IAT 2015

INTEGRABLE SYSTEMS AND ALL THAT 2015

. . . waiting for PMNP 2015!

15 June 2015

Program

Last updated: June 10, 2015

Monday 15

9.30 – 10.15	Ferapontov	*Dispersionless integrable systems in 3D and Einstein–Weyl geometry*
10.15 – 11.00	Winternitz	*Symmetry preserving discretization of* $SL_x(2) \times SL_y(2)$ *invariant ODEs*
11.00 – 11.30	*Coffee Break*	
11.30 – 12.15	Kodama	*Confluence of hypergeometric functions and hydrodynamic systems*
12.15 – 13.00	Prinari	*Dark-bright soliton solutions with nontrivial polarization interactions for the three-component defocusing nonlinear Schrodinger equation*

15.00 – 15.45	Bogdanov	*Six-dimensional heavenly equation and similar systems*
15.45 – 16.30	Pavlov	*Three-dimensional quasilinear integrable equations of second order and Hamiltonian hydrodynamic chains*
16.30 – 17.00	*Coffee Break*	
17.00 – 17.45	Martina	*Symmetry Properties of the discretization procedures of the Liouville equation*
17.45 – 18.15	Vitolo	*On the projective geometry of Hamiltonian operators of differential-geometric type*

Abstracts [1]

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Dispersionless integrable systems in 3D and Einstein–Weyl geometry

E.V. Ferapontov
Department of Mathematical Sciences, loughborough University, UK

Talk Summary: For several classes of second-order dispersionless PDEs, we show that the symbols of their formal linearizations define conformal structures which must be Einstein-Weyl in 3D (or self-dual in 4D) if and only if the PDE is integrable by the method of hydrodynamic reductions. This demonstrates that the integrability of dispersionless PDEs can be seen from the geometry of their formal linearizations.
Reference: E. V. Ferapontov and B. Kruglikov, Dispersionless integrable systems in 3D and Einstein-Weyl geometry, J. Diff. Geom. **97** (2014) 215-254.

Symmetry preserving discretization of $SL_x(2) \times SL_y(2)$ invariant ODEs

P. Winternitz
Centre de recherches mathematiques, Universite de Montreal, Canada

Talk Summary: Ordinary differential equations (ODE) and ordinary difference systems ($O\Delta S$) invariant under the actions of the Lie groups $SL_x(2)$, $SL_y(2)$ and $SL_x(2) \times SL_y(2)$ of projective transformations of the independent variables x and dependent variables y are constructed. The ODE are continuous limits of the $O\Delta S$, or conversely, the $O\Delta S$ are invariant discretizations of the ODE. The invariant $O\Delta S$ are used to calculate numerical solutions of the invariant ODE of order up to five. The solutions of the invariant numerical schemes are compared to numerical solutions obtained by standard Runge-Kutta methods and to exact solutions, when available. The invariant method performs at least as well as standard ones and much better in the vicinity of singularities of solutions.
Joint work with R. Campoamor-Stursberg and M.A. Rodriguez.

Confluence of hypergeometric functions and hydrodynamic systems

Y. Kodama
Ohio State University, USA

[1] TBA = to be announced

Talk Summary: We start with a brief introduction of the Gelfand hypergeometric (GH) functions and their confluences by means of the action of the centralizers of regular elements. The confluence then implies that the GH functions are now defined in a degenerate cell of the Grassmannian. We construct hydrodynamic systems defined on such cells. This is a joint work with Boris Konopelchenko.

Dark-bright soliton solutions with nontrivial polarization interactions for the three-component defocusing nonlinear Schrodinger equation

B. Prinari
Università del Salento, Lecce, Italy

Talk Summary: In this talk we will present novel dark-bright soliton solutions for the three-component defocusing nonlinear Schrodinger equation with nonzero boundary conditions. The solutions are obtained within the framework of a recently developed inverse scattering transform for the underlying nonlinear integrable PDE, and unlike dark-bright solitons in the two component (Manakov) system in the same dispersion regime, their interactions display non-trivial polarization shift for the two bright components.

Six-dimensional heavenly equation and similar systems

L.V. Bogdanov
Landau Institute for Theoretical Physics of the Russian Academy of Sciences, Moscow, Russia

Talk Summary: Dressing method and the hierarchy for the six-dimensional heavenly equation are constructed through the reduction of general six-dimensional case. The immersion of six-dimensional heavenly equation to the four-dimensional second heavenly equation hierarchy is discussed. Similar system in the context of universal hierarchy (Martinez Alonso - Shabat) are also considered.

Three-dimensional quasilinear integrable equations of second order and Hamiltonian hydrodynamic chains

M.V. Pavlov
Lebedev Institute of Theoretical Physics, Moscow

Talk Summary: We consider hydrodynamic chains associated with Gelfand–Dorfman Poisson brackets.
We construct three-dimensional quasilinear equations of second order connected with these hydrodynamic chains

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Symmetry Properties of the discretization procedures of the Liouville equation

L. Martina
Università del Salento, Lecce, Italy

Talk Summary: The Liouville equation is well known to be linearizable by a point transformation. It has an infinite dimensional Lie point symmetry algebra isomorphic to a direct sum of two Virasoro algebras. We show that it is not possible to discretize the equation keeping the entire symmetry algebra as point symmetries. We do however construct a difference system approximating the Liouville equation that is invariant under the maximal finite subalgebra $SL_x(2) \times SL_y(2)$. The invariant scheme is an explicit one and provides a much better approximation of exact solutions than comparable standard (non invariant) schemes. Comparisons with the generalized symmetry invariant scheme and integrable discrete scheme are discussed.

Joint work with D. Levi and P. WInternitz.

Reference: D. Levi *et al.* , *Lie-point symmetries of the discrete Liouville equation*, J. Phys A: Math. Theor. **48**, 2 (2015) 025204, 2 (2010), pp. 123–456.

On the Hamiltonian structure of systems of conservation laws

R.F. Vitolo
Università del Salento, Lecce, Italy

Talk Summary: The theory of quasilinear equations of first order is one of the most developed parts in integrable systems. However their Hamiltonian formulation is an open question in general. B.A. Dubrovin and S.P. Novikov introduced the concept of homogeneous differential-geometric Poisson brackets in 1983-1984. A subclass of the hydrodynamic type systems can be equipped by first order homogeneous Hamiltonian operators. However, some hydrodynamic type systems can be equipped by third order homogeneous Hamiltonian operators.
In this talk we find a new criterion which allows us to effectively reconstruct such a Hamiltonian operator if the corresponding hydrodynamic type system is written as a system of conservation laws. Conversely, the criterion can be used to describe all possible hydrodynamic type systems for each given third order homogeneous Hamiltonian operator. We solve this problem using our classification of 3-component homogeneous Hamiltonian operators.

Joint work with M.V. Pavlov and E.V. Ferapontov.