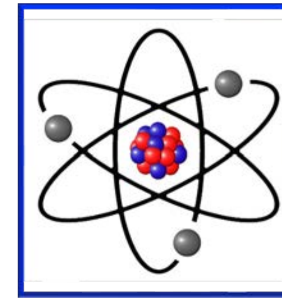


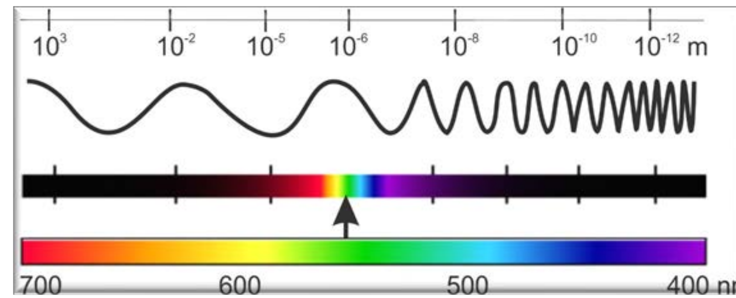
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$






- It is responsible for a variety of phenomena, from electromagnetic waves, electronics, binding electrons in atoms, atoms in molecules and molecules in a liquid or solid,…

- The electric force, like gravity, also follows an inverse square law (the Coulomb law). But unlike gravity, it is attractive between opposite charges and repulsive between same-sign charges

# Weak force

- Discovered in the 20° century (as the strong force)

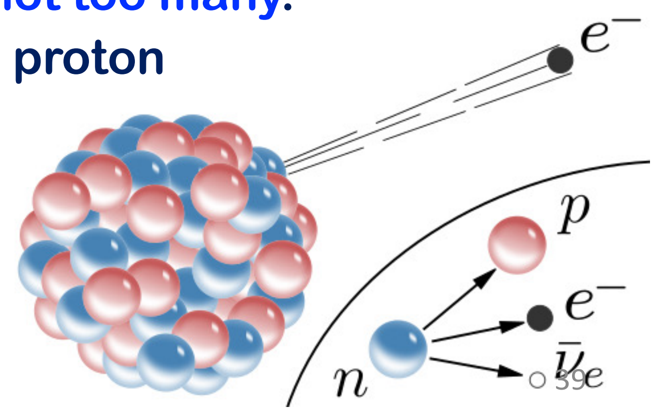
- It is not detectable in our daily lives due to its very short range:

$$R \approx 10^{-16} \div 10^{-15} \text{ cm}$$

At distance short compared to their ranges, it obeys the inverse-square law. Beyond these distances, it becomes extremely small

- It is responsible for the instability of the neutron, that decays by radioactive beta decay within a long time (weak force) of ~15 minutes

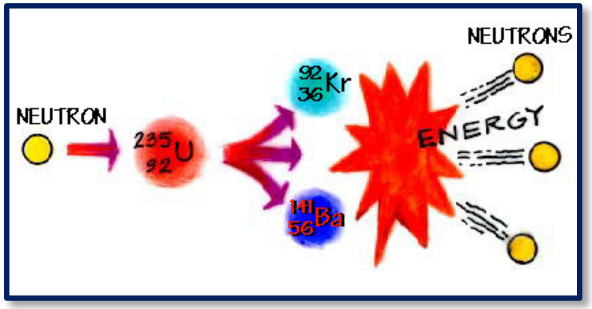
- Neutrons inside nuclei remain stable if they are not too many. With many neutrons, a neutron is changed into a proton if the released energy ($m_n$-$m_p$-$m_e$≈0.8 MeV/c²) wins over the additional electrostatic repulsion in the nucleus with the additional proton



# Strong (or nuclear) force

- The strong force is the strongest of the four forces (in its range)

The great strength of the nuclear force can be seen from the enormous amount of energy that can be derived from a small quantity of fissionable material in a nuclear power plant or in an atomic bomb



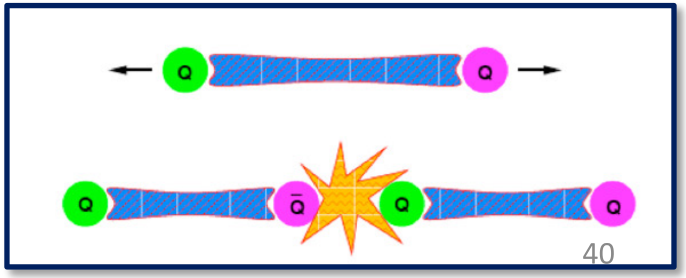

- Between two nucleons it is an inverse-square law force with range

$$R \approx 10^{-13}\ cm$$

- Between two quarks appear to be peculiar. No matter how far apart two quarks are, there is always a constant force to pull them back. The energy needed to pull the two quarks apart is proportional to their separation.

An infinite energy is needed to separate and isolate them, it is more convenient to produce couples of quark-antiquark

⇒ CONFINEMENT



# Theory of Forces

- The electromagnetic and gravitational forces have long ranges and obey the inverse-square law, while the weak and strong forces have only short ranges

**What determines the range of a force ?**

# Theory of Forces

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**What determines the range of a force ?**

- The gravitational force is always attractive, while the electromagnetic force is attractive between opposite charges and repulsive between same-sign charges

**Why are some of the forces attractive and other repulsive ?**

# Theory of Forces

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**What determines the range of a force ?**

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**Why are some of the forces attractive and other repulsive ?**

**These questions are unanswerable in both classical physics and non-relativistic quantum theory**

# The Theory of Forces

- However, when Special Relativity is incorporated into Quantum Mechanics, the above question become answerable

➡ **Quantum Field Theory provides a Theory of Forces**

1) Special Relativity tells us that, provided an energy

$$E = mc^2$$

is available, a particle of that mass can be created

2) Quantum Mechanics tells us that the energy of a closed system may fluctuate by an amount

$$\Delta E \sim \hbar/\Delta t$$

during a time interval $\Delta t$. It is the time-energy Heisenberg's uncertainty relation

# Theory of Forces

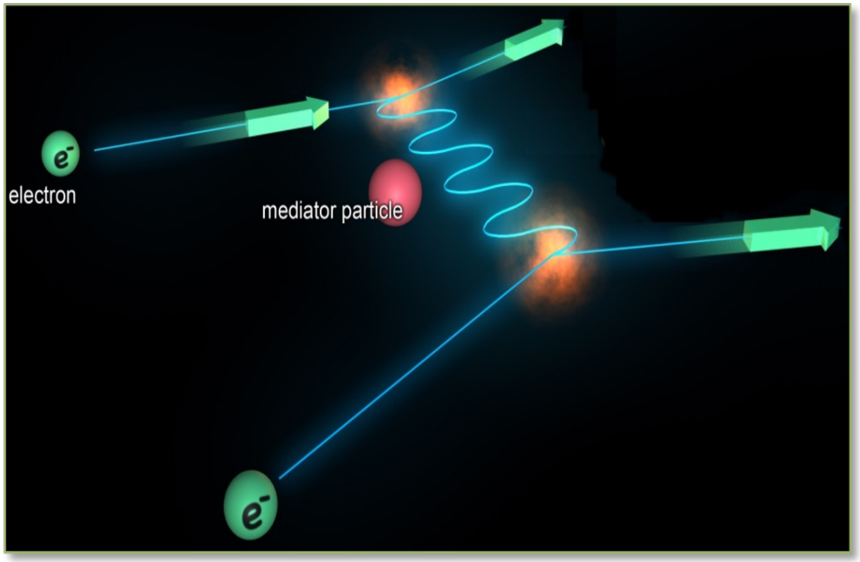
- Thus, at short time intervals, enough energies are available to create particles
- A created particle of mass m can last only a time interval

$$\Delta t \sim \hbar/\Delta E \sim \hbar/mc^2$$

- Such a particle is known as **virtual particle** (or exchange or mediator particle)

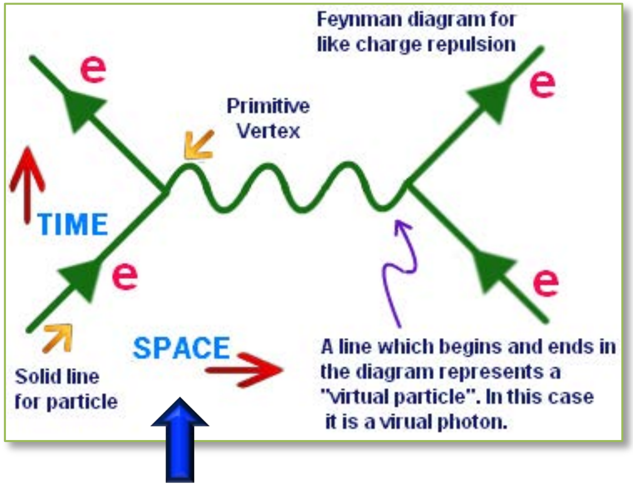
This is how the force arises in a Quantum Field Theory:

A virtual particle created by particle A at one location and annihilated by particle B at another transmits a force between A and B




- Since the interaction consists in the exchange of virtual particles, the force cannot be transmitted instantaneously

# Theory of Forces

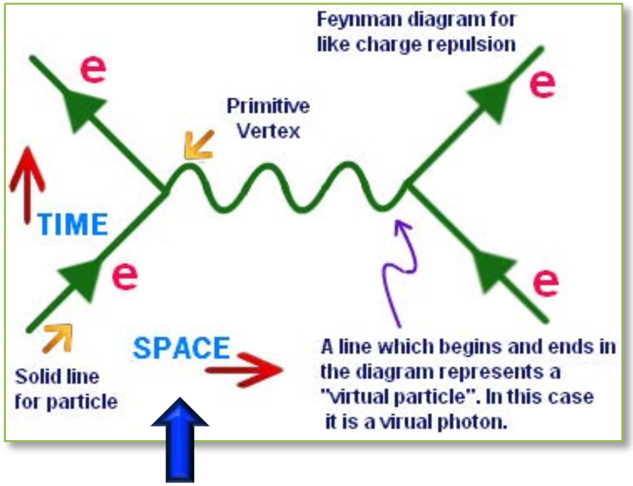



- The intensity of the interaction is determined by the coupling of the interacting particles with the mediator

In the case of the electromagnetic interaction the mediator is the photon, and the coupling is the electric charge e of the particle

- This graph is called a Feynman diagram. It corresponds to a well-defined mathematical expression (Feynman rules) for the amplitude

# Theory of Forces

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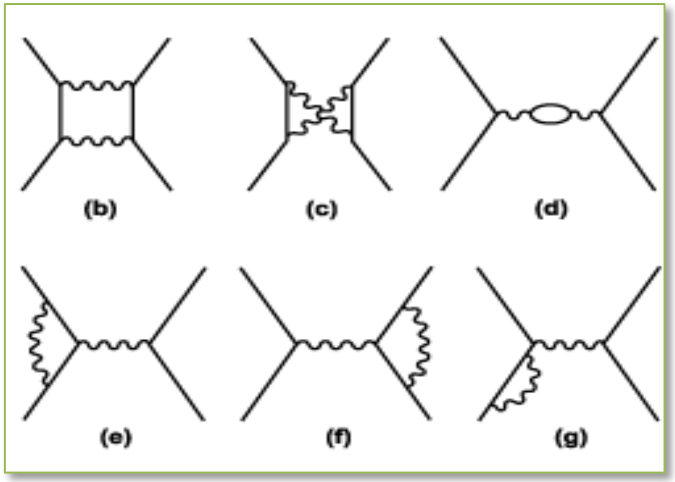
- For a more accurate prediction, higher-order contributions must be considered

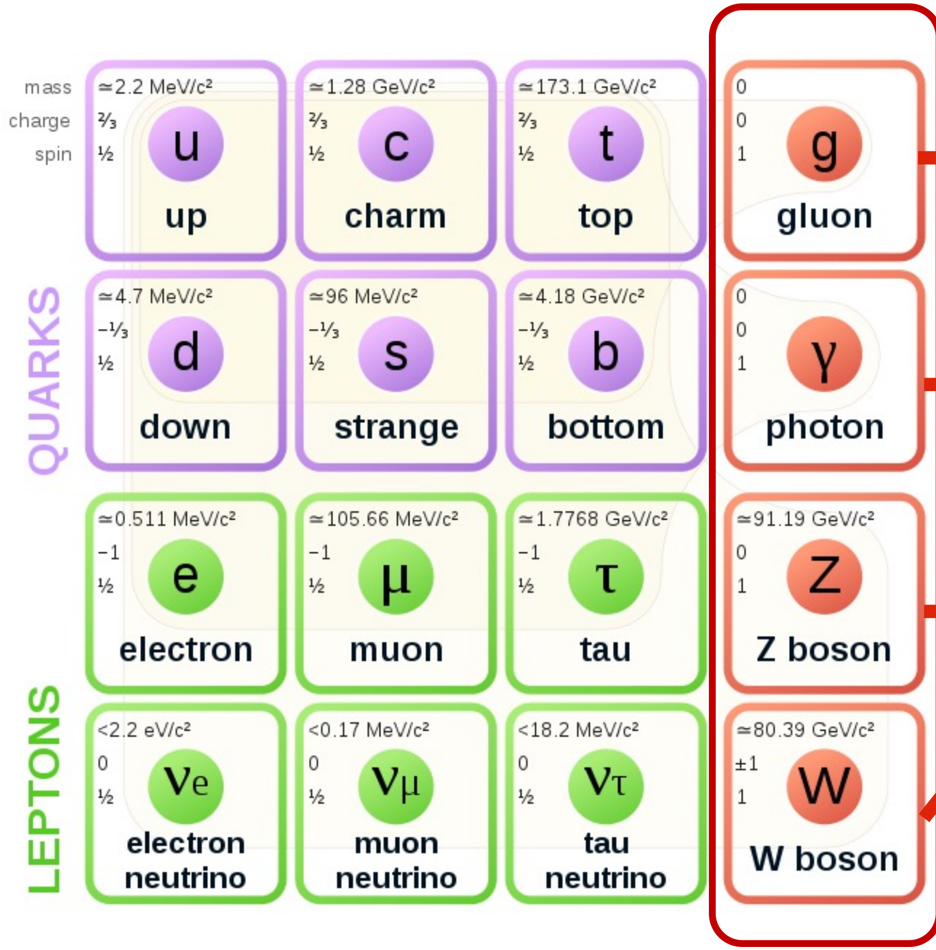
This approach is called Perturbation Theory



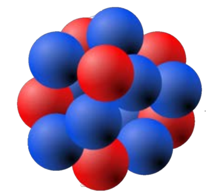

- If the coupling is too strong (like for strong interactions at small energies), the perturbative approach cannot be applied ➡ Numerical lattice QCD simulations

# Mediators of Forces

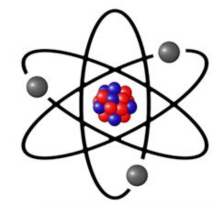
- In the list of elementary particles, a set of bosons are the mediators of fundamental forces



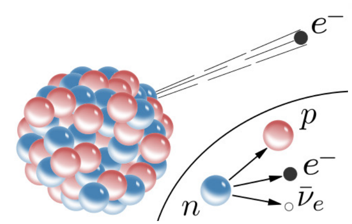

Strong



Electromagnetic



Weak




These are the mediators we know today

# Mediators of Forces

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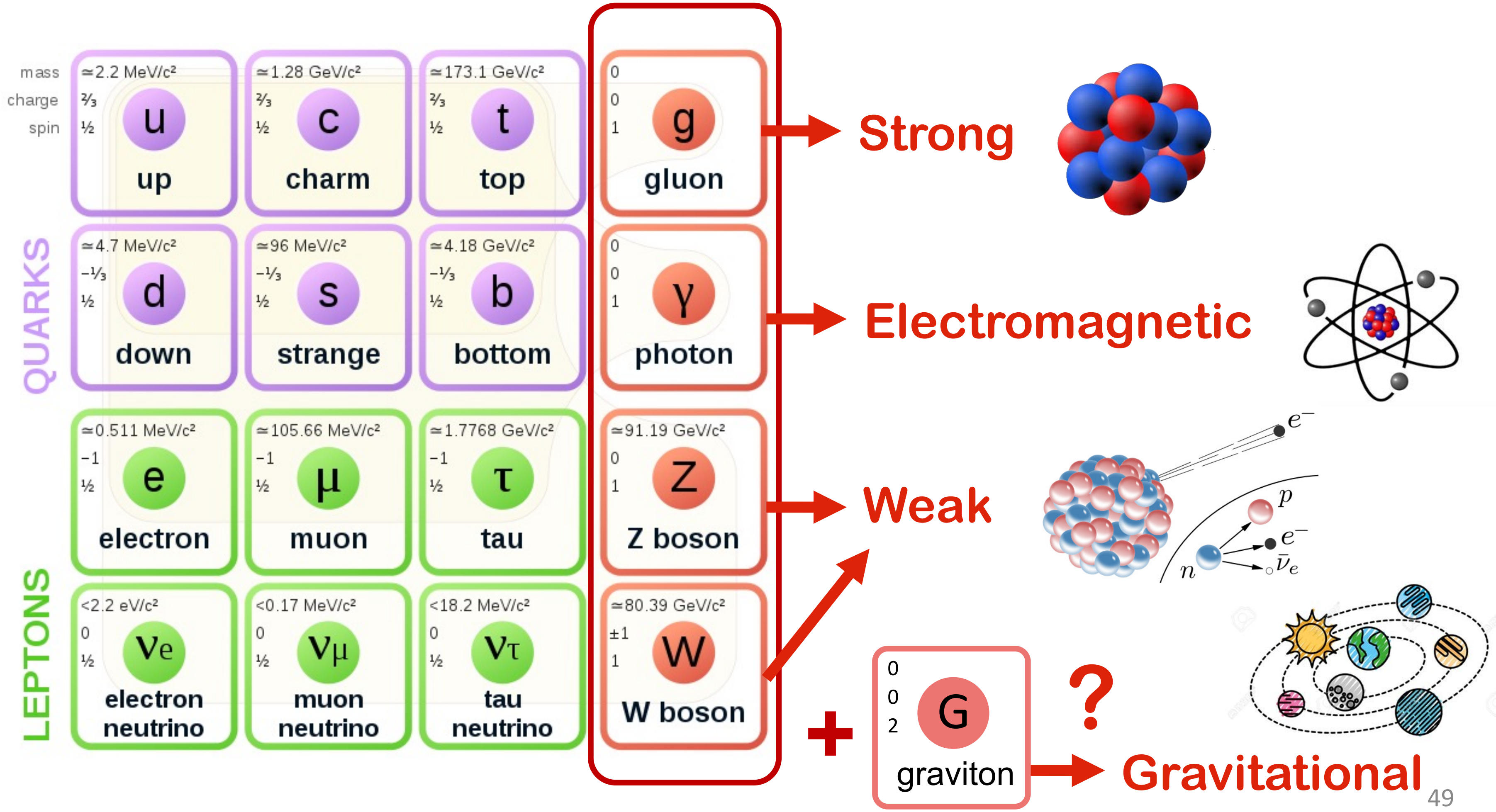

# Range and Mass

- The range of the force is related to the mass of the exchange particle

- Even moving at the speed of light, the virtual particle, which exists for a time interval Δt, can travel only a distance R ~ c Δt before it is absorbed:

$$\Delta t \sim \hbar/\Delta E \sim \hbar/mc^2 \quad \Rightarrow \quad R \sim c\,\Delta t \sim \hbar/mc$$

Range of the force

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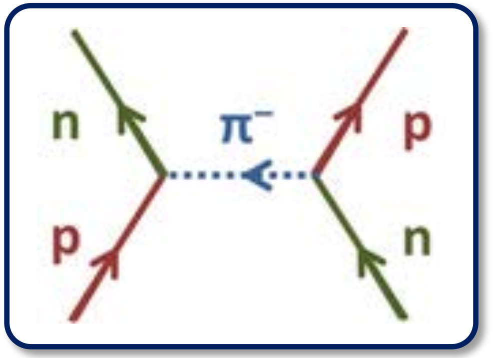
Range of the force

- This argument was firstly proposed by Hideki Yukawa in 1935 and led him to predict the existence of a new particle, the pion




- The range of the strong force between two nucleons was known to be:

$$R \sim 10^{-13}\ cm$$ ➡ $$m_\pi c^2 \sim \hbar c/R \sim 200\ \mathrm{MeV}$$

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- The range of the force is related to the mass of the exchange particle
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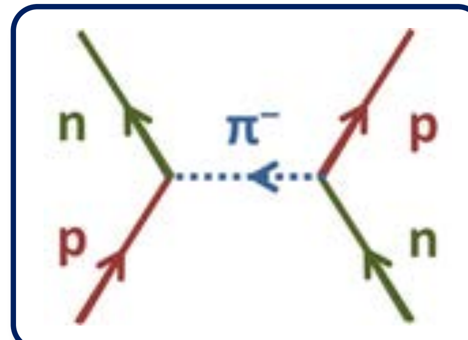


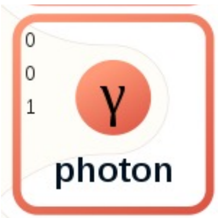
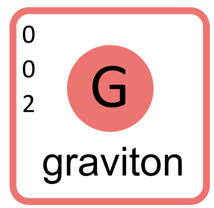
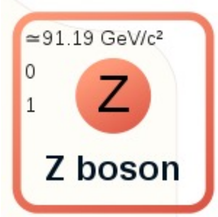
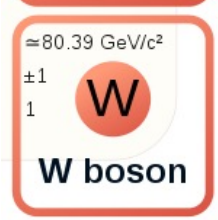
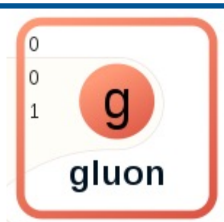

- The range of the strong force between two nucleons was known to be:

$$R \sim 10^{-13}\ cm \quad \Rightarrow \quad m_\pi c^2 \sim \hbar c/R \sim 200\ \text{MeV}$$

- After the discovery of the pion ($m_\pi \approx 140$ MeV/$c^2$) Yukawa was awarded a Nobel prize
- We know that the pion is not an elementary particle, but that does not affect its ability to transmit a force

# Range and Mass

$R \sim \hbar c / mc^2$

| | | | |
|---|---|---|---|
| **Electromagnetic** |  | $m_\gamma = 0$ | $R = \infty$ |
| **Gravitational** |  | $m_G = 0$ | $R = \infty$ |
| **Weak** | <br> | $m_Z \simeq 90\ \mathrm{GeV}/c^2$<br>$m_W \simeq 80\ \mathrm{GeV}/c^2$ | $R \sim 10^{-16}\ cm$ |
| **Strong** |  | $m_g = 0$ | Confined<br>The Yukawa argument does not apply |

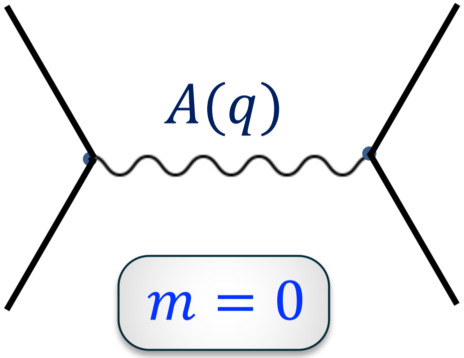
# Inverse-square law

**At variance with classical and non-relativistic quantum theories, in relativistic quantum field theory the expression of the force can be predicted**

- **The potential is the Fourier transform of the amplitude A(q) for the propagation of the virtual particle**

$$V(r) = \int \frac{d^3q}{(2\pi)^3} A(q)\, e^{\frac{i}{\hbar}\vec{q}\cdot\vec{r}}$$

- **For a massless mediator, like the photon:**




$$A(q) = \frac{e^2}{q^2}$$

**It follows from the field equations**

$$V(r) = \frac{e^2}{4\pi\, r}$$

**Coulomb potential**

**It is the inverse-square law**

# Inverse-square law

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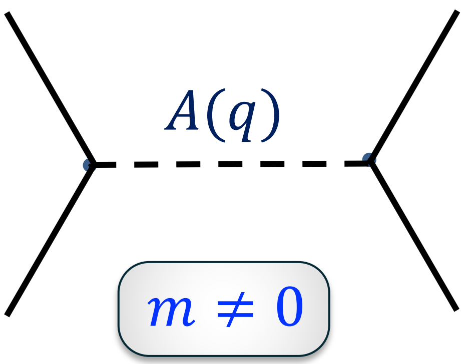
- For a **massive mediator**, like the W or Z bosons :




$$A(q) = \frac{g^2}{q^2 + m^2c^2} \Rightarrow$$

**Yukawa potential**

$$V(r) = \frac{g^2}{4\pi\, r} e^{-\frac{mc}{\hbar} r}$$

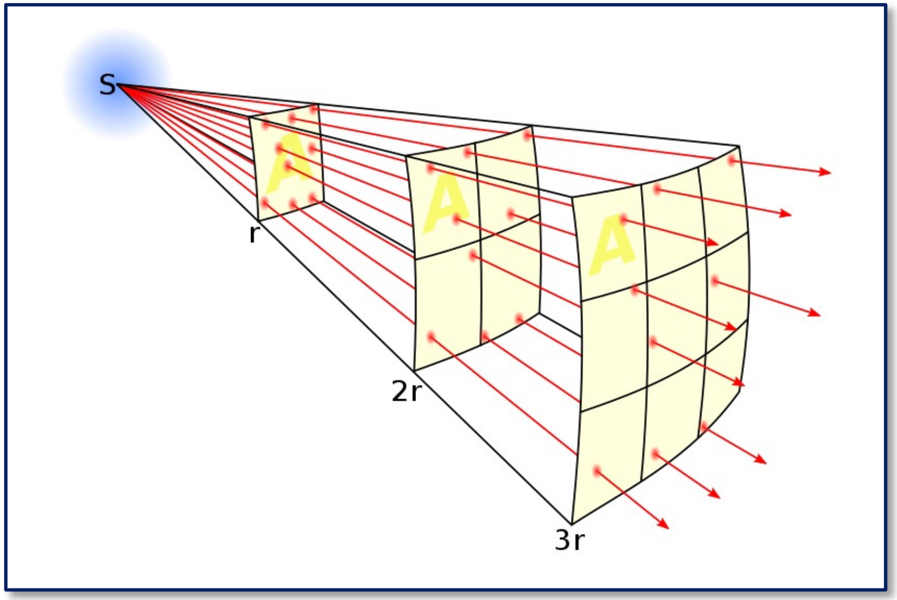
The **range of the interaction** is $R \sim \hbar/mc$ as predicted by Yukawa

# Inverse-square law

- The inverse-square law has a simple physical explanation

- A force becomes zero beyond its range, because the virtual particle cannot propagate any longer

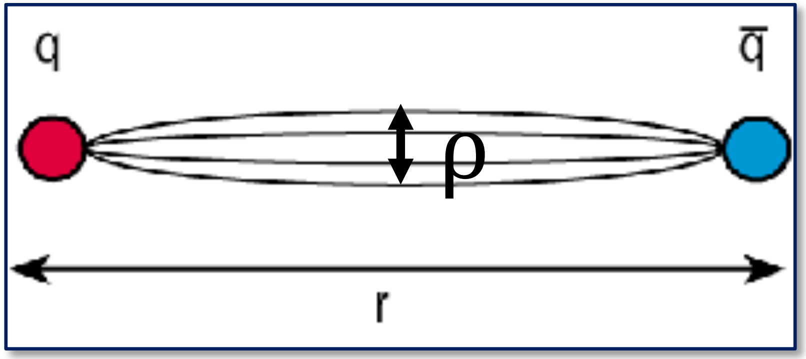
- But at distance r from the emitting particle, well within the range of the virtual particle, the surface area of the portion of sphere seen by a virtual particle that moves in a given solid angle increase like $r^2$, and the virtual particle can be anywhere on this surface.




- Thus, the probability of having the virtual particle along some fixed direction is proportional to $1/r^2$, which is the origin of the inverse square-law

# Range of strong interactions

- The argument that explains the inverse-square law fails if the emission of the virtual particle is confined to a cylinder of constant radius $\rho$
- At a distance $r \ll \rho$, the same argument leads to the inverse-square law
- But at a distance $r \gg \rho$ the story is different. The exchange particle is guided down the cylinder, where the cross-sectional area is constant, and the force remains independent of $r$.




This is what seems to happen for quarks, with $\rho \approx 1$ fm ($10^{-13}$ cm)

**Strong interactions between quarks are independent of distance. This is at the origin of CONFINEMENT**

- A constant force independent of distance at large distance is confirmed by numerical lattice simulation: $V(r) \sim \sigma r$ at large distance

# Range of strong interactions

- So, the interactions between quarks are surely long-ranged
- On the other hand, interactions between two nucleons have range R ~ 1 fm, as predicted by Yukawa, that is nuclear interactions are short-ranged
- How can we explain the long-ranged interactions between quarks with the short-ranged interaction between nucleons?

The short-ranged nuclear force is presumably a result of cancelation between the long-ranged forces from the different quarks inside a nucleon

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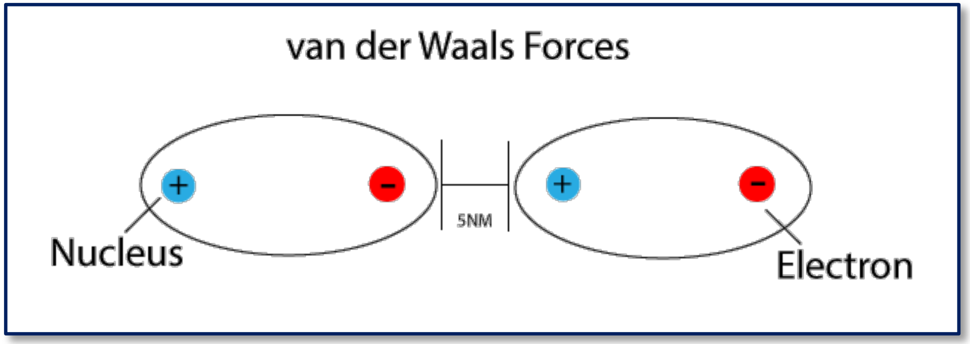
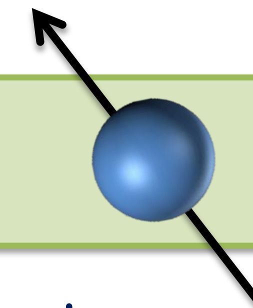
The short-ranged nuclear force is presumably a result of cancelation between the long-ranged forces from the different quarks inside a nucleon

This is analogous to the force between neutral atoms (van der Waals force) which has a range much shorter than the range of the Coulomb forces between electrons and/or protons.
This is because an electron in one atom sees both repulsive and attractive forces from the electrons and the protons of another atom.
These almost equal and opposite forces tend to cancel each other



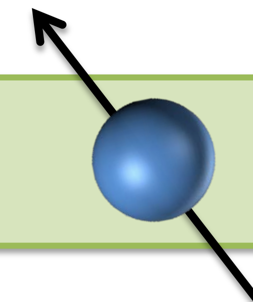
# Spin and the Nature of Forces



- The spin of the exchange particle conveys important information about the nature of the force that the particle transmits

- Quantum Field Theory dictates that the nature of the force between two particles with same- or different-sign charges depends on the spin of the mediator according to
  - s is an even integer ⟹ same-sign charges ($q_1 q_2 > 0$) : attractive; different-sign charges ($q_1 q_2 < 0$) : repulsive
  - s is an odd integer ⟹ same-sign charges ($q_1 q_2 > 0$) : repulsive; different-sign charges ($q_1 q_2 < 0$) : attractive

# Spin and the Nature of Forces



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  - s is an odd integer ⟹ same-sign charges ($q_1 q_2 > 0$) : repulsive; different-sign charges ($q_1 q_2 < 0$) : attractive

- The charge of the gravitational force is the mass, which is always positive, and the spin of the graviton is s=2 ⟹ gravity is always attractive
- The spin of the photon is s=1 ⟹ Coulomb force is repulsive for same-sign charges and attractive for opposite sign
- Pion have s=0 ⟹ The nuclear force between two protons is attractive

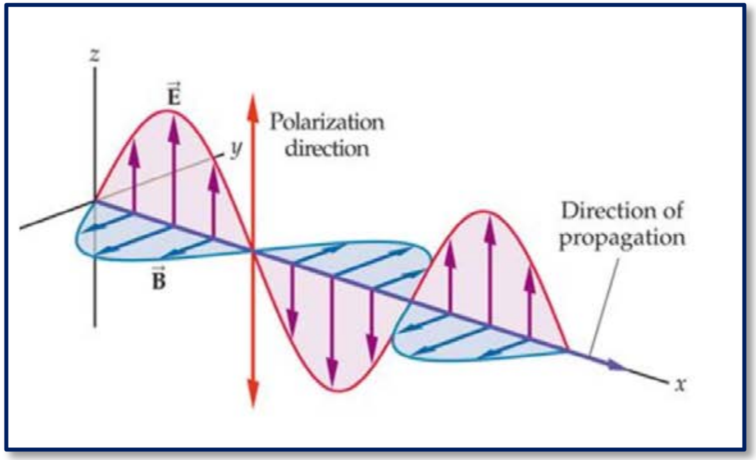
# Spin and the Nature of Forces

**The spin of the exchange particle also determines the complexity of the force**

For a given spin s, the z-component $m_s$ may assume 2s+1 discrete values, i.e. –s, -s+1,…, s-1, s.

An exchange particle carries with it 2s+1 bits of information on its orientation. These different bits can trigger different forces, so the number and complexity of the force increases with the value of the spin. For example:

- The Yukawa nuclear force, transmitted by the s=0 pion, is a single attractive force (the field is a scalar)
- The electromagnetic force, carried by the s=1 photon, is electric and magnetic (the field is a vector)
- The gravitational force, transmitted by the s=2 graviton, is even more complex (the field is a tensor)



# Scale-dependent couplings

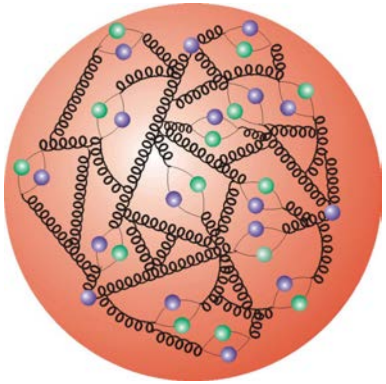
A new prediction of quantum field theory is that the couplings, i.e. charges and masses, depends on the scale (distance or energy) at which they are probed

- A particle is never an isolated object, since it emits continuously virtual particles

If there is no other particle nearby, a created virtual particle is reabsorbed in the same point

Consequently, every particle is surrounded by a Yukawa cloud of virtual particles

An electron, for instance, is surrounded by a cloud of virtual photons and virtual $e^+e^-$ pairs emitted and reabsorbed by the virtual photons

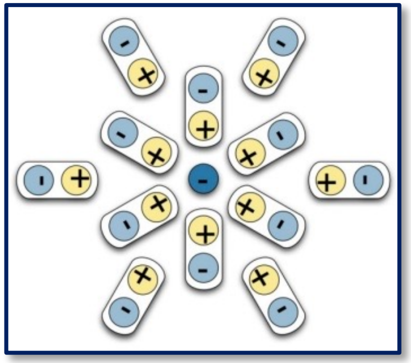


A proton is surrounded by a cloud of virtual gluons and virtual $q\text{-}\bar{q}$ pairs

# Scale-dependent couplings

- By looking at an electron from different distances, one sees different amounts of the Yukawa cloud around the electron, hence a different energy or mass of the electron (mass is the energy at rest):

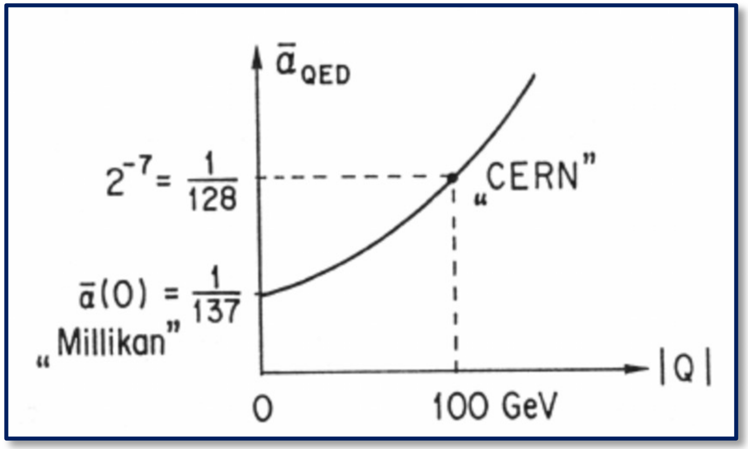
$$m = m(\mu) \quad , \quad \text{with } \mu = r \text{ or } E$$



- Similarly, by looking at the electron from different distances, a different electric charge is seen, due to the screening effect of the $e^+e^-$ pairs

Therefore, the electric charge of the electron decreases at large distances or, equivalently, at small energies:

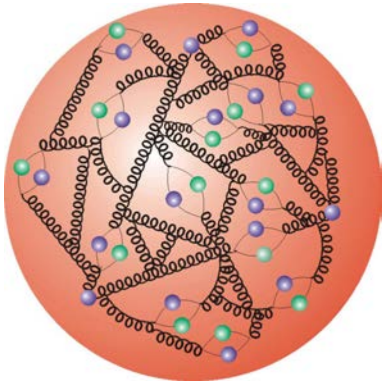
$$e = e(\mu) \quad , \quad \text{with } \mu = r \text{ or } E$$



# Scale-dependent couplings

- An important difference between strong and electromagnetic interactions is that gluons are “colored”, i.e. they have strong charge

Therefore, for hadrons, like the proton, due to the strong interactions, the cloud is no longer color neutral and there are two competing effects due to gluons and $\bar{q}$-q pairs.



Going to shorter distances, i.e. penetrating the cloud, we are seeing less q-$\bar{q}$ shielding, and this effect increases the coupling strength $g_s$

On the other hand, we are also seeing less gluons which are colored objects

Calculations show that gluons represent the main contribution so that $g_s(\mu)$ is an increasing function of the distance (opposite to the electromagnetic case)

At short distance, or large energies, $g_s(\mu)$ becomes quite small, leading to phenomenon of

**ASYMPTOTIC FREEDOM**

# Elementi di Fisica Teorica Contemporanea

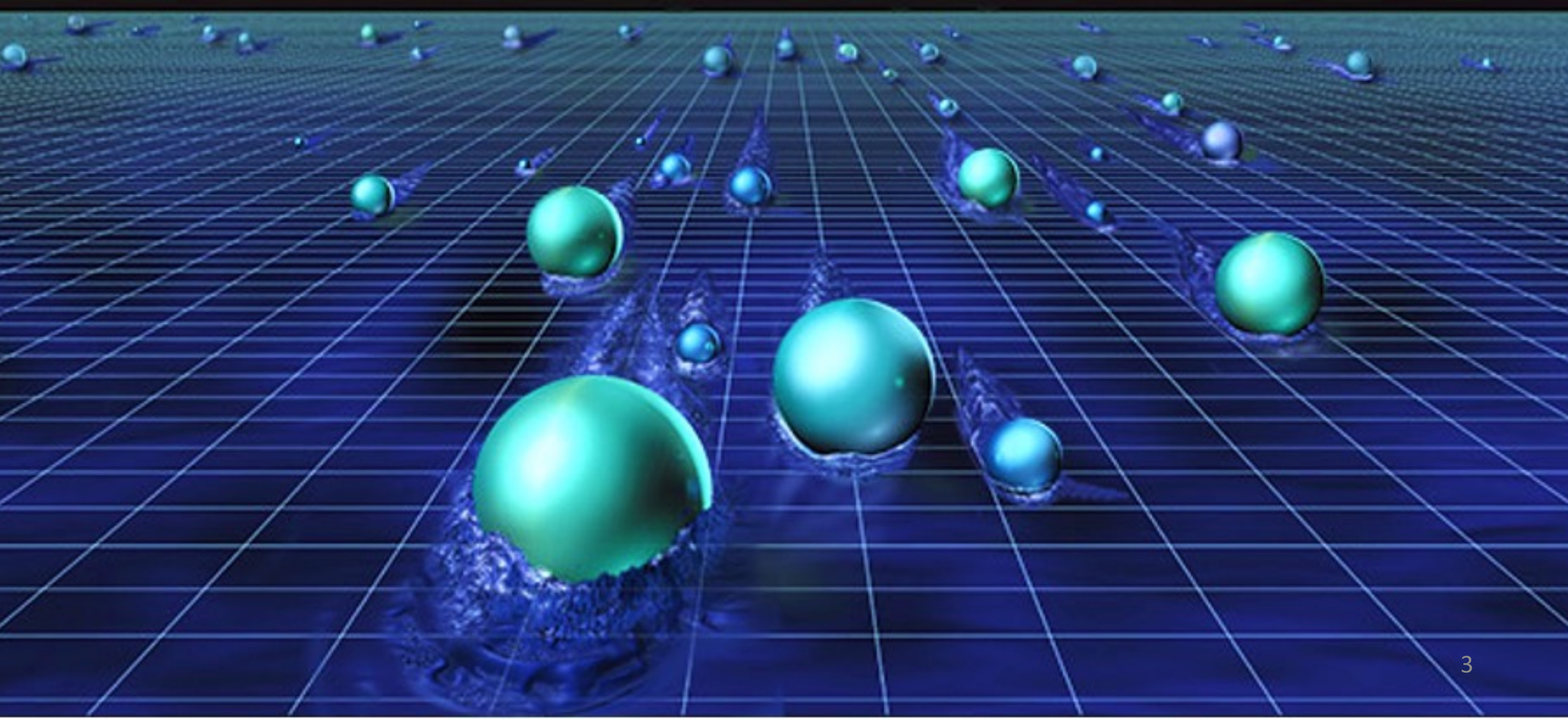
1. Teoria della Relatività
2. Meccanica quantistica
3. Particelle e campi (V. Lubicz / C. Tarantino)*
4. Gravità quantistica

(*) Testo consigliato:
Q. Ho-Kim, N. Kumar, C.S. Lam: *Invitation to Contemporary Physics*

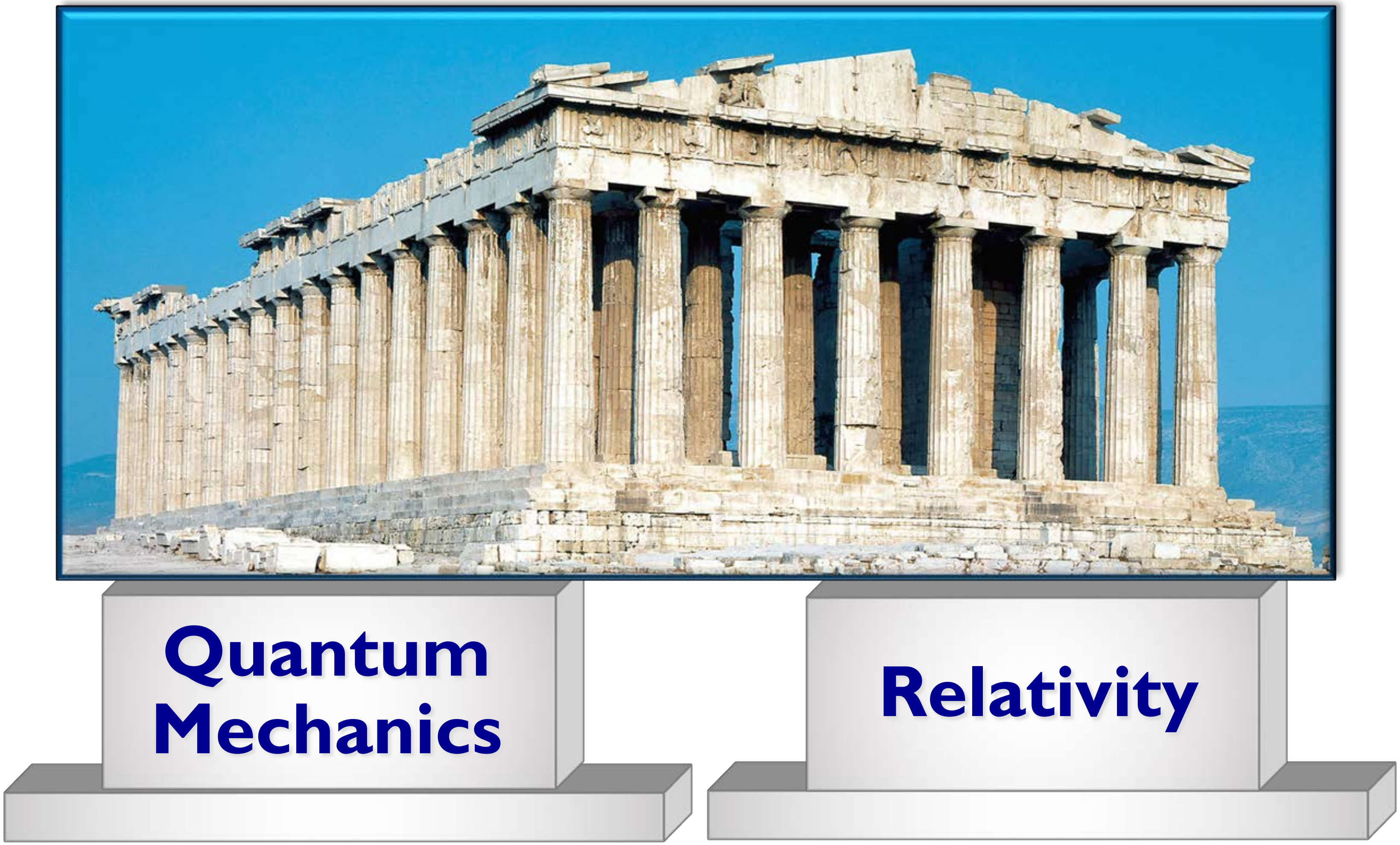
# Particelle e campi
# Parte 3

## Sommario

# 4) Il Modello Standard



# STANDARD MODEL



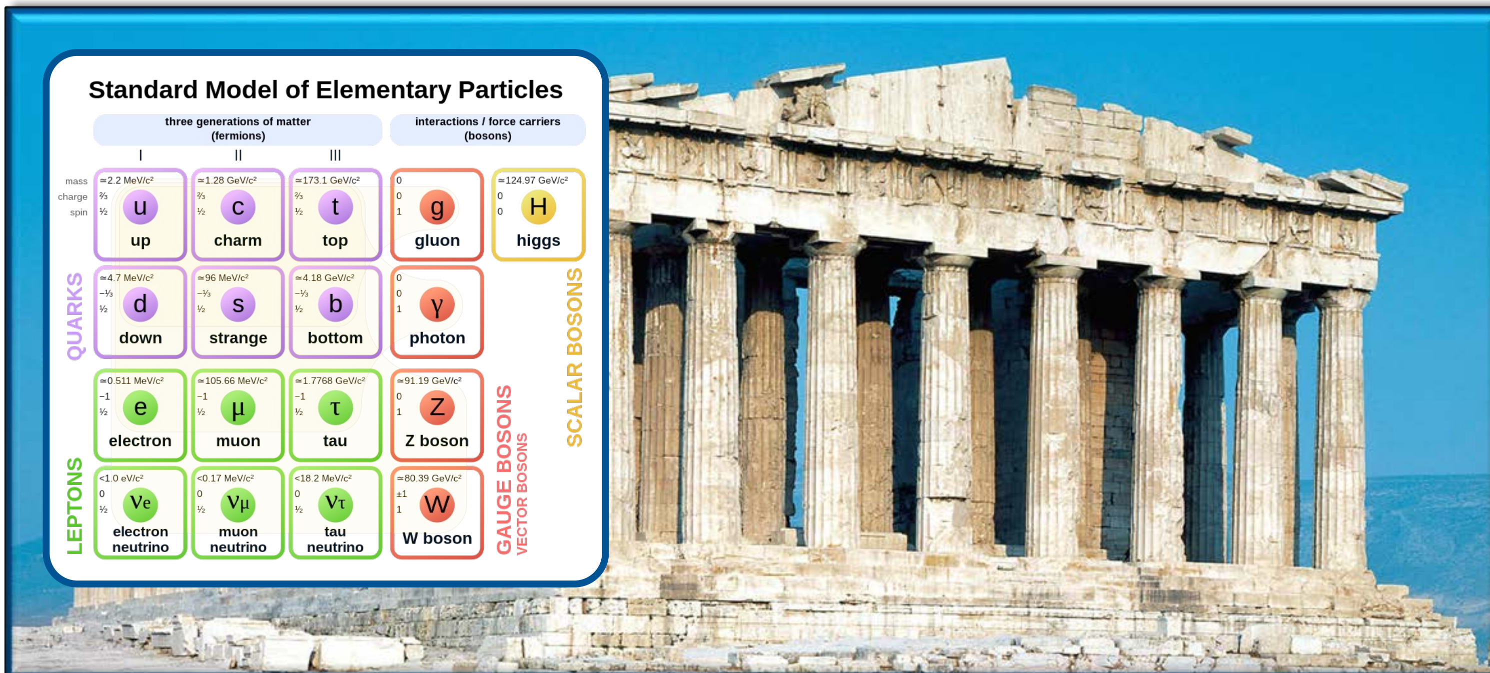
# STANDARD MODEL

**Quantum Mechanics**

**Relativity**

The Standard Model is the current theory for strong, weak and electromagnetic interactions

# STANDARD MODEL

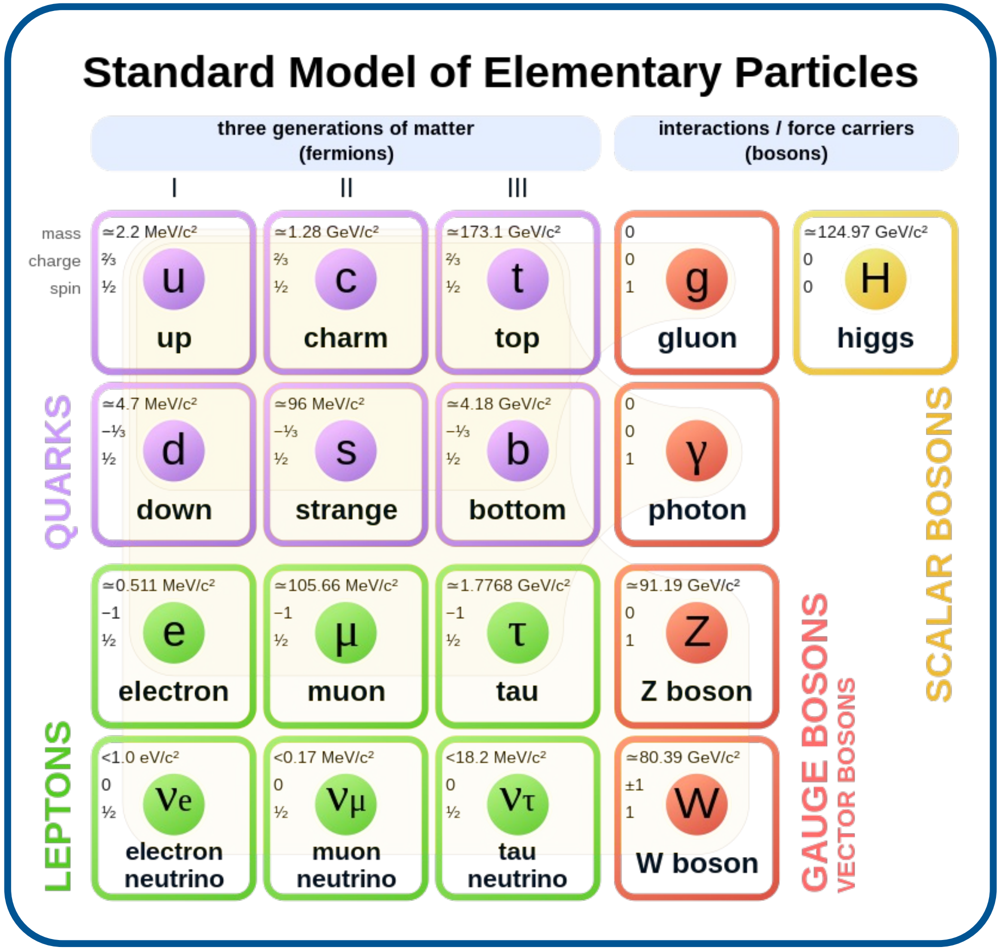



**Quantum Mechanics**

**Relativity**

Everything exists is constituted by a relatively small number of elementary particles

# STANDARD MODEL



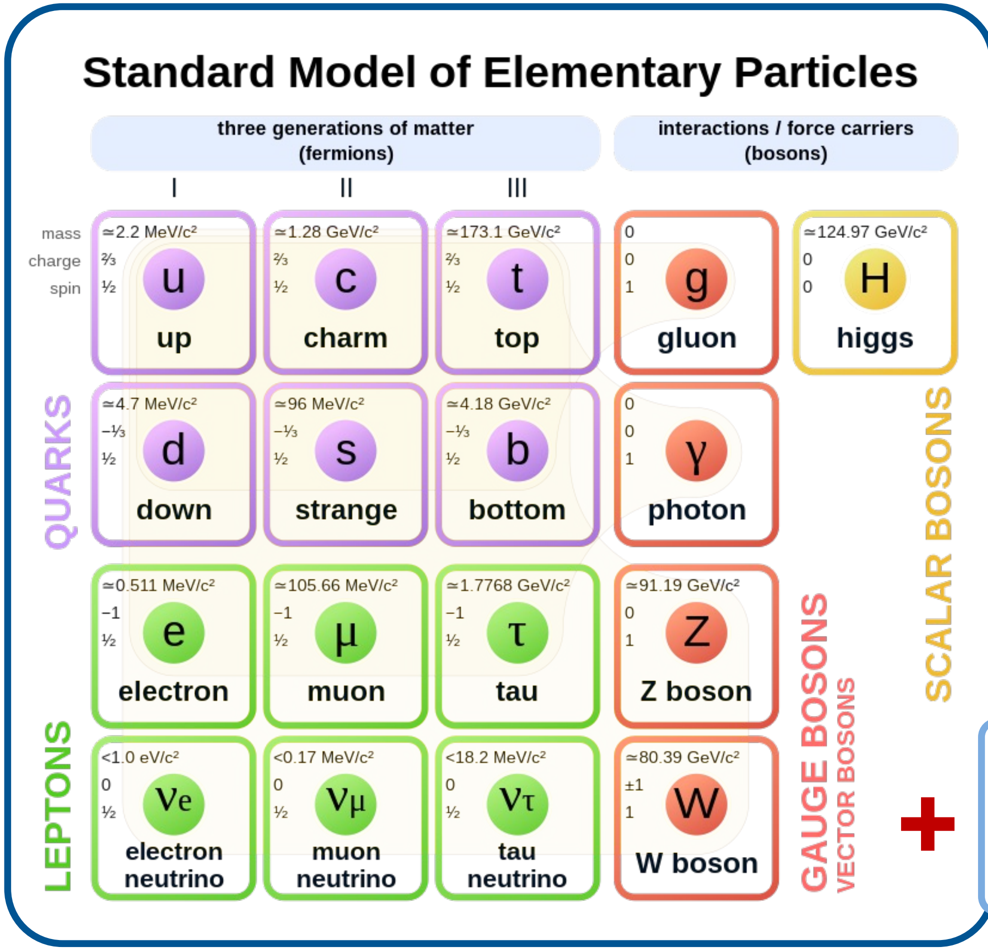

- Being elementary, they have no internal structure, i.e. they are dimensionless pointlike particles

- The complete list contains
  6 quarks , 6 leptons ,
  4 gauge bosons,
  1 Higgs boson

We have already discussed all of them except for the Higgs boson

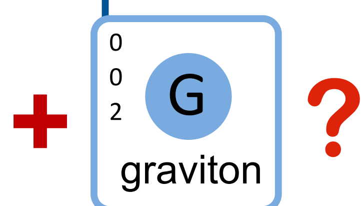
Everything exists is constituted by a relatively small number of elementary particles

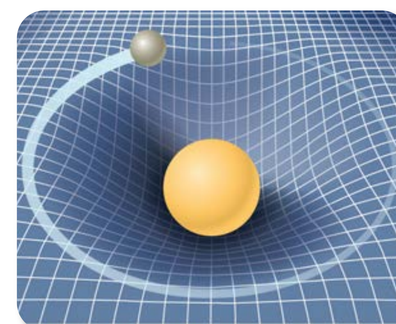
# STANDARD MODEL




- We know that **the Standard Model cannot be the ultimate theory** (at least) since **it does not contain quantum gravity.**

- **Quantum gravity** effects become **relevant** at **very short distance or very high energy**, $M_{Planck} \sim 10^{19}$ GeV



**Everything exists** is constituted by a relatively **small number of elementary particles**

# STANDARD MODEL


**Quantum Mechanics**

**Relativity**

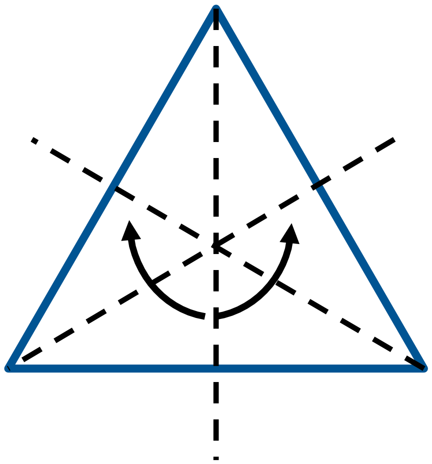
The form of the interactions is completely fixed by a symmetry principle: the gauge symmetry

# SYMMETRY

"A thing is symmetrical if there is something you can do to it so that after you have finished doing it, it looks the same as it did before"

Hermann Weyl

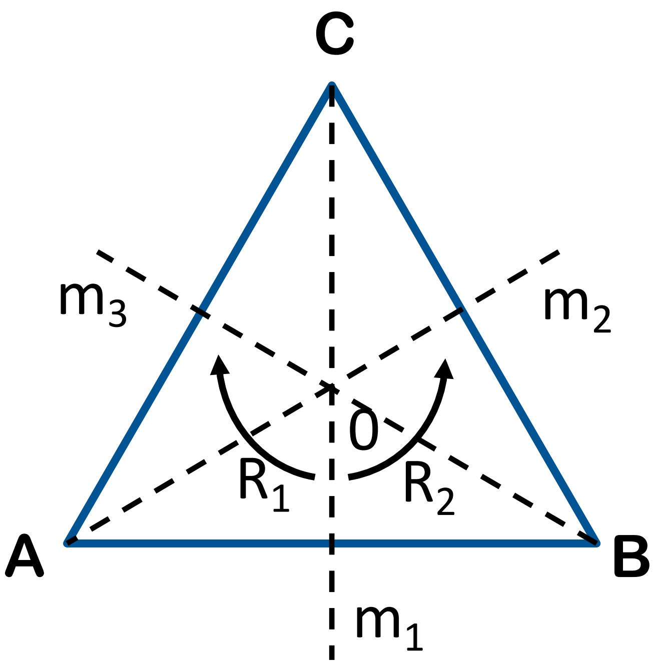
- The thing here is the object of interest
- What you do to it is called the symmetry operation or transformation
- Looks the same is another name for invariance



- The object can be anything: a geometrical figure, a mathematical equation, a physical system, etc.
- The transformation can be a rotation by 120°, a translation in space by a repeat distance, a mirror reflection, a Lorentz transformation in an equation, an exchange of particles, a "gauge" transformation, …

# SYMMETRY

- Consider the symmetry of an equilateral triangle living on a plane
- There are 6 symmetry operations:
  - The rotations $R_1$ and $R_2$ (clockwise and anti-clockwise by 120 degrees respectively, about the threefold symmetry axis) as well as the identity E (rotation by 0 degrees)
  - The reflections $m_1$, $m_2$, $m_3$ in the three mirror lines (medians)



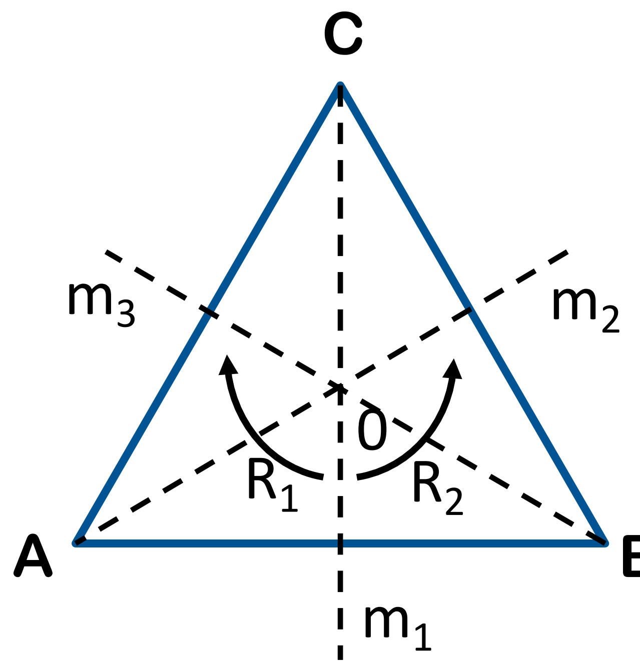

- The successive applications of any two operations leaves the object invariant. Therefore, the combined operation is one of the symmetry operations. For instance:

$$R_2 \cdot R_1 = E \qquad (ABC \xrightarrow{R_1} BCA \xrightarrow{R_2} ABC)$$

$$R_2 \cdot m_1 = m_3 \qquad (ABC \xrightarrow{m_1} BAC \xrightarrow{R_2} CBA)$$

# SYMMETRY

- Consider the symmetry of an equilateral triangle living on a plane

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$R_2 \cdot R_1 = E \quad (ABC \xrightarrow{R_1} BCA \xrightarrow{R_2} ABC)$

$m_1 \cdot R_2 = m_2 \quad (ABC \xrightarrow{R_2} CAB \xrightarrow{m_1} ACB)$

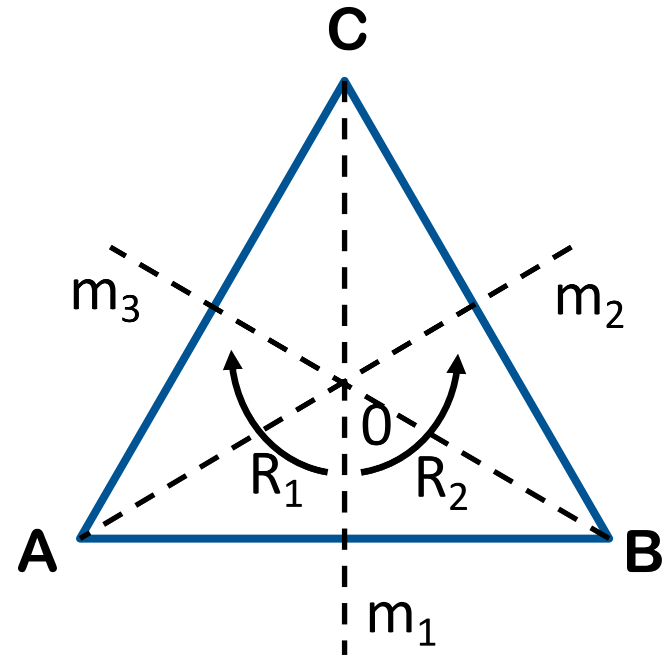
$R_2 \cdot m_1 = m_3 \quad (ABC \xrightarrow{m_1} BAC \xrightarrow{R_2} CBA)$

$R_2 \cdot m_1 \neq m_1 \cdot R_2$ **Non-abelian symmetry**

# SYMMETRY

- There is a one-to-one correspondence between the symmetry operations of the equilateral triangle and the permutations on three objects

| Triangle | Permutations |
|---|---|
| E | ABC ⟶ ABC |
| $R_1$ | ABC ⟶ BCA |
| $R_2$ | ABC ⟶ CAB |
| $m_1$ | ABC ⟶ BAC |
| $m_2$ | ABC ⟶ ACB |
| $m_3$ | ABC ⟶ CBA |




- The two sets of elements contain objects of different type but the transformations rules under the symmetry operations are the same

$$R_2 \cdot m_1 = m_3 \quad \Longleftrightarrow \quad ABC \longrightarrow BAC \longrightarrow CBA \;=\; ABC \longrightarrow CBA$$

- The multiplication table is like the fingerprint of the symmetry. Identical multiplication tables imply identical symmetry structures

# GROUP THEORY

The proper language for a systematic study of symmetry is the group theory

- A group G is a set of elements a, b, c, … (transformations) for which is defined a composition or multiplication law (successive operations) which satisfies the following properties:

- Closure: the set is closed under the multiplication

  If $a \in G$, $b \in G$ then $a \cdot b \in G$

- Associative: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

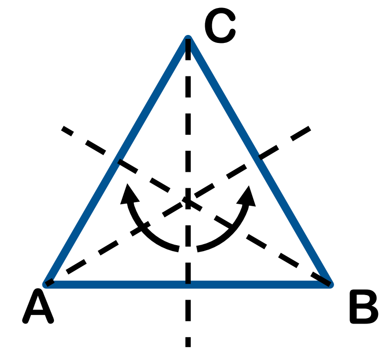
- Identity: there exists an identity element e (or 1) such that

  $a \cdot e = e \cdot a = a \quad \forall\, a \in G$

- Inverse: for each element $a \in G$ there exists the inverse $a^{-1}$ such that

  $a^{-1} \cdot a = a \cdot a^{-1} = e$

# GROUP THEORY




- The group of permutations of 3 objects is called the symmetric group $S_3$. This is also the symmetry group of the equilateral triangle

- $S_3$ is a non-abelian group, because its multiplication law is not commutative (it is the smallest non-abelian group; it contains 6 elements)

- A group can be finite, if it contains a finite number of elements, or infinite. The number of elements of a group is called the order of the group.

- The simplest group consists of just a single element, $G = (e)$.

  The next-to-simplest group contains two elements, $G = (e, a)$, with $a \cdot a = e$.
  It is called $Z_2$, the cyclic group of order 2.
  In general, $Z_n = (a, a^2, a^3, \dots, a^n = e)$

# GROUP THEORY

- If group is finite, we can denote its elements with $a_\theta$, where $\theta$ assumes a finite number of values.

  We can generalize this idea to a countable infinite group, with an infinite number of discrete values of $\theta$, or to a group with a continuous of elements, where $\theta$ is a continuous parameter: a($\theta$).

  Finally, $\theta$ can stand for a set of continuous parameters: $a(\vec{\theta}) = a(\theta_1, \theta_2, \dots, \theta_n)$

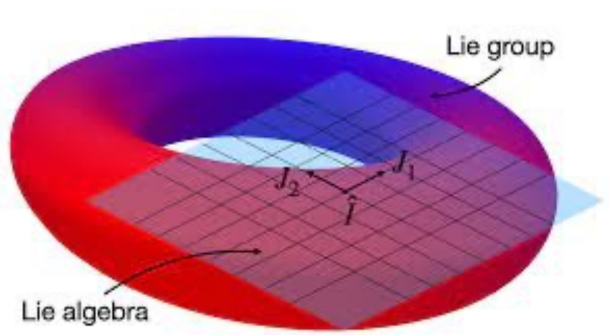
- Consider the product of two elements of the group

$$a(\vartheta) \cdot a(\varphi) = a(\xi) \quad \text{with} \quad \xi = \xi(\vartheta, \varphi)$$

If $\xi(\theta, \varphi)$ is a continuous function of $\theta$ and $\varphi$, then the group is a continuous group.

If $\xi(\theta, \varphi)$ is also an analytic function (infinitely differentiable), then the group is called a

**Lie group**



# Examples of Lie groups

- **Group of translations**

$a(\theta)$: $x' = x + \theta$

**(in 1 dimension)**

$$a(\varphi) \cdot a(\theta):\quad x'' = x' + \varphi = x + \theta + \varphi \equiv a(\xi) \;\Rightarrow\; \xi(\theta,\varphi) = \theta + \varphi$$

**It is an abelian group:** $\xi(\theta,\varphi) = \xi(\varphi,\theta)$

**The identity is** $e = a(0)$ **and the inverse is** $a(\theta)^{-1} = a(-\theta)$

# Examples of Lie groups

**• Group of translations** $a(\theta)$: $x' = x + \theta$ **(in 1 dimension)**

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It is an **abelian group**: $\xi(\theta, \varphi) = \xi(\varphi, \theta)$

The **identity** is $e = a(0)$ and the **inverse** is $a(\theta)^{-1} = a(-\theta)$

**• Group of affine transformations** $a(\theta_1, \theta_2)$: $x' = \theta_1 x + \theta_2$

$$a(\vec{\varphi}) \cdot a(\vec{\theta}): \quad x'' = \varphi_1\, x' + \varphi_2 = \varphi_1\, (\theta_1\, x + \theta_2) + \varphi_2 \equiv a(\vec{\xi})$$

$$\Rightarrow \quad \xi_1(\vec{\theta}, \vec{\varphi}) = \theta_1 \varphi_1 \quad , \quad \xi_2(\vec{\theta}, \vec{\varphi}) = \theta_2 \varphi_1 + \varphi_2$$

It is a **non-abelian group**: $\xi_2(\theta, \varphi) \neq \xi_2(\varphi, \theta)$

The **identity** is $e = a(1,0)$ and the **inverse** is $a(\vec{\theta})^{-1} = a(1/\theta_1, -\theta_2/\theta_1)$

# Examples of Lie groups

**• Group of rotations SO(n) or R(n)**

$x' = R\,x$ $R^T R = 1$ $\mathrm{Det}(R) = 1$

It is the group of orthogonal matrices with unitary determinant
(→ reflections are not included)

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \qquad R = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{n1} & \cdots & r_{nn} \end{pmatrix}$$

Under the transformation, the scalar product is invariant: $x'^2 = x\,R^T R\,x = x^2$

The orthogonality condition $R^T_{ik} R_{kj} = \delta_{ij}$ corresponds to a set of

$$n(n-1)/2 + n = n(n+1)/2$$

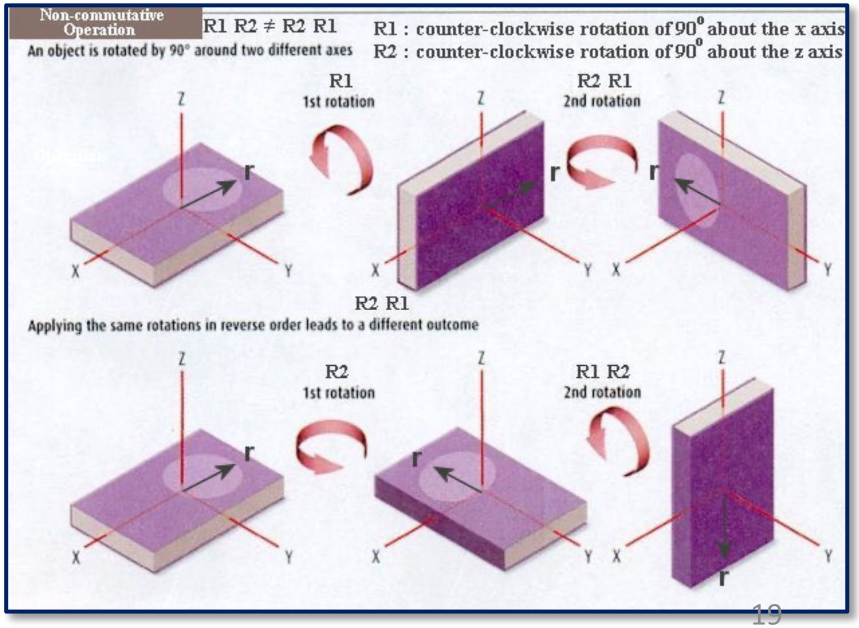
conditions.

The group is thus characterized by

$$n^2 - n(n+1)/2 = n(n-1)/2$$

real parameters (3 parameters in D=3)

The group is non-abelian ➡



# Examples of Lie groups

- **Unitary group U(n)**

$$\psi' = U\,\psi \qquad U^{+}U = 1 \qquad [\,(U^{+})_{ij} = U^{*}_{ji}\,]$$

It is the group of unitary matrices

Under the transformation, the scalar product in the complex vector space is invariant:

$$|\psi'|^2 = \psi^*\,U^{+}U\,\psi = |\psi|^2$$

The unitarity condition, $U^{*}_{ki}U_{kj} = \delta_{ij}$, corresponds to a set of $2n(n-1)/2 + n =$ $= n^2$ real conditions.

The group is thus characterized by $2n^2 - n^2 = n^2$ real parameters

$$U(\theta_0, \theta_1, \cdots, \theta_{n^2-1}) = exp[i(\theta_0 I + \theta_1 T_1 + \cdots + \theta_{n^2-1}T_{n^2-1})] \qquad T_i = T_i^{+}$$

The $n^2$-1 linearly independent matrices $T_i$ are called the generators of the group

- For n=1: **U(1)** $\quad U(\theta) = e^{i\theta}$

the group consists of all complex numbers with absolute value 1

# Examples of Lie groups

- **Special unitary group SU(n)**

$\psi' = U\,\psi$

$U^{+}U = 1$

$\mathrm{Det}(U) = 1$

**It is the group of unitary matrices with determinant 1**

**It Is characterized by $n^2 - 1$ real parameters (one less than the unitary group)**

$$U(\vec{\theta}) = exp[i(\theta_1 T_1 + \cdots + \theta_{n^2-1} T_{n^2-1})] = exp(i\vec{\theta} \cdot \vec{T})$$

# Examples of Lie groups

- **Special unitary group SU(n)**

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It is the group of unitary matrices with determinant 1

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$$U(\vec{\theta}) = exp[i(\theta_1 T_1 + \cdots + \theta_{n^2-1} T_{n^2-1})] = exp\left(i\vec{\theta}\cdot\vec{T}\right)$$

# Lie algebra

- It can be shown that the commutators of the generators of a Lie group are linear combinations of generators

$$\left[T_i\,,T_j\right] = i\,C_{ij}^{k}\,T_k$$

$C_{ij}^{k}$ are called structure constants

Like the multiplication table for a finite group, for the Lie group the Lie algebra is the fingerprint of the symmetry

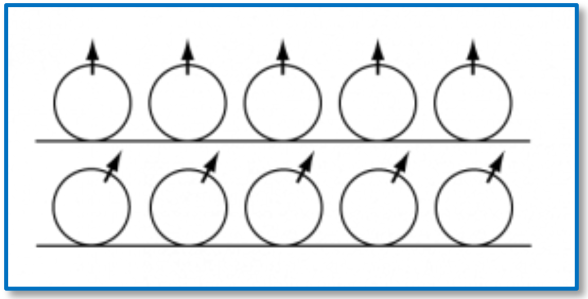
# GAUGE THEORY

- Many theories in physics are described by Lagrangians that are invariant under some symmetry transformation group

- When a theory is invariant under a transformation identically performed at every point in the spacetime, it is said to have a global symmetry

E.g. $$\psi' = exp\left(i\vec{\theta} \cdot \vec{T}\right) \psi$$

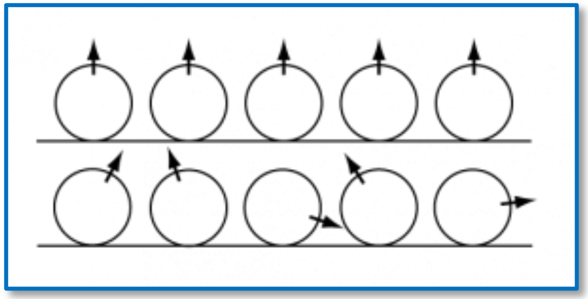
**Global**



- A symmetry transformation which is performed differently at every point in the spacetime, it is called a local symmetry

E.g. $$\psi' = exp\left(i\vec{\theta}(x) \cdot \vec{T}\right) \psi$$

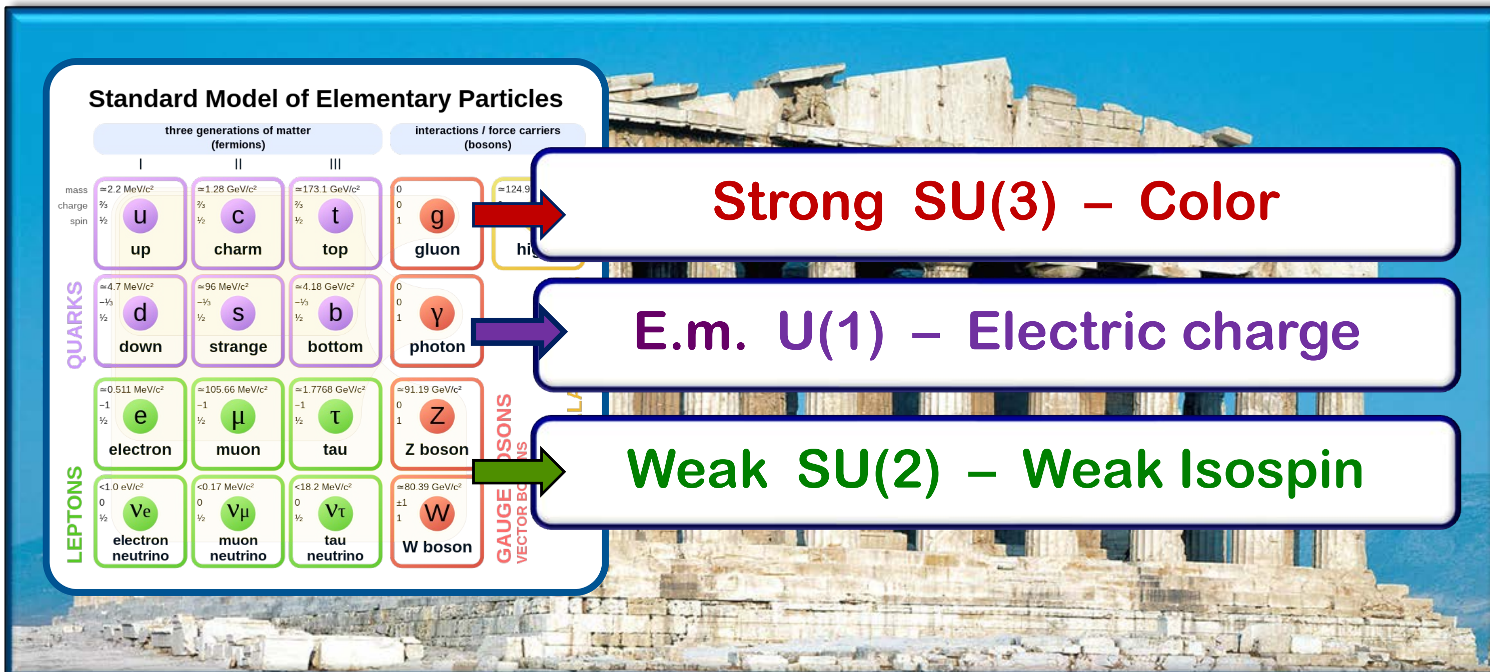
**Local**



**A gauge theory is a field theory based on a local symmetry**

The simplest and oldest example of gauge theory is Maxwell's electromagnetism

# STANDARD MODEL

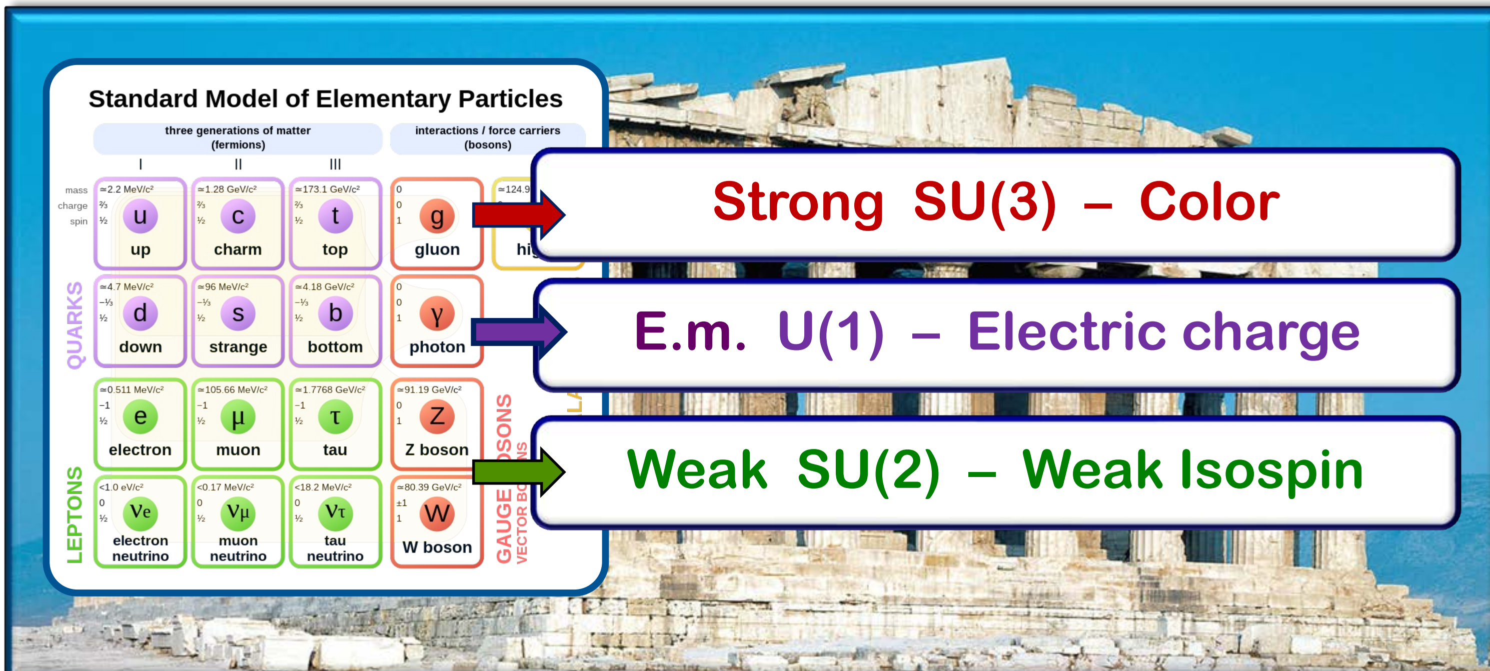



**Quantum Mechanics**

**Relativity**

The Standard Model is a gauge theory based on the group
SU(3) x SU(2) x U(1)

# STANDARD MODEL




Strong SU(3) – Color

E.m. U(1) – Electric charge

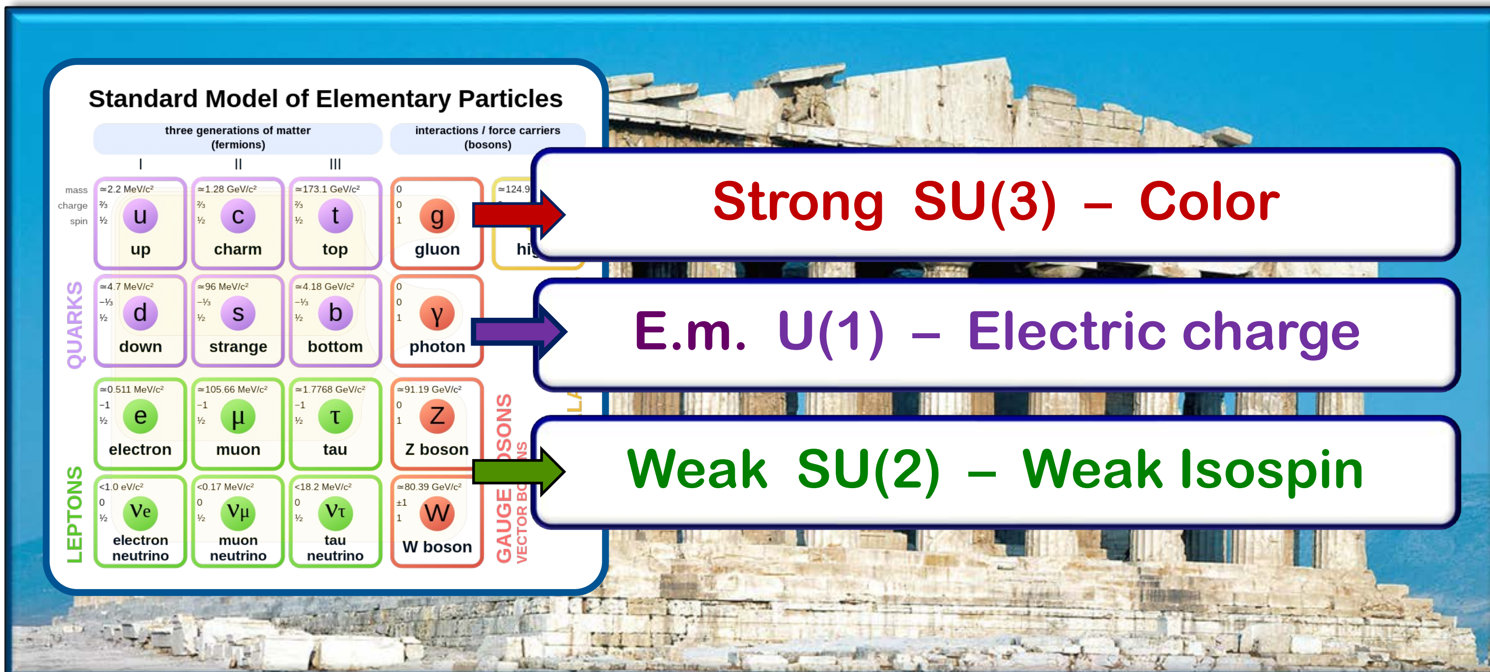
Weak SU(2) – Weak Isospin

**Quantum Mechanics**

**Relativity**

A gauge theory is at present the most elegant formulation of a force theory

# STANDARD MODEL

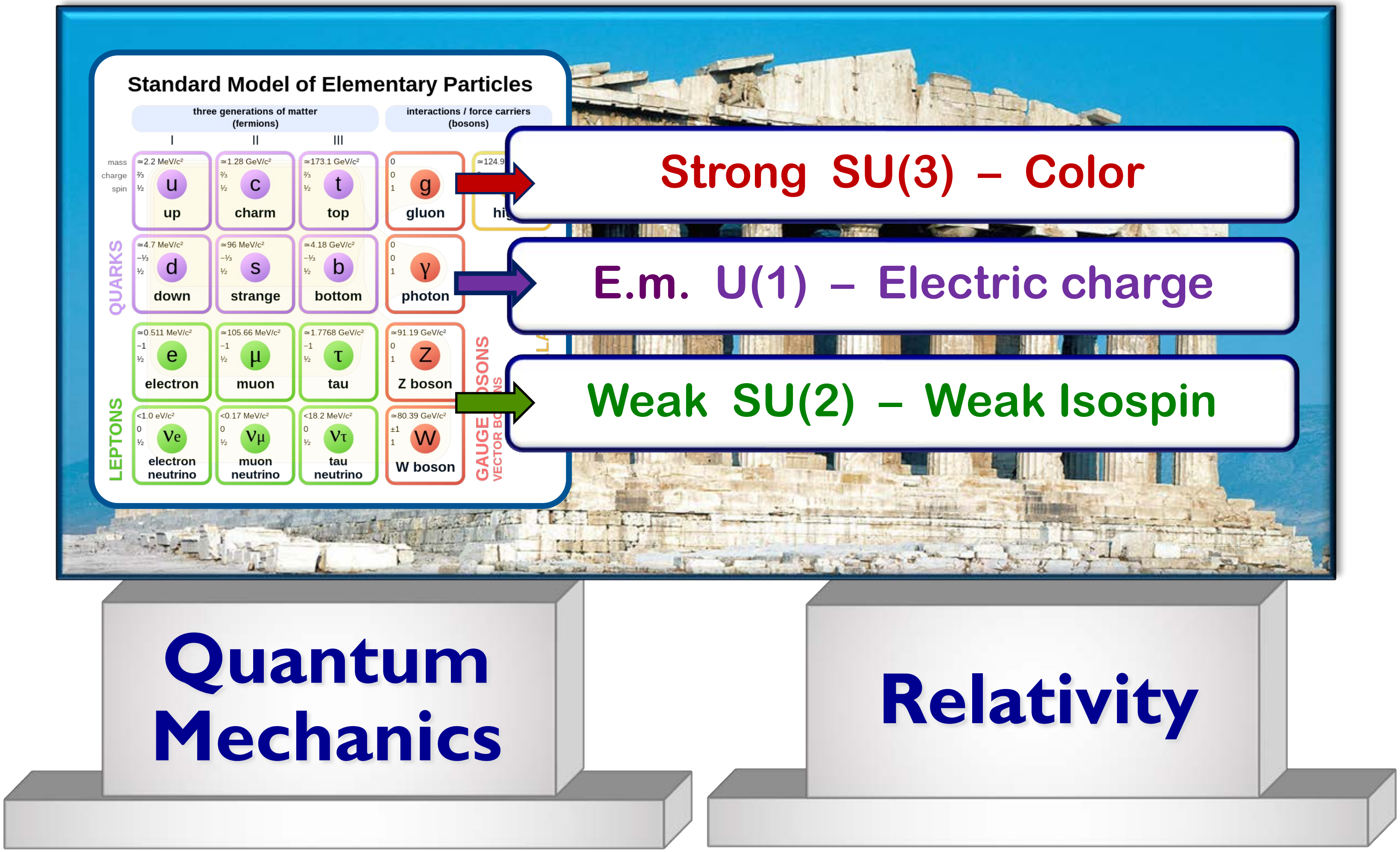



**Quantum Mechanics**

**Relativity**

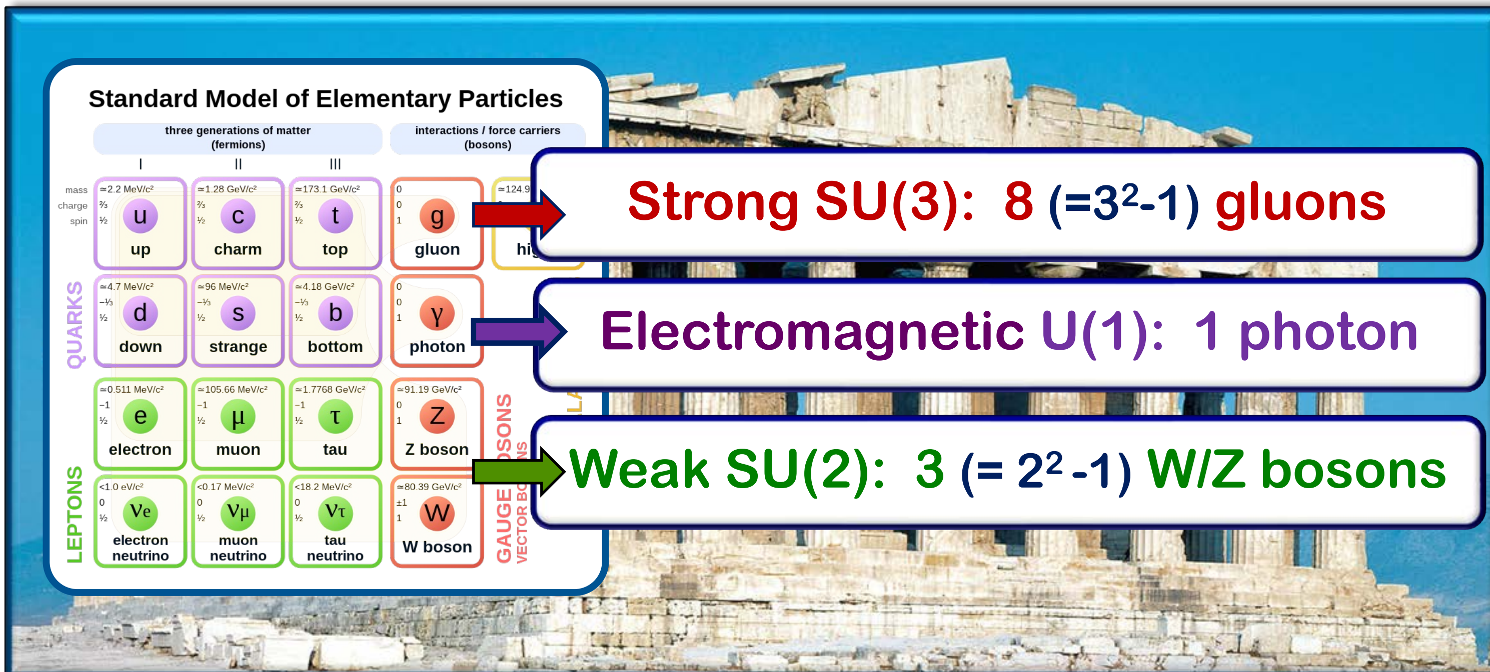
The form of the interactions is completely fixed by gauge symmetry

# STANDARD MODEL




Note: Einstein's theory of gravity is also a (classical) gauge field theory

# STANDARD MODEL




**Quantum Mechanics**

**Relativity**

The number of gauge boson for each group is equal to the dimension of the group, i.e. the number of generators

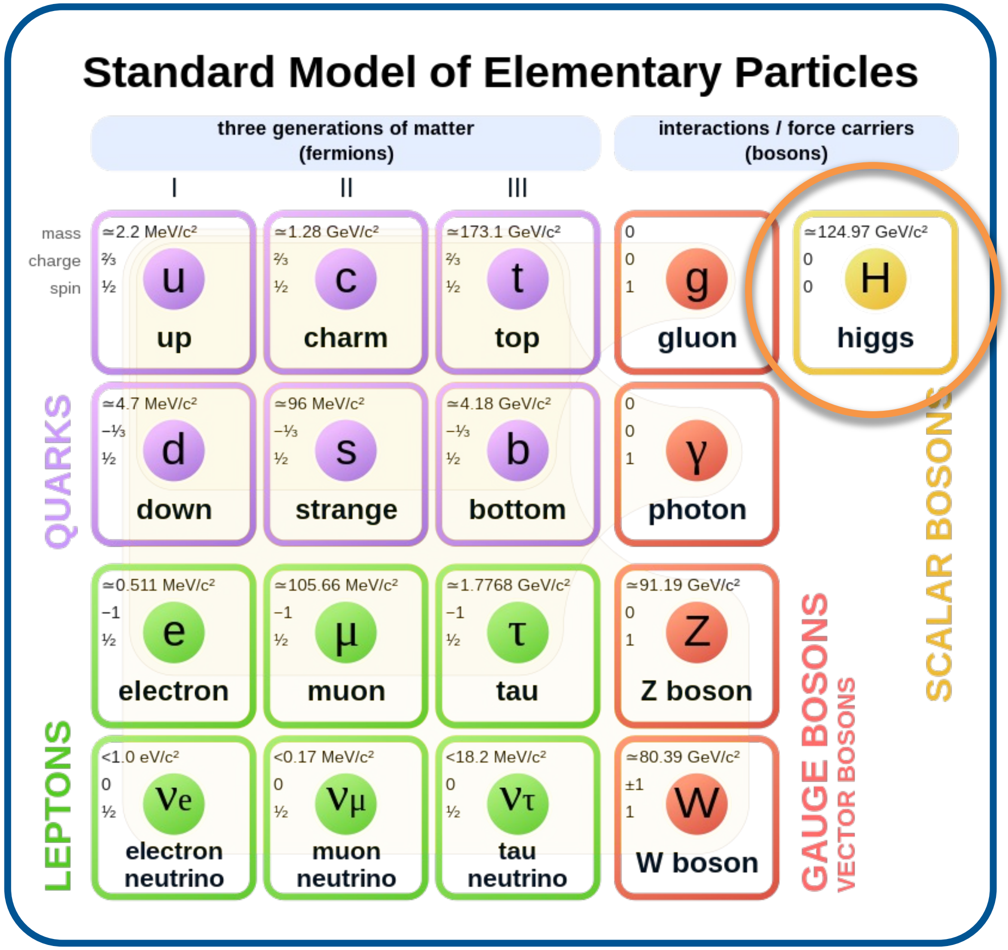
# Symmetries and Conservation Laws

- **Emmy Noether's theorem: there exists a deep correspondence**

| Symmetries | ⟷ | Conservation Laws |
| --- | --- | --- |

- **Time-translational invariance leads to energy conservation**
- **Space-translational invariance leads to momentum conservation**
- **Rotational symmetry leads to angular momentum conservation**
- **$U(1)_{EM}$ gauge symmetry leads to electric charge conservation as well as $SU(3)_C$ and $SU(2)_W$ lead to color and weak isospin conservations**

# Higgs Mechanism




- We have discussed all the elementary particles except for the Higgs boson

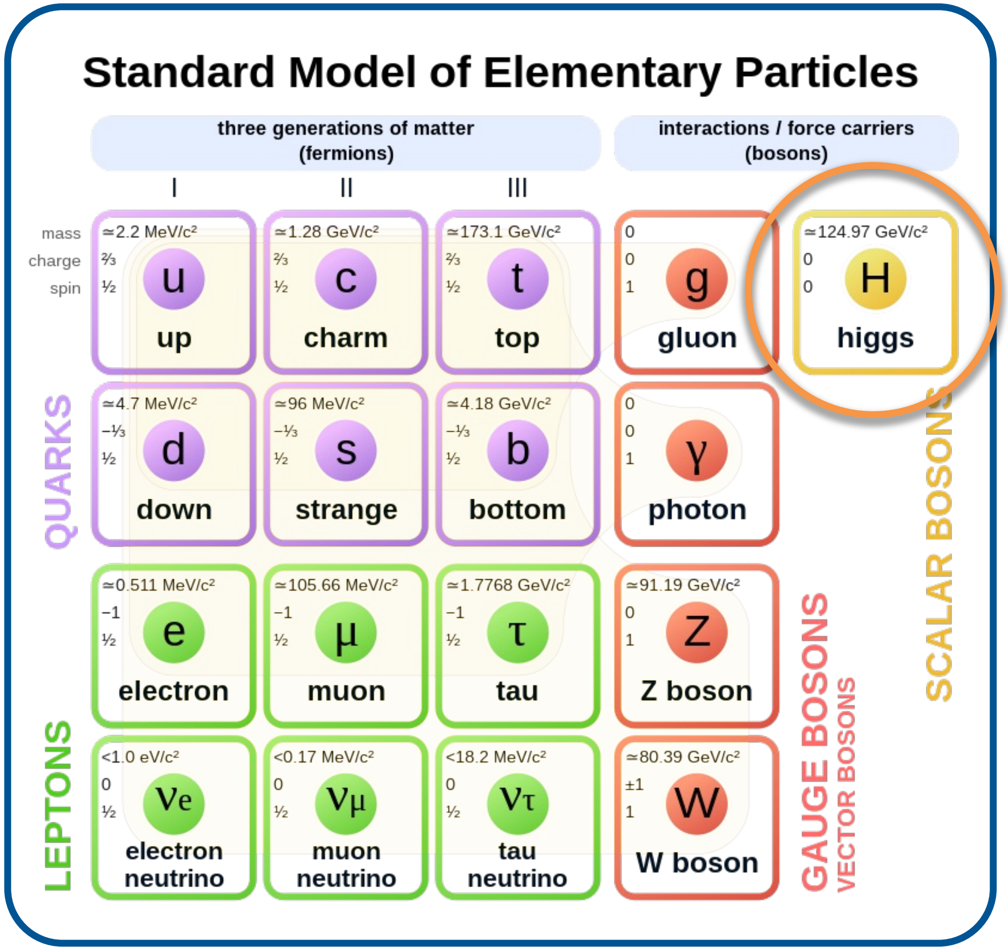
- On the one hand the gauge particles have to be massless to allow for gauge invariance.

  On the other hand, weak forces are found to be extremely short-range and the mass of exchange gauge particles should be very large.

  The two requirements seem to be incompatible

**The dilemma is solved by the Higgs mechanism**

# Higgs Mechanism



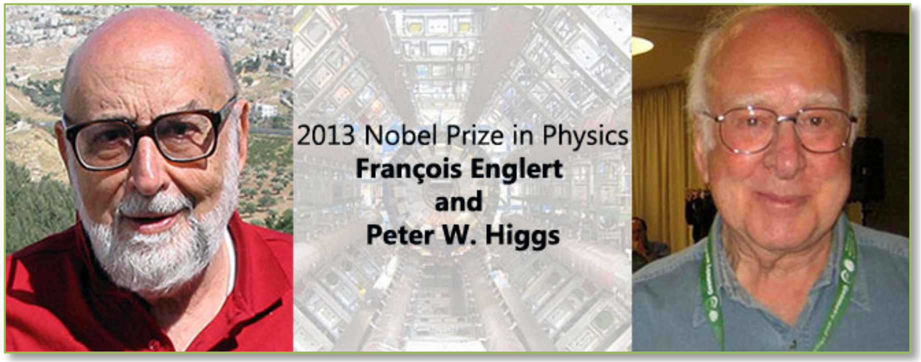

- The **Higgs mechanism** enables a **gauge particle** to become **massive**
- The **Higgs mechanism** is rather technical. It is based on the presence of a vacuum condensate of a scalar (s=0) particle, the **Higgs boson**
- The **Higgs boson** interacts with the W and Z bosons as well as with the matter particles and slow down them from the speed of light

**The interaction with the Higgs boson gives mass to the interacting particle**

# Higgs Mechanism


R. Brout, F. Englert (1964)

P. Higgs (1964)





**The Higgs boson has been discovered in 2012 at CERN**

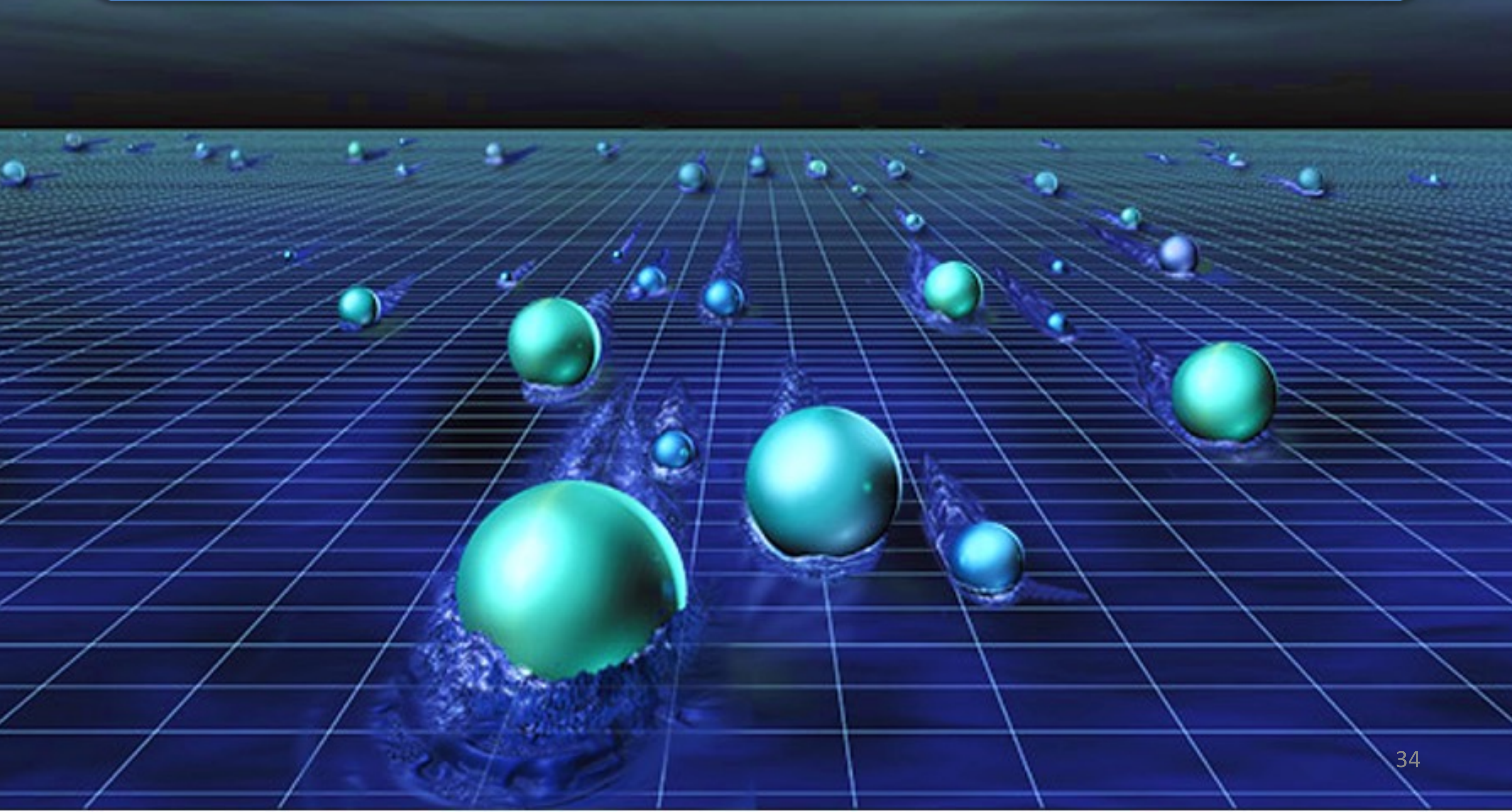
# STANDARD MODEL

**Quantum Mechanics**

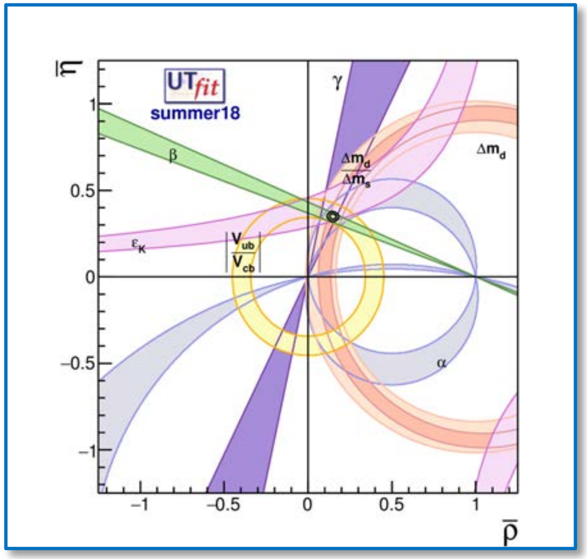
**Relativity**

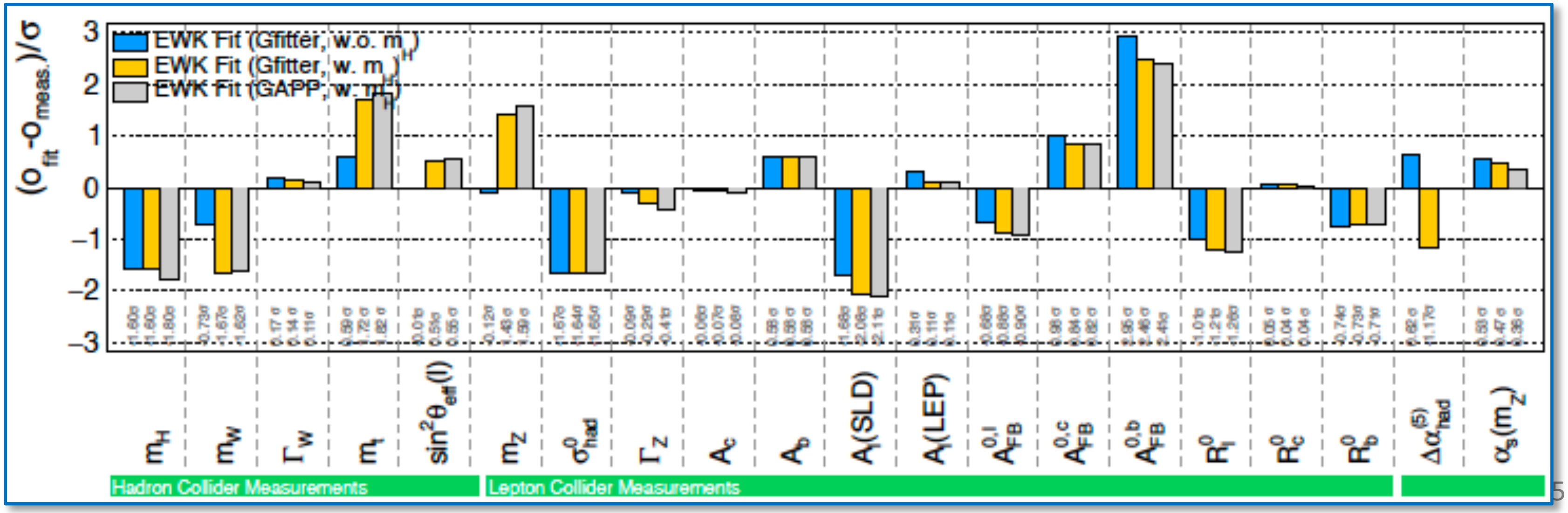
**The SM has passed so far all the experimental tests**

# 5) Fisica oltre il Modello Standard



## Il Modello Standard ha avuto grande successo nello spiegarci tutti i dati di precisione ad oggi disponibili



**Il Modello Standard ha avuto grande successo nello spiegarci tutti i dati di precisione ad oggi disponibili**

**Tuttavia i fisici sono convinti che esista Fisica oltre il Modello Standard, la cosiddetta**

**NUOVA FISICA**

Il Modello Standard ha avuto grande successo nello spiegarci tutti i dati di precisione ad oggi disponibili

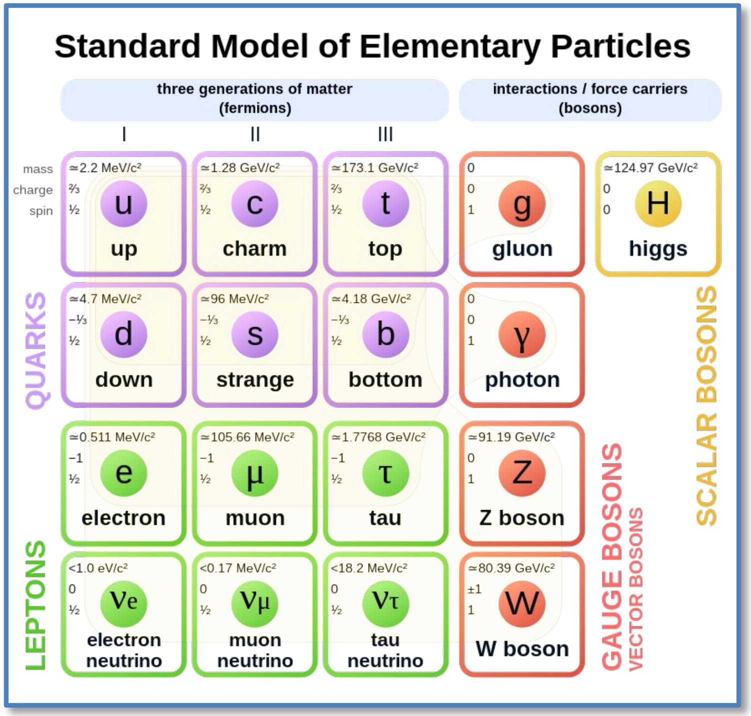
Tuttavia i fisici sono convinti che esista Fisica oltre il Modello Standard, la cosiddetta

**NUOVA FISICA**

**Perché?**

# Nuova Fisica: motivazioni teoriche

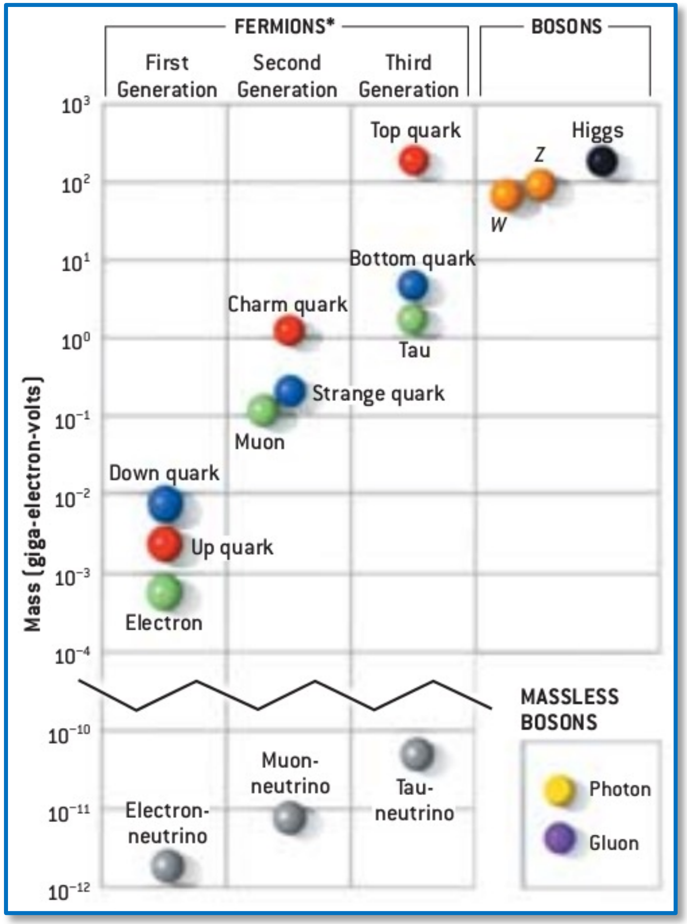
## Oltre alla gravità quantistica esistono altre motivazioni teoriche:




- Perché esistono 3 famiglie di particelle?

# Nuova Fisica: motivazioni teoriche

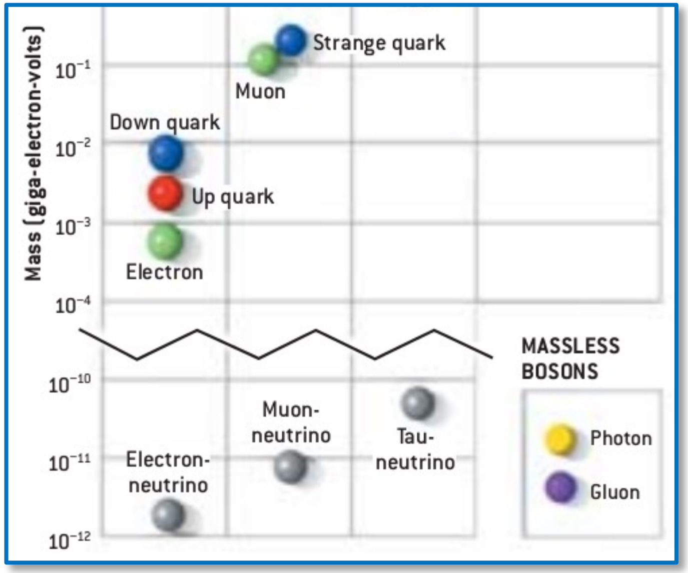
**Oltre alla gravità quantistica esistono altre motivazioni teoriche:**




- **Perché esistono 3 famiglie di particelle?**
- **Perché le masse variano su circa 6 ordini di grandezza?**

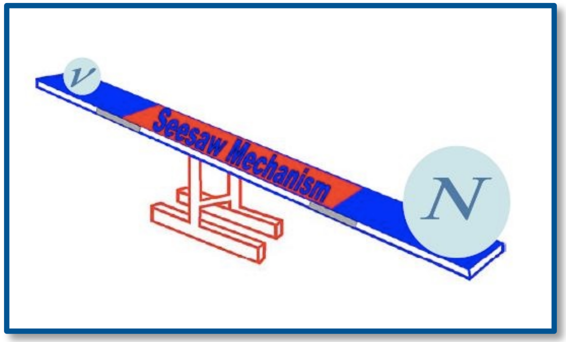
# Nuova Fisica: motivazioni teoriche

**Oltre alla gravità quantistica esistono altre motivazioni teoriche:**




- **Perché esistono 3 famiglie di particelle?**
- **Perché le masse variano su circa 6 ordini di grandezza?**
- **Perché i neutrini hanno massa quasi nulla?**

**Il meccanismo più accreditato per generare le masse dei neutrini è il meccanismo see-saw, in cui le masse dei neutrini sono piccole perché inversamente proporzionali ad una grande scala (M ~ $10^{15}$ – $10^{16}$ GeV)**



# Nuova Fisica: motivazioni teoriche

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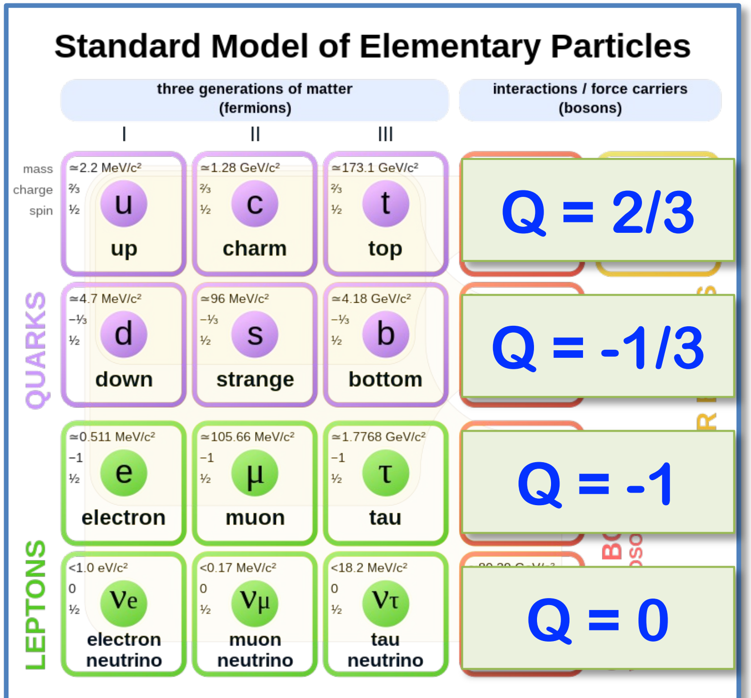


- Perché esistono **3 famiglie** di particelle?
- Perché le **masse** variano su circa **6 ordini di grandezza**?
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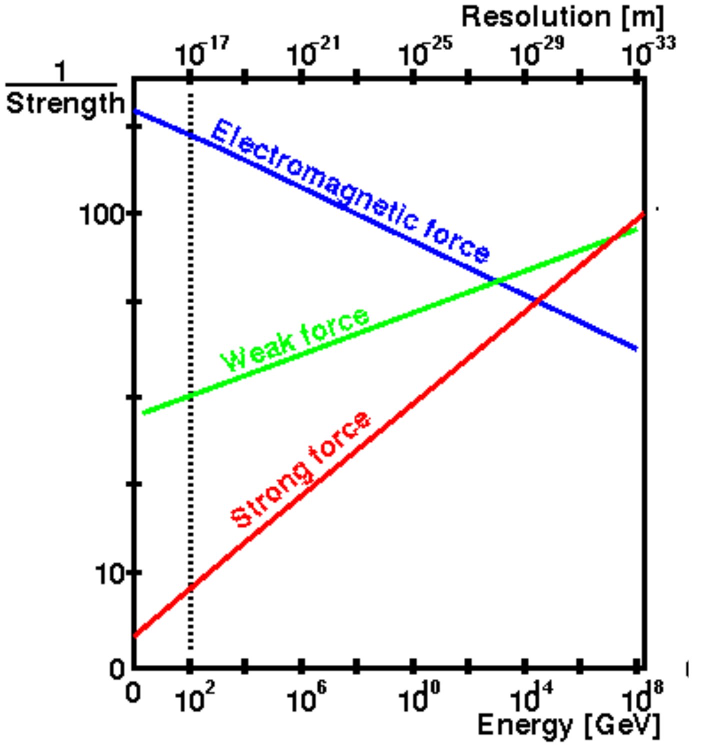
# Nuova Fisica: motivazioni teoriche

**Oltre alla gravità quantistica esistono altre motivazioni teoriche:**

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Le costanti di accoppiamento delle interazioni forti, elettromagnetiche e deboli variano con l'energia e suggeriscono l'unificazione delle tre forze ad una scala ($M_{GUT} \sim 10^{16}$ GeV)

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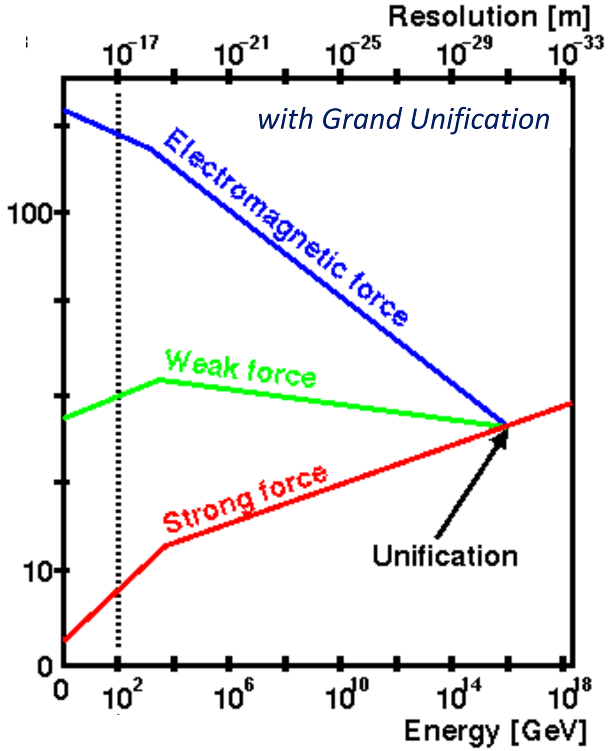


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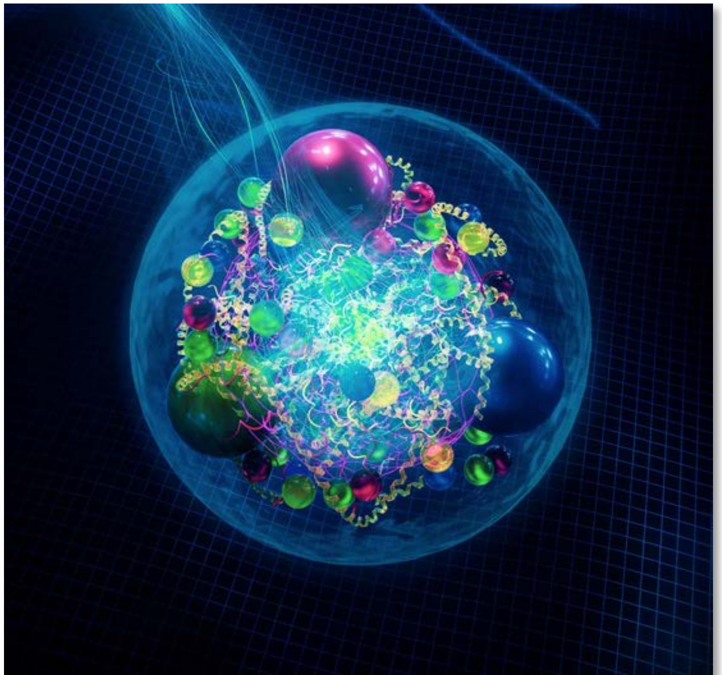

La nuvola virtuale di una particella scalare (s=0) come l'Higgs accresce significativamente la sua massa, fino alla più grande scala della teoria

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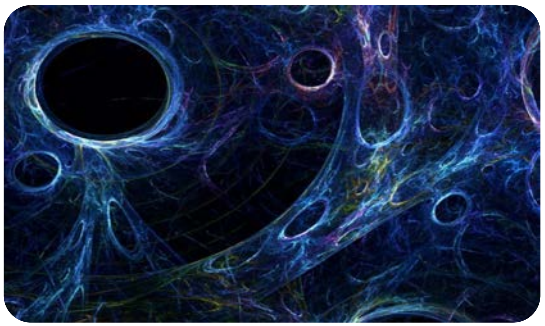
- Perché la massa del bosone di Higgs è così piccola rispetto alla scala di Planck? Problema della «gerarchia»

# Nuova Fisica: evidenze

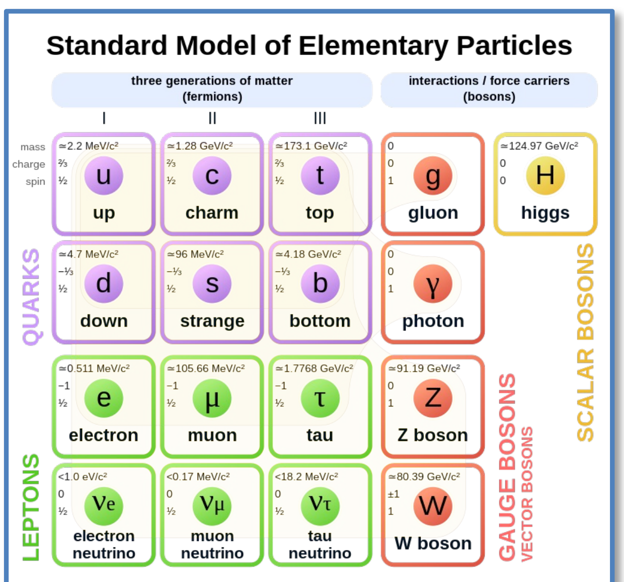
## Oltre alle motivazioni teoriche ci sono anche i “fatti” :



- **La materia oscura**

  È 5 volte più abbondante della materia ordinaria

  Ne abbiamo evidenza dai suoi effetti gravitazionali (lenti gravitazionali, velocità di rotazione)





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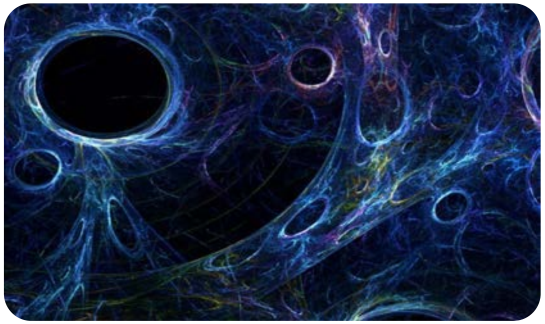

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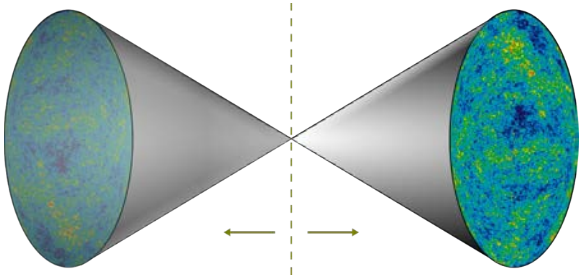

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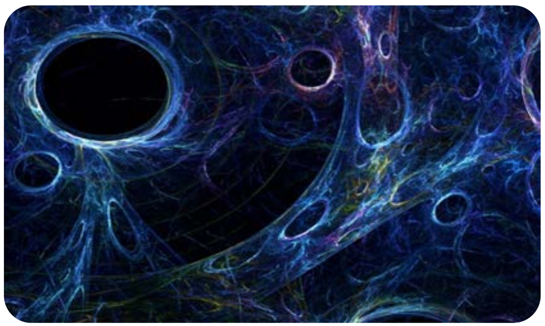

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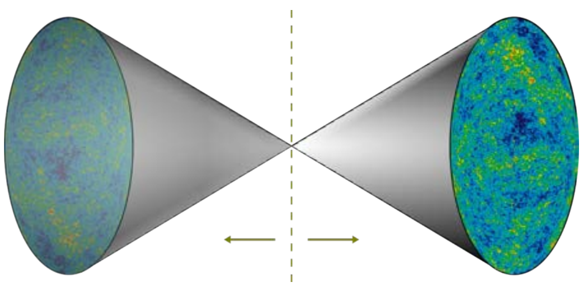

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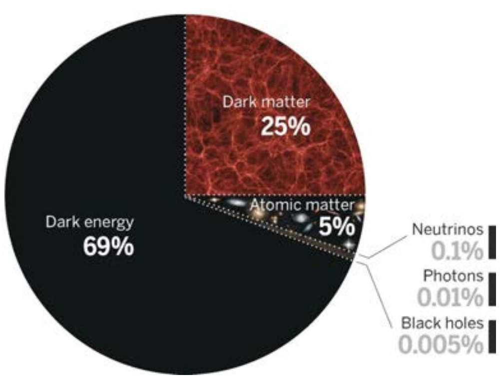



- **L’energia oscura**

  Ne abbiamo evidenza perché l’espansione dell’Universo sta accelerando

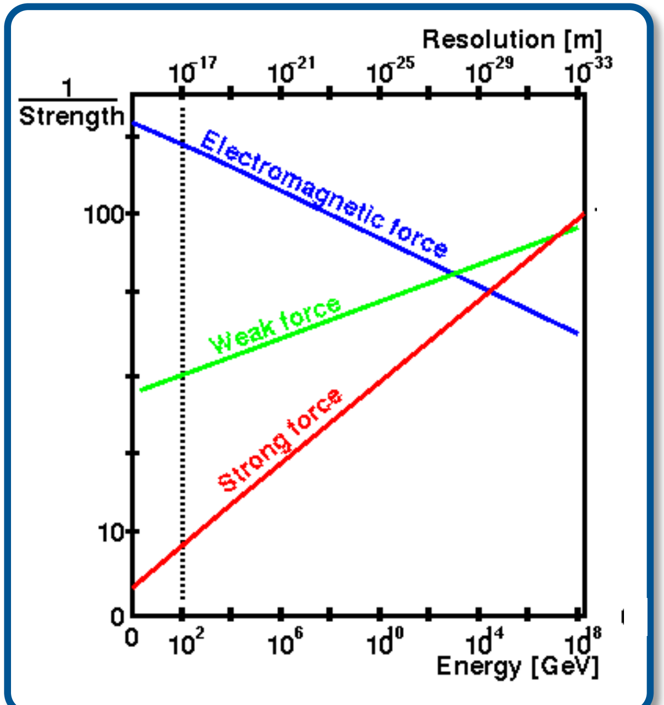
  L’energia del vuoto prevista dal Modello Standard è sbagliata di 120 ordini di grandezza

# Modelli di Nuova Fisica

Anche se siamo convinti che esista **Nuova Fisica oltre il Modello Standard**, la sua forma ci è **ancora ignota**

- Teorie di Grande Unificazione
- Supersimmetria
- Dimensioni extra
- ...

# Teorie di Grande Unificazione




- Ci sono 3 diverse interazioni nel Modello Standard, una per ciascun gruppo di gauge, SU(3), SU(2) e U(1)
- Queste interazioni hanno diversa intensità e dunque diverse costanti di accoppiamento
- Tuttavia le costanti di accoppiamento variano con l'energia e c'è una certa evidenza che potrebbero essere uguali ad una scala di energia $E_{GUT} \sim 10^{16}$ GeV

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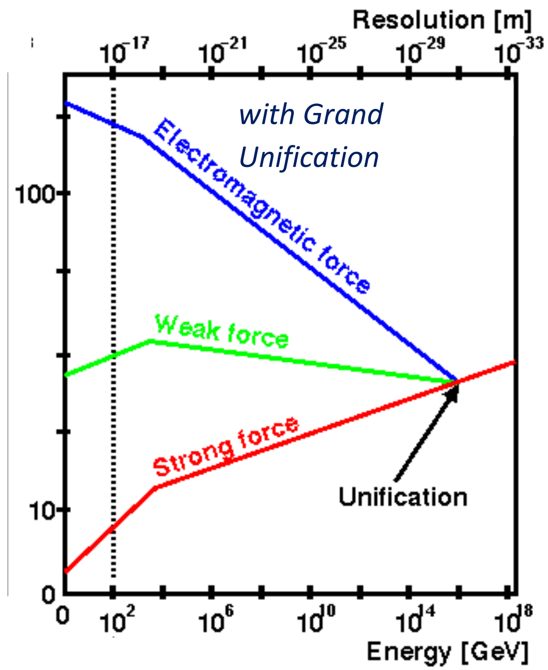


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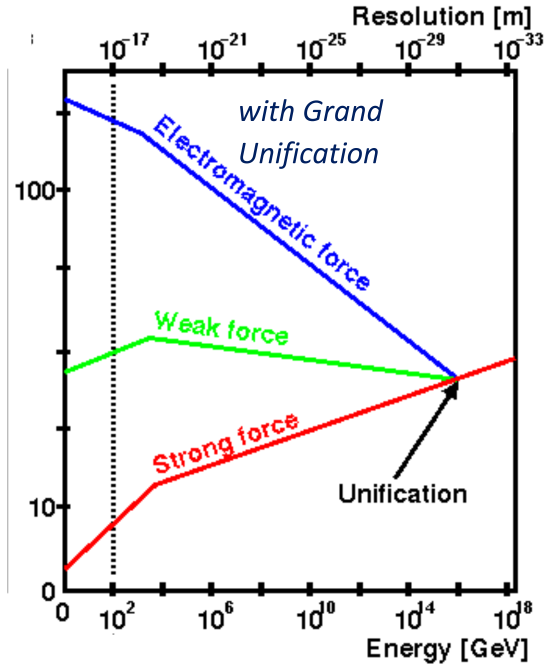
- Se questo accadesse, le 3 forze potrebbero essere 3 diverse manifestazioni di una stessa forza. Una teoria di questo tipo è nota come Teoria di Grande Unificazione (o GUT)
- Il gruppo di gauge del Modello Standard dovrebbe essere un sottogruppo del gruppo di gauge della GUT. Una possibilità realistica sembra essere

$$SO(10) \supset SU(3) \times SU(2) \times U(1)$$

# Teorie di Grande Unificazione

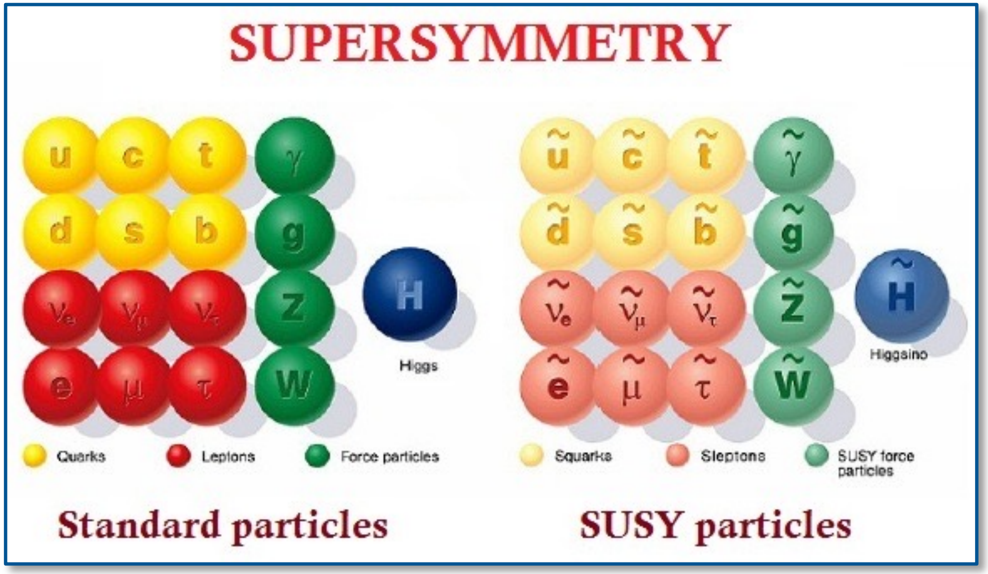



- Nel gruppo di gauge del Modello Standard, SU(3) x SU(2) x U(1) vi sono 8 + 3 + 1 = 12 bosoni di gauge. Una GUT ne prevede molti di più (ad esempio 45 in SO(10))

- Alcuni dei bosoni di gauge della GUT (leptoquark) possono trasformare un quark in un leptone ed indurre il decadimento del protone (per esempio $p \rightarrow e^+ \pi^0$).
  I limiti sperimentali sulla vita media del protone ($10^{34} - 10^{35}$ anni) escludono il gruppo GUT più semplice, SU(5)

- L'unificazione di quark e leptoni in stessi multipletti del gruppo GUT predice la relazione tra carica elettrica dei quark e dell'elettrone (la frazione 1/3 è determinata in particolare dal numero di colori, $N_C$=3)

# Supersimmetria




- È una simmetria che collega le particelle elementari del Modello Standard ad altre particelle che differiscono per mezza unità di spin (dette superpartner)
- Se la superimmetria fosse esatta, partner e superpartner avrebbero la stessa massa.

Ma i superpartner non sono stati mai osservati, dunque la superimmetria deve essere rotta

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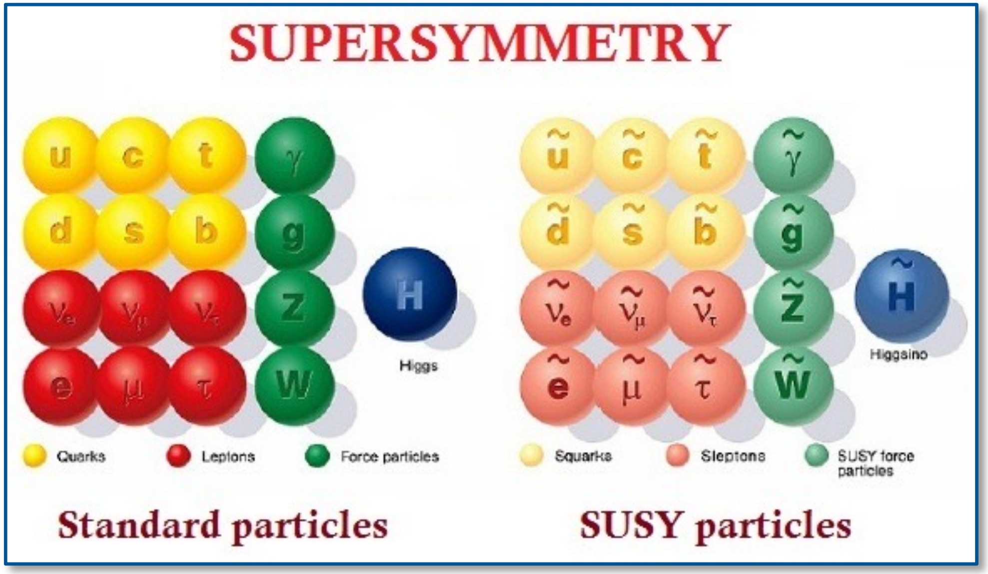


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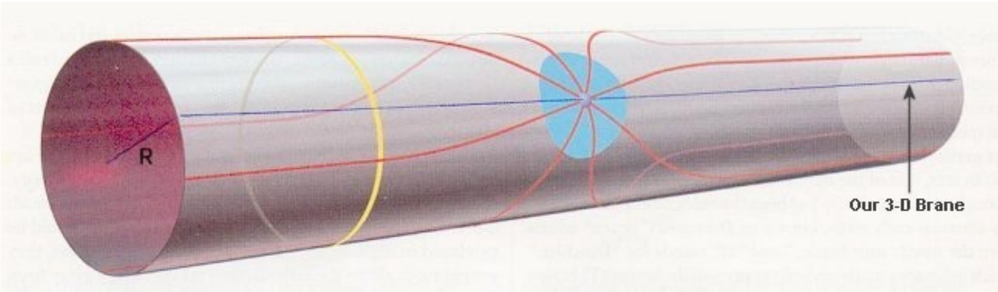
Ma i superpartner non sono stati mai osservati, dunque la superimmetria deve essere rotta

- La presenza dei superpartner risolve il problema della gerarchia
- Il superpartner più leggero può essere un candidato per la materia oscura
- La supersimmetria consente l'unificazione delle costanti di accoppiamento e fornisce dunque un contesto in cui le GUT funzionano bene
- La supersimmetria è richiesta nella teoria delle stringhe

# Dimensioni extra




- La teoria è basata sull' idea che esistano più di 3 dimensioni spaziali e le dimensioni extra siano "compattificate" (raggio R)

- La teoria consente di spiegare l'estrema debolezza della gravità.

La gravità sarebbe la sola interazione che si propaga anche nelle dimensioni extra.

Con 3 dimensioni spaziali, la legge dell'inverso del quadrato si spiega con la superficie $4\pi r^2$ di una sfera di raggio r. Con 3+n dimensioni spaziali, la superficie della sfera è proporzionale a $r^{2+n}$ per distanze $r \ll R$.
Ma a distanze $r \gg R$, la sfera di raggio r viene schiacciata a lunghezza R nelle dimensioni extra, cosicché la superficie corrispondente sarà $\sim R^n r^2$.

La legge di gravitazione ha pertanto la forma:

$$F = G' \frac{m_1 m_2}{r^{2+n}} \quad , \quad \text{per } r \ll R \qquad\qquad F = k\, G' \frac{m_1 m_2}{R^n r^2} \quad , \quad \text{per } r \gg R$$

( $k$ fattore geometrico)

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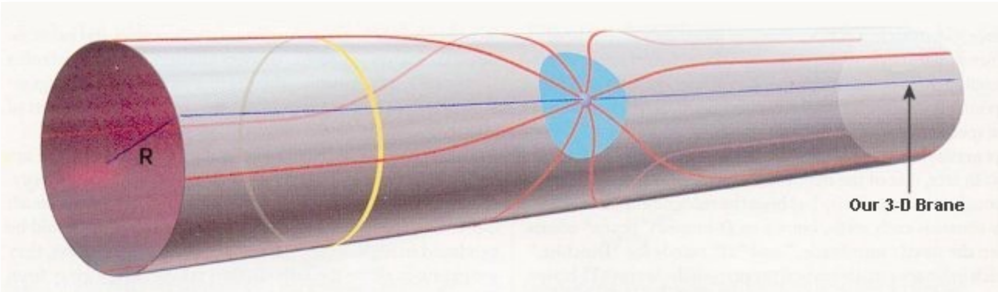


Dal confronto di

$$F = k\,G' \frac{m_1 m_2}{R^n r^2} \quad , \text{ per } r \gg R$$

con la legge di gravitazione in 3 dimensioni spaziali, si trova

$$G = \frac{k\,G'}{R^n}$$

Dunque G' può diventare grande per R e/o n grandi.
Ad esempio, per R~0.1 mm e n=2, la gravità diventa forte (e confrontabile con le altre forze) alla scala del TeV o, equivalentemente a distanze $10^{-19}$ m.

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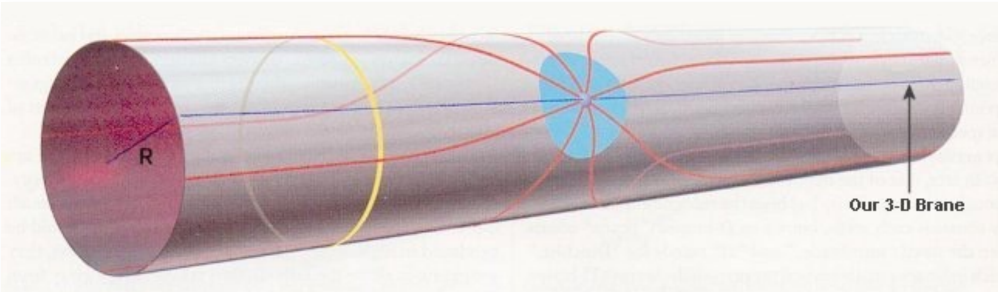


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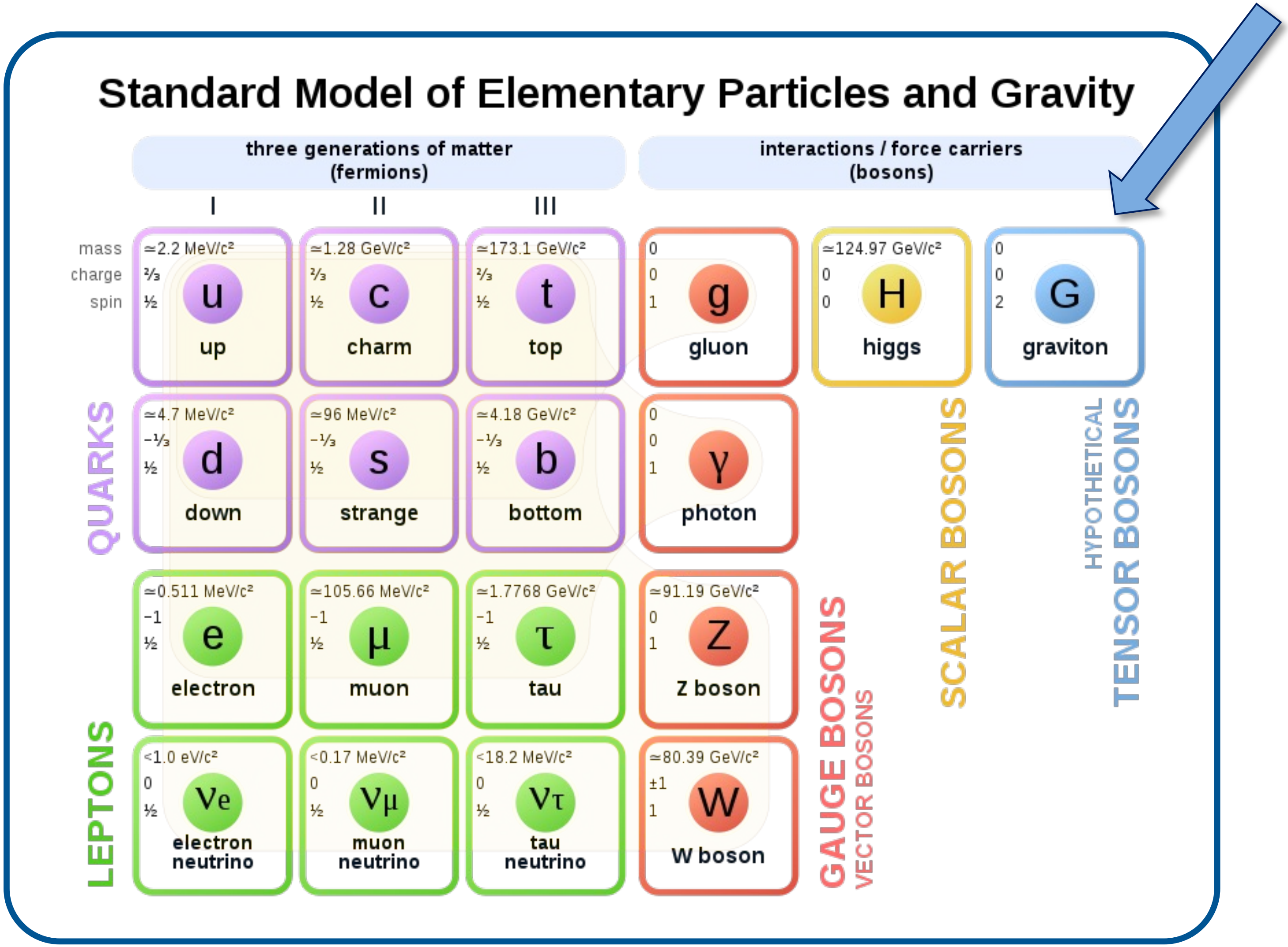
- Un valore di G′ più grande corrisponde ad una scala di Planck più piccola. Viene così risolto il problema della gerarchia

- Nelle 3 dimensioni spaziali che noi percepiamo, le dimensioni extra si manifestano in modo effettivo con la comparsa di nuove particelle (dette di Kaluza-Klein). La più leggera delle particelle di Kaluza-Klein può essere un candidato per la materia oscura

- Dimensioni extra sono richieste nella teoria delle stringhe

Le teorie di Nuova Fisica che abbiamo discusso (e anche altre) risolvono i problemi a cui il Modello Standard non dà risposta. Con una grande eccezione:




La gravità quantistica sarà l'argomento della prossima (ultima) lezione