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Book of Abstracts

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Day/Time	Monday May 25	Tuesday May 26	Wednesday May 27
08:30 - 09:00	Opening Remembrance of A. Verganelakis		University of Cagliari Vice-Chancellor
09:00 - 09:30	F. Calogero	R. Halburd	A.P. Veselov
09:30 - 10:00	N. Atakishiyev	A. Dzhamay	O. Chalykh
10:00 - 10:30	M. Semenov-Tian-Shansky	P. Winternitz	A. Kasman
10:30 - 11:00	Coffee Break	Coffee Break	Coffee Break
11:00 - 11:30	J. Sanders	Y. Zarmi	B.A. Dubrovin
11:30 - 12:00	A.V. Mikhailov	S. Leble	F. Leyvraz
12:00 - 12:30	M. Gekhtman	V. Hussin	O. Bihun
12:30 - 13:00	L. Martina	T. Ioannidou	G. Tondo
13:00 - 13:30	Lunch	Lunch	V.E. Zakharov
13:30 - 14:00			Lunch
14:00 - 14:30			
14:30 - 15:00			
15:00 - 15:15	**MW1** D. Gomez-Ullate (*Chair*)	**MW2** N. Joshi (*Chair*)	S. Carillo
15:15 - 15:30			
15:30 - 15:45		M. Mazzocco (*Chair*)	O. Chvartatskyi
15:45 - 16:00			
16:00 - 16:15	Y. Grandati		S. Rauch-Wojciechowski
16:15 - 16:30			
16:30 - 17:00	Coffee Break	Coffee Break	Coffee Break
17:00 - 17:15	A. Kuijlaars	O. Lisovyy	**Poster**
17:15 - 17:30			
17:30 - 17:45	S. Tsujimoto		
17:45 - 18:00		Y. Yamada	
18:00 - 18:15	R. Milson		
18:15 - 18:30			
18:30 - 19:00			
	Dinner	Dinner	Gala Dinner & Sardinian Night

Day/Time	Thursday May 28	Friday May 29	Saturday May 30
08:30 - 09:00			
09:00 - 09:30	C. Rogers	P.M. Santini	G. Falqui
09:30 - 10:00	S. Kamvissis	V.B. Matveev	M. Pedroni
10:00 - 10:30	V. Caudrelier	M. Onorato	B. Huard
10:30 - 11:00	Coffee Break	Coffee Break	Coffee Break
11:00 - 11:30	A.K. Pogrebkov	J. Åkerman	A. Bilge
11:30 - 12:00	C. Schiebold	A.N. Slavin	R. Hernandez Heredero
12:00 - 12:30	R. Buckingham	M. Conforti	M.L. Gandarias
12:30 - 13:00	L. Fermo	L. Kovachev	C. Scimiterna
13:00 - 13:30	Lunch	Lunch	M. Petrera
13:30 - 14:00			Lunch
14:00 - 14:30			
14:30 - 15:00			
15:00 - 15:15	**MW3** G. Biondini (*Chair*)	**MW4** (*Chair*: M. Onorato & S. Lombardo) A. Degasperis	Free afternoon (Excursion)
15:15 - 15:30			
15:30 - 15:45			
15:45 - 16:00	P.A. Miller		
16:00 - 16:15		A. Chabchoub	
16:15 - 16:30			
16:30 - 17:00	Coffee Break	Coffee Break	
17:00 - 17:15	P.A. Perry	T. Horikis	
17:15 - 17:30			
17:30 - 17:45		F. Baronio	
17:45 - 18:00	T. Aktosun		
18:00 - 18:15		D.A. Georgieva	
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18:30 - 19:00			
	Dinner	Dinner	Dinner

Contents

1 Abstracts of Contributions

Magnetic droplets and dynamical skyrmions

J. Åkerman[a]

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Nanocontact spin-torque oscillators (NC-STOs) provide an excellent environment for studying nano-magnetic phenomena such as localized and propagating auto-oscillatory spin wave (SW) modes [1]. The recent experimental observation of magnetic droplet solitons in NC-STOs with perpendicular magnetic anisotropy (PMA) free layers [2], and the numerical [3] and experimental [4] demonstrations of spin transfer torque (STT) nucleated skyrmions in similar magnetic thin films add two interesting and useful nanoscale magnetic objects. Due to the competition between exchange, anisotropy, and, in the case of skyrmions, the Dzyaloshinskii-Moriya interaction (DMI), the droplet and the skyrmion are extremely compact, on the order of 10-100 nm. One of the main differences between a magnetic dissipative droplet soliton and a skyrmion is that the former is a dynamical object with all its spins precessing around an effective field and stabilized by STT, exchange, and PMA, while the latter has static spins and an internal structure stabilized by DMI, exchange, and PMA. The dissipative droplet is furthermore a non-topological soliton, while the skyrmion is topologically protected. In this work we report on our most recent droplet experiments, including droplet collapse at very high fields, droplets excited in nano-wire based NC-STOs, and studies of the field-current droplet nucleation boundary. We also demonstrate numerically and analytically that STT driven precession can stabilize so-called dynamical skyrmions even in the absence of DMI, and we describe their very promising properties in detail. From a more fundamental perspective, precession is hence a third independent possibility to stabilize a skyrmion, without the need for the conventional stabilization from either dipolar energy or DMI [5].

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Non-local and Local Nonlinear Shrödinger Equation from Geometric Curve Flows in some low dimensional Hermitian symmetric space $Sp(2)/U(2)$ and $SU(3)/U(3)$

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In recent work of [1], several nonlocal generalization of the nonlinear Shrödinger equation have been obtained together with their Lax pair, bi-Hamiltonian operators which are $U(1)$- invariant. These nonlocal Schrödiner generalization are derived from a geometrical flows utilized by a geometrical parallel moving frame on the flows in the Riemannian symmetric space $SO(2n)/U(n)$. In this set up, which is adaption of a more general result [3], the Cartan structure equation encode bi-Hamiltonian structure and Lax pair for the nonlinear nonlocal Schrödineger.

In the Hermitian symmetric spaces $SP(2)/U(2)$ and $SU(3)/U(3)$ there are a natural complex structure compatible with the Riemannian metric. The frame bundle of these spaces have subgroup $U(2)$ and $U(3)$ as a gauge group respectively. For arclength parametrized curves in this geometry, there is a natural parallel frame whose equivalence group is $U(1)$. The components of Cartan connection matrix of this frame, known as Hasimoto variables, yield a real-valued differential covariant of the curve in addition to a complex valued differential covariant. The resulting generalization of the NLS equation are $U(1)-$ invariant integrable systems in which a real variable is coupled to a complex scaler variable. We use the Hermitian structure to complexify the Hasimoto real varianble in a natural way.

The main result is utilizing two representation of singular elements in a Cartan subspaces of $\mathfrak{sp}(2)/\mathfrak{u}(2)$ and one singular element in $\mathfrak{su}(3)/\mathfrak{u}(3)$ generating center respectively in the gauge Lie subalgebras $\mathfrak{u}(2)$ and $\mathfrak{u}(3)$ of maximal dimension. The parallel frame araising in this manner in the first case yields two different new generalized non-local nonlinear Schroödinger equations for a real variable u and complex Hasimoto like variable $\mathbf{u}$ in the first case and local nonlinear Schroödinger equations in the second case. The nonlocal one is given as

$$\mathrm{u}_t = \mathrm{u}|\mathbf{u}|^2 + Re(\bar{\mathbf{u}}_x D_x^{-1}(\mathrm{u}\mathbf{u})) \tag{1}$$

$$\mathbf{u}_t = \mathrm{i}\big(\frac{1}{4}\mathbf{u}_{xx} + \frac{1}{2}\mathbf{u}|\mathbf{u}|^2 + +\mathrm{u}^2\mathbf{u} + \mathrm{u}_x D_x^{-1}(\mathrm{u}\mathbf{u}) + 2\mathbf{u}|D_x^{-1}(\mathrm{u}\mathbf{u})|^2\big) \tag{2}$$

We also give the flow equations which is variant of Schrödinger map equation and is invariant under isometry group.

These result may also viewed as a generalization as well as a geometric interpretation of AKNS method [4] on Hermitian symmetric spaces in which they only have found local generalization of Schrödinger equation, while by construction we obtain a nonlocal one in the first case.

Toward the classification of all integrable hierarchies like generalized Sine-Gordon, local and nonlocal NLS and mKdV system by studying the geometric curve flows in all irreducible symmetric spaces, which are classified by Satake diagram, one can illustrate the whole picture and classification, by attaching to a specific vertex of each Satake diagaram all these integrable systems in each case. The present work is an example of attempt to build this correspondence in general case. The idea goes back to the different approaching the classification of KdV type equations corresponding to an arbitrary simple Lie algebras of Drinfeld and Sokolov [5] , which is actually is using normalized Lax pair in the context of Kac-Moody algebra associated to the simple Lie algebra under consideration

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On a discrete number operator and its eigenvectors associated with the 5D discrete Fourier transform

M.K. Atakishiyeva, **N.M. Atakishiyev**[a], and J. Méndez Franco

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We construct an explicit form of a difference analogue of the quantum number operator in terms of the raising and lowering operators that govern eigenvectors of the 5D discrete (finite) Fourier transform. Eigenvalues of this difference operator are represented by distinct nonnegative numbers so that it can be used to sistematically classify, in complete analogy with the case of the continuous classical Fourier transform, eigenvectors of the 5D discrete Fourier transform, thus resolving the ambiguity caused by the well-known degeneracy of the eigenvalues of the discrete Fourier transform.

Optical solitons of NLSM Systems in $\mathcal{PT}$-Symmetric lattice with a vacancy defect

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In many applications the leading nonlinear polarization effect in optical materials are quadratic; they are referred to as $\chi^{(2)}$ materials. The pulse dynamics in multidimensional nonresonant $\chi^{(2)}$ materials can be described by generalized nonlinear Schrödinger (NLS) equation with coupling to a mean term (hereafter denoted as NLSM Systems) [1].

Recently, the regions of collapse and collapse dynamics in the NLSM systems have been investigated [2, 3]. Also, it was pointed out that NLSM collapse can be arrested by small nonlinear saturation [4].

Another way of arresting wave collapse is adding an external potential (lattice) to the governing equation. The purpose of this study is to investigate soliton properties in NLSM Systems with Parity-Time ($\mathcal{PT}$) Symmetric external potentials (which include a vacancy defect or not). The model is given by

$$iu_z + \frac{1}{2}\Delta u + |u|^2 u - \rho\phi u - V(x,y)u = 0, \quad \phi_{xx} + \nu\phi_{yy} = (|u|^2)_{xx}. \tag{1}$$

where $u(x,y,z)$ is the normalized amplitude of the envelope of the electric field (which associated with the first-harmonic), $V(x,y)$ is external potential, $\phi(x,y,z)$ is the normalized static field, ρ is a coupling constant, and ν is the coefficient that comes from the anisotropy of the material [1, 3].

In this study, we consider a $\mathcal{PT}$-symmetric lattice with a vacancy defect as external potential (which should satisfy $V(x,y) = V^*(-x,-y)$ [5]),

$$V(x,y) = \frac{V_0}{25}\left|2cos(k_x x) + 2cos(k_y y) + e^{i\theta(x,y)}\right|^2 + iV_0W_0[sin(2x) + sin(2y)]$$

where $V_0 > 0$ is the peak depth of the potential, $\vec{k} = (k_x, k_y)$ is a wave vector, W_0 is the relative magnitude of the imaginary component and, $\theta(x,y)$ is a phase function that is given by $\theta(x,y) = tan^{-1}(\frac{y-y_0}{x}) - tan^{-1}(\frac{y+y_0}{x})$.

Physically, $\theta(x,y)$ corresponds to two first order phase dislocations displaced in the y direction by a distance of $2y_0$. A vacancy defect can thus be obtained using $y_0 = \pi/K$ where $K = k_x = k_y$ [6]. By setting $\theta(x,y) = 0$, the vacancy defect in the $\mathcal{PT}$-symmetric lattice is removed and the periodic lattice counterpart of this lattice obtained.

Using a modification of spectral renormalization method [7], we numerically find the fundamental and dipole solitons in a $\mathcal{PT}$-Symmetric lattice with a vacancy defect. The linear and nonlinear (in)stabilities are also examined for these localized structures by direct computations of the NLSM System and its linearized equation.

The results of stability analysis show that the fundamental and dipole solitons in a lattice with a vacancy defect can be nonlinearly stable under suitable conditions (in the absence of $\mathcal{PT}$-Symmetry), but none of fundamental and dipole solitons in $\mathcal{PT}$-Symmetric lattices (with or without a vacancy defect) are found stable.

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Discrete integrable equations from the Bäcklund transformations of the self-dual Yang-Mills equations

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It is known that many continuous integrable systems may be obtained via reduction of the self-dual Yang-Mills (SDYM) equation. In this talk we show how, by use of a Darboux matrix with affine dependence on the spectral parameter, one can construct a general class of Bäcklund-Darboux transformations for the SDYM equations. We find that Pohlmeyer's form of the SDYM equation is the natural setting for such construction with the resulting Bäcklund transformation (BT) having a very symmetric form and depending on two matrices which, among other things, are responsible for the injection of the Bäcklund parameters in the system. One then recovers the BTs of the various reduced equations by reduction of this system.

The Bäcklund transformations are then used to construct, via Bianchi permutability, a very rich discrete integrable system which can be shown to have reductions to many important discrete equations. The richness of the discrete reductions arises from the choice of the Yang matrix and the two matrices containing the Bäcklund parameters. The latter play a fundamental role in the whole construction.

Integrable systems related to the tetrahedral reduction group

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In this talk we will present integrable systems related to the tetrahedral reduction group. Our starting point will be a 3×3 Lax operator with tetrahedral symmetry. One can find a detailed account regarding the theory of Reduction groups in [1]. By considering Darboux transformations of this operator we will arrive at systems of differential-difference equations as well as fully discrete systems. By construction these systems will possess Lax representations and so are integrable. We shall see that the differential-difference equations will constitute non-local symmetries of the fully discrete systems, as well as Bäcklund transformations of a related continuous system. Local symmetries for the discrete systems will also be derived. Lastly, various reductions, potentiations and Miura transformations of the found systems will be presented.

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Delay-differential equations of Painlevé

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Delay-differential equations are differential equations in which the argument of the unknown function may be shifted by discrete values. Reductions of integrable lattice equations have furnished nonlinear examples of such equations of Painlevé type. These equations have continuum limits to differential Painlevé equations and possess special properties associated with integrable purely differential or discrete equations.

The six nonlinear Painlevé equations were originally identified based on their simple singularity structure. Later they were found to possess many other special properties including representation as isomonodromy deformation problems and special (algebraic, rational, and special function) solutions for certain choices of parameters. The philosophy behind the methods used to isolate the Painlevé equations (singularity analysis) has more recently been applied to discrete equations. Here the singularity confinement criterion was used to identify discrete nonlinear equations and place them in correspondence with differential Painlevé equations. Other methods of detecting integrable equations include algebraic entropy-based approaches and Nevanlinna theory.

There are natural analogues of integrability-detection methods for discrete equations that can be applied to the study of delay-differential equations. We discuss these methods and give examples of the equations they isolate. For these integrable equations, we discuss the relationship with differential and discrete Painlevé equations as well as special solutions.

Properties of the Zeros of the Generalized Hypergeometric, Askey and q-Askey Scheme Polynomials

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The properties of the zeros of polynomials are a core problem of mathematics to which, over time, an immense number of investigations have been devoted. Nevertheless, new findings in this area continue to emerge, e.g. [1, 2, 3]. We extend a known method for construction of solvable N-body problems to discover new and remarkable properties of several important polynomials [4]. Generalized hypergeometric, Wilson and Racah (top of the Askey scheme) polynomials are defined in terms of the *generalized hypergeometric function*

$$ {}_{r+1}F_s(\alpha_0, \alpha_1, \ldots, \alpha_r; \beta_1, \ldots, \beta_s; z) = \sum_{j=0}^{\infty} \frac{(\alpha_0)_j (\alpha_1)_j \cdots (\alpha_r)_j}{j!(\beta_1)_j \ldots (\beta_s)_j} z^j, $$

where the Pochhammer symbol $(\alpha)_j = \alpha(\alpha+1)\cdots(\alpha+j-1)$ with $(\alpha)_0 = 1$. Because the quantities playing the role of the arguments are defined quite differently for these polynomials, they are not special cases of each other. The Askey-Wilson and q-Racah (top of the q-Askey scheme) polynomials are defined in terms of the *generalized basic hypergeometric function*

$$ \begin{aligned} &{}_{r+1}\phi_s(a_0, a_1, \ldots, a_r; b_1, \ldots, b_s; q; z) \\ &= \sum_{j=0}^{\infty} \frac{(a_0;q)_j (a_1;q)_j \cdots (a_r;q)_j (-1)^{(s-r)j} q^{(s-r)j(j-1)/2}}{(q;q)_j (b_1;q)_j \ldots (b_s;q)_j} z^j, \end{aligned} $$

where the q-Pochhammer symbol $(c;q)_j = (1-c)(1-cq)\cdots(1-cq^{j-1})$ with $(c;q)_0 = 1$.

We identify new and remarkable *nonlinear algebraic relations* satisfied by the zeros of the generalized hypergeometric, Wilson, Racah, Askey-Wilson and q-Racah polynomials. We express these zeros as the *equilibria* of certain solvable many-body problems – solvable by the virtue of their equivalent formulation as *linear* partial, differential difference or differential q-difference equations satisfied by polynomials with time-dependent coefficients. This method is an extension of the technique pioneered by Stieltjes and Szëgo. By linearizing the many-body problems in the vicinity of their equilibria, we obtain interesting matrices defined in terms of the zeros of these polynomials [5, 6, 7, 8]. These are *isospectral matrices* because their eigenvalues are given by neat expressions independent of many of the parameters of the polynomials. These eigenvalues are integer or rational – a *Diophantine property* – provided that certain parameters of the polynomials are integer or rational. Of course, these findings generate multiple theorems on the properties of the zeros of those polynomials that are special cases of the generalized hypergeometric polynomials or belong to the "lower levels" of the Askey or the q-Askey scheme. It is interesting to note that the orthogonality properties of the polynomials play no role in the proofs.

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Integrable Equations of the Sawada-Kotera and Kaup Type: A New Hierarchy Starting with an Essentially Nonlinear 5th Order Equation

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In a series of papers we have studied the integrability of scalar evolution equations in one space dimension, that are of the form $u_t = F$, using the "formal symmetry" method of [1], that is based on the existence of a formal Laurent series R, in inverse powers of the derivative operator D, satisfying the operator equation $R_t + [R, F_*] = 0$, where F_* is the Frechet derivative of F. The existence of such a formal series leads to an infinite sequence of conservation laws for the so called "canonical densities", denoted here as ρ_i, $i = -1, 0, 1, \ldots$. In particular, for any mth order evolution equation, the quantity $\rho_{(-1)} = \left(\frac{\partial F}{\partial u_m}\right)^{-1/m}$ is conserved.

In [2] we have obtained the explicit expression of the canonical densities $\rho_{(i)}$, $i = 1, 2, 3$ for arbitrary m. Using these, we have shown that integrable equations of order greater than or equal to 7, are quasi-linear. There is however is a non quasi-linear candidate of integrable equation of order 5.

Later on in [3] we have shown that if the canonical densities $\rho_{(i)}$, $i = 1, 2, 3$ are non-trivial, then any evolution equation of order $m \geq 7$ is polynomial in top three derivatives. In [4], we have studied quasi-linear integrable fifth order equations $u_t = Au_5 + B$, where A and B are independent of u_5. Using the non-triviality of the canonical densities $\rho_{(i)}$, $i = 1, 2, 3$ we showed that these equations are polynomial in $a = A^{1/5}$ (hence in the inverse of $\rho_{(-1)}$) and a has the form $(\alpha u_3^2 + \beta u_3 + \gamma)^{-1/2}$. The u_2 dependency of a is obtained in terms of $P = 4\alpha\gamma - \beta^2 > 0$ and we obtained an explicit quasi-linear but non-polynomial fifth order equation.

In the present work we study those evolution equations for which the canonical density $\rho_{(3)}$ is trivial. We call such equations as "Sawada-Kotera and Kaup type" equations because for these two hierarchies the conserved densities of orders multiples of 3 are trivial.

For generic quasi-linear equations of order m, $u_t = a^2 u_m + B$, there is a dichotomy characterized by the form of a; for non-trivial $\rho_{(3)}$ $a = (\alpha u_3^2 + \beta u_3 + \gamma)^{-1/2}$ while for trivial $\rho_{(3)}$, $a = (\mu u_3 + \nu)^{-1/3}$. We have obtained a preliminary classification of integrable equations up to order $m = 17$ up to their top "top level" [3], our results suggesting that they belong to certain hierarchies. The "essentially non-linear third order equation" of [1] has the form $u_t = F = (\alpha u_3^2 + \beta u_3 + \gamma)^{-1/2}(2\alpha u_3 + \beta) + \eta$. Computing the partial derivative of F with respect to u_3 one can see that $\rho_{(-1)} = (\alpha u_3^2 + \beta u_3 + \gamma)^{1/2}(-1/2\beta 2 + 2\alpha\gamma)^{-1/3}$, hence higher order generic equations with non-trivial $\rho_{(3)}$, (that we call "KdV-like"), belong possibly to a hierarchy starting at the essentially nonlinear third order equation.

We have shown that the non quasi-linear equation obtained in [2] is characterized by the triviality of $\rho_{(3)}$. This equation is of the form $u_t = -\frac{3}{2A}(Au_5 + B)^{-2/3} + C$, where the functions A, B and C are independent of u_5, and A and C are independent of u_4. It is easy to see that, for this equation the canonical density $\rho_{(-1)}$ is $(Au_3 + B)^{1/3}$ and this equation is interpreted as an essentially nonlinear starting symmetry of a hierarchy with trivial conserved densities at orders that are multiples of 3.

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An integrable Hénon–Heiles system on the sphere and the hyperboloid

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The integrable Hénon–Heiles systems can be written as particular cases of the multiparametric family of two-dimensional Hamiltonian systems given by [1]

$$\mathcal{H} = \frac{1}{2}(p_1^2 + p_2^2) + \Omega_1\, q_1^2 + \Omega_2\, q_2^2 + \alpha\left(q_1^2 q_2 + \beta\, q_2^3\right),$$

where Ω_1, Ω_2, α and β are real constants.

In particular, the only known Liouville integrable cases are [2, 3]:

- The Sawada–Kotera Hamiltonian ($\beta = 1/3$, $\Omega_1 = \Omega_2$).
- The Korteweg–de Vries (KdV) Hamiltonian ($\beta = 2$).
- The Kaup–Kupershmidt Hamiltonian ($\beta = 16/3, \Omega_2 = 16\,\Omega_1$).

In this contribution we present a constant curvature analogue $\mathcal{H}_\kappa$ of the integrable Euclidean KdV Hénon–Heiles Hamiltonian and its invariant $\mathcal{I}_\kappa$, whose construction hinges on the appropriate formulation for the curved version [4] of the Ramani homogeneous potentials [5]. The starting point is the new curved anisotropic oscillator proposed in [6], for which the special tuning in the frequencies ($\Omega_2 = 4\Omega_1$) coincides with the well known superintegrable curved $1:2$ oscillator system obtained in [1].

Our approach is based on the use of the constant Gaussian curvature of the underlying spaces as an explicit deformation parameter κ [6, 1, 8], thus connecting the Euclidean and the curved systems (on the sphere and hyperbolic spaces) in a smooth way. This allows us to present the integrable curved KdV Hénon–Heiles Hamiltonian in a geometric unified approach, so covering, simultaneously, the Euclidean, spherical and hyperbolic cases. Hence, the Euclidean system is obtained as the flat limit $\kappa \to 0$ performed over the new integrable curved system $\mathcal{H}_\kappa$ and its invariant $\mathcal{I}_\kappa$.

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Long-time transition asymptotics for the Camassa-Holm equation with non-decaying initial data

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The Camassa-Holm equation

$$u_t - u_{txx} + 2\omega u_x + 3uu_x = 2u_x u_{xx} + uu_{xxx}, \quad x \in \mathbb{R},\, t \geq 0,\, \omega > 0$$

is a completely integrable model for dispersive shallow water waves. For initial data that decays sufficiently fast as $x \to \pm\infty$, it is known [3] that in the long-time limit the solution exhibits qualitatively different behavior in each of four regions: a solitonic sector, two sectors of slowly decaying modulated oscillations, and a sector of rapid decay. The behavior of the solution in the solitonic-to-oscillatory and oscillatory-to-decaying transition regions is also known [2] and can be expressed in terms of Ablowitz-Segur solutions to the Painlevé-II equation.

Recently, A. Minakov [6] derived the Riemann-Hilbert problem associated to step-like initial data

$$u(x,0) \to \begin{cases} c > 0, & x \to -\infty, \\ 0, & x \to +\infty. \end{cases}$$

A. Minakov and D. Shepelsky subsequently computed the long-time asymptotic behavior for this initial data. In particular, for $c > 3\omega$, there are five long-time sectors: convergence to c, genus-two hyperelliptic oscillations, elliptic oscillations plus a decaying self-similar wave, elliptic oscillations, and decay to zero. We present current progress on the asymptotic behavior in the transition regions via the Deift-Zhou nonlinear steepest-descent method for Riemann-Hilbert problems. This is the first investigation of the long-time transition asymptotics in a nonlinear wave equation with non-decaying initial data.

There are certain novel features that distinguish the analysis from that of decaying initial data. Most significantly, the non-decaying analysis requires the use of a so-called g-function, leading to a dressed Riemann-Hilbert problem with jumps that do not decay on bands instead of simply points. The transition between the elliptic and decay-to-zero regions is characterized by the "birth of a cut" phenomenon observed in random matrix theory, small-dispersion nonlinear wave equations, and other fields (see, for example, [4] and [5]). On the other hand, the transition between the convergence-to-c and hyperelliptic regions is characterized by the splitting of one band into two (see, for example, [1]).

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On a coupled system of shallow water equations admitting travelling wave solutions

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A non-local formulation of water waves for both $(1+1)$ and $(2+1)$ dimensions was presented in [1], where the original equations with unknown boundary conditions are replaced by an integro-differential equation and a nonlinear partial differential equation, both of which are formulated in

a known domain. A generalization of the results obtained in [1] was presented in [2], where a non-local formulation was derived, governing two ideal fluids separated by a free interface and bounded above either by a rigid lid or by a free surface [2]. Due to the dependence on a free spectral parameter, the corresponding equations are usually called the non-local spectral (NSP) equations of the two-fluid system. The NSP equations were particularly useful for deriving asymptotic approximations: we wish to point out an asymptotically (2+1)-dimensional generalization of the intermediate long wave (ILW) equation reported in [2] which includes the KP equation and the Benjamin-Ono equation as limiting cases. Numerical investigations indicated the existence of lump type solutions, with a speed versus amplitude relationship shown to be linear in the shallow, intermediate and deep water regime.
However, to the best of our knowledge the phenomenological models for more than two fluids mentioned at the beginning of this section have not been paralleled by any analytical study. This prompted us to develop a generalization of the NSP formulation to the case of three ideal fluids, separated by two free interfaces and limited above by a rigid lid. Namely, we consider three inviscid, incompressible, irrotational fluids that are confined between the rigid lids $y = -h_1$ and $y = h + H$, and are separated by two free interfaces $\eta_1(x,t)$ and $\eta_2(x,t)$. We derive the NSP equations governing the evolution of the three-fluid system. Then, after a suitable nondimensionalization of the variables of the six equations of the NSP formulation, we obtain the reduction to a system of shallow water equations in the weakly nonlinear limit. Finally, under the assumption of maximal balance, we introduce travelling wave variables and study only the $(1+1)$-dimensional case. In terms of the new variables we obtain a system of coupled nonlinear shallow water equations which we study numerically in terms of the parameters enter in the theory and that we show to admit solitary wave solutions.

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Finite-dimensional representations of shift operators, remarkable matrices and matrix functional equations

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In this talk I plan to report—to the extent time will permit—quite recent results. (i) The identification of $(N \times N)$-matrices providing *finite-dimensional representations* of two types of "shift" operators, $\check{\delta}(x)$ respectively $\hat{\delta}(y)$, acting as follows on functions $f(z)$ of the variable z, $\check{\delta}(x) f(z) = f(xz)$ respectively $\hat{\delta}(y) f(z) = f(z+y)$; representations which are *exact*—in a sense that shall be explained—in the functional space spanned by polynomials of degree less than (the arbitrary positive integer) N. [1] (ii). The identification of $(N \times N)$-matrices which are explicitly expressed in terms of N arbitrary numbers or in terms of the N zeros of named polynomials of degree N and which feature *remarkable properties*, such as eigenvalues which are *explicitly known* and have *Diophantine* characteristics. [1] (iii). The identification of *matrix functional equations*, such as, for instance, $\mathbf{G}(y)\,\mathbf{F}(x) = \mathbf{F}(x)\,\mathbf{G}(xy)$—where $\mathbf{F}(x)$ respectively $\mathbf{G}(y)$ are $(N \times N)$-matrix-valued functions of the scalar variables x respectively y—and of a class of *nontrivial* solutions of these functional equations [2] [3].

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The symmetry approach for higher Lagrangian systems

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In this poster we will show results on the classification of integrable Lagrangian systems of the form

$$\mathcal{L} = \frac{1}{2} L_2(u, u_x, u_{xx}) u_t^2 + L_1(u, u_x, u_{xx}) u_t + L_0(u, u_x, u_{xx}) \tag{1}$$

using the symmetry approach of Shabat et al. [1] as extended in [2] and with the adaptations explained in an oral presentation in this Conference by the second author. This family of systems includes relevant examples such as

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left(u_t + u_{xx} - u_x^2\right)^2 && \text{generalised NLS} \\
\mathcal{L} &= \frac{1}{2}\left(u_t + u_{xx}\right)^2 + \frac{1}{2} u_x^3 && \text{Boussinesq} \\
\mathcal{L} &= \frac{\left(u_t + u_{xx} - \frac{1}{2}R'(u)\right)^2}{4\left(u_x^2 - R(u)\right)} + \frac{1}{12} R''(u) && \text{Landau} - \text{Lifshitz}
\end{aligned}$$

The poster will summarize results on a complete classification of integrable Lagrangians (1) where we found additional integrable Lagrangian systems possessing interesting symmetry properties.

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Non-commutative Bäcklund Charts[1]

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The term *Bäcklund Chart* was devised back in 1987, at the IV NEEDS Conference in Balaruc Les Bains, when a wide net of links represented by different Bäcklund transformations was depicted to relate hierarchies of non-linear evolution equations admitting 5th order base members [1]. Subsequently, the same idea was adopted to relate well known hierarchies such as KdV, mKdV and Harry Dym [3, 4]. Since then, a wide variety of interesting results were obtained on application of Bäcklund transformations. Here generalizations to non-commutative equations and hierarchies are considered. Specifically, our attention is focussed on our latest results [2] wherein the study on non-commutative hierarchies, started in [5, 6], is further developped and deals, in particular, with properties of recursion operators in this non-commutative setting.

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On the inverse scattering method for the nonlinear Schrödinger equation on a star-graph

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We present a framework to solve the open problem of formulating the inverse scattering method (ISM) for an integrable PDE on a star-graph. The idea is to map the problem on the graph to a matrix initial-boundary value (IBV) problem and then to extend the unified method of Fokas to such a matrix IBV problem. The nonlinear Schrödinger equation is chosen to illustrate the method. The framework unifies all previously known examples which are recovered as particular cases. The case of general Robin conditions at the vertex will be used to introduce the notion of linearizable initial-boundary conditions. For such conditions, the method is shown to be as efficient as the ISM on the full-line, in analogy to linearizable boundary conditions in the Fokas method.

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[1]dedicated to Francesco Calogero on his 80th birthday, with reverence and admiration

KP hierarchy for a cyclic quiver and Calogero–Moser system

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We introduce a generalization of the KP hierarchy, intimately related to the cyclic quiver on m vertices; the case $m = 1$ corresponds to the usual KP hierarchy. Generalizing the result of [1], we show that our hierarchy admits special solutions parameterised by suitable quiver varieties. Using a link to Cherednik algebras [2], we identify the dynamics of the poles of these solutions with the classical Calogero–Moser system for the complex reflection groups $G(m, 1, n) = \mathbb{Z}_m \wr S_n$. The constructed solutions are closely related to the bispectral operators from the work [3], which are obtained by Darboux transformations applied to higher Bessel operators. As a result of our work, the bispectral families from [3] are given a nice parameterization by the points of completed phase spaces for the Calogero–Moser system of type $G(m, 1, n)$, $n = 1, 2, \ldots$. This is joint work with Alexey Silantyev (Leeds).

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'Riemann Equations' in Bidifferential Calculus

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Given an associative algebra $\mathcal{A}$ and two derivations $\mathrm{d}, \bar{\mathrm{d}} : \mathcal{A} \to \Omega^1$ into an $\mathcal{A}$-bimodule, the equation

$$\bar{\mathrm{d}}\phi - (\mathrm{d}\phi)\,\phi = 0 \tag{1}$$

resembles a Riemann (or Hopf) equation.

Depending on the choice of the bidifferential calculus ($\mathcal{A}$, Ω^1, d, $\bar{\mathrm{d}}$), besides a semi-discrete and a fully discrete version of the matrix Riemann equation, the latter leads to quite different examples. We show that realizations of (1) share a simple 'linearization method', which in some cases turns out to be a (continuous or discrete) Cole-Hopf-type transformation. Such realizations are in the class of 'C-integrable equations' [1, 2].

If there is an extension of the derivations d and $\bar{\mathrm{d}}$ to maps $\mathcal{A} \overset{\mathrm{d},\bar{\mathrm{d}}}{\to} \Omega^1 \overset{\mathrm{d},\bar{\mathrm{d}}}{\to} \Omega^2$, with another $\mathcal{A}$-bimodule Ω^2 such that $\mathrm{d}^2 = \bar{\mathrm{d}}^2 = \mathrm{d}\bar{\mathrm{d}} + \bar{\mathrm{d}}\mathrm{d} = 0$, then (1) yields

$$\mathrm{d}\bar{\mathrm{d}}\phi + \mathrm{d}\phi\,\mathrm{d}\phi = 0 \tag{2}$$

as an integrability condition. By choosing appropriate bidifferential calculi, this equation leads to a number of prominent integrable equations, like self-dual Yang-Mills, matrix versions of the two dimensional Toda lattice, Hirota's bilinear difference equation, (2+1)-dimensional NLS, KP and

Davey-Stewartson equations. The abovementioned 'linearization method' does not extend to (2), for which, however, there is another universal method [3], representing an abstract version of binary Darboux transformations.

The talk is based on a joint work with F. Müller-Hoissen and N. Stoilov [4].

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Emission of radiation from perturbed dispersive shock waves

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Dispersive shock waves (DSWs) are expanding regions filled with fast oscillations that stem from the dispersive regularization of classical shock waves (SWs) [1]. Originally introduced in collisionless plasmas and water waves, it is only recently that they have been the focus of intense multidisciplinary efforts that have established their universal role in atom condensates, light pulse (temporal) and beam (spatial) propagation, oceanography, quantum liquids, electron beams, magma flow, granular materials, and wave or material disorder. The dynamics of DSWs is understood in terms of a weakly dispersive formulation of integrable models (and their deformations) such as the Korteweg–De Vries, the Benjamin–Ono, or the defocusing nonlinear Schrödinger equation (dNLSE) [2]. However, since the leading-order dispersion of such models must be extremely weak for the phenomenon to take place, one is naturally led to wonder about the effects of higher-order dispersion (HOD), which must be accounted for to describe the actual dispersion in many physical situations. The aim of this work is to show that HOD corrections lead DSWs to emit resonant radiation (RR) due to a specific phase matching with linear waves, which can ultimately alter the shock dynamics itself. We devote our attention to the study of fiber optic systems modeled dy dNLSE perturbed by higher order dispersion. We show that emission of radiation is quite a general phenomenon that can take place during the propagation of bright pulses [3], or continuous waves at different frequencies experiencing four wave mixing [4]. Moreover, we show that emission of radiation can happen also in non Hamiltonian system described by driven and damped dNLS, being the fiber ring cavity a representative example [5] .

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Expanded Lie group transformations and conservation laws of a generalized variable-coefficient Gardner equation

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In recent years, there is widespread interest in solving group classification problems for several classes of partial differential equations which are of current relevance or potential interests for applications, nonlinear equations being prominent among these. Furthermore, nonlinear equations with variable coefficients have become increasingly important over the last years due to these describe many nonlinear phenomena more realistically than equations with constant coefficients.

Symmetry group analysis has numerous well-known applications. They enable us to obtain exact solutions of partial differential equations directly or via similarity solutions, classify invariant equations, reduce the number of independent variables or determine conservation laws. The analysis of Lie symmetries of equations involving arbitrary functions seems rather difficult. Equivalence transformations allow us to reduce of a class to its subclass with fewer number of arbitrary functions.

Conservation laws play an important role in physics and mathematics. These describes that a certain measurable property of an isolated physical system does not change over time. In mathematics, the integrability of a partial differential equation is strongly related with the existence of conservation laws. Moreover, they can be used to obtain exact solutions of a partial differential equation.

In this work we study a generalized variable-coefficient Gardner equation with nonlinear terms of any order and forcing term. The Gardner equation is widely used in diverse fields of physics including fluid dynamics, quantum field theory, plasma physics and many wave phenomena in plasma and solid state. The equation under consideration generalizes substantially interesting equations, such as KdV, mKdV and Gardner equation [1, 2, 3, 4, 5, 6]. We perform an analysis of Lie symmetries of the equation. We obtain the continuous equivalence transformations of the equation in order to reduce the number of arbitrary functions. We determine the subclasses of the equation which are nonlinearly self-adjoint. Using nonlinearly self-adjointness we construct conservation laws of the considered equation.

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On quantum integrable systems and Schur polynomials

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We consider commuting operators obtained by quantization of Hamiltonians of the Hopf (aka dispersionless KdV) hierarchy. Such operators naturally arise in the setting of Symplectic Field Theory (SFT), see [2, 3]. A complete set of common eigenvectors of these operators is given by Schur polynomials. Applications to SFT will also be discussed.

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Higher-rank Schlesinger transformations and difference Painlevé equations

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The relationship between isomonodromic deformations and differential Painlevé equations is quite well-known: Schlesinger equations describing isomonodromic deformations of a two-dimensional Fuchsian system with four poles on the Riemann sphere reduce to the most general Painlevé VI equation, and other differential Painlevé equations can be obtained from this by coalescence. In fact, the isomonodromic approach is one of the most powerful ways to study properties of Painlevé equations and their solutions, the Painlevé transcendents. Similar isomonodromic representation exists for difference Painlevé equations as well. In this case, instead of continuous deformations of the poles of our Fuchsian system we consider Schlesinger transformations, which are a special kind of gauge transformations that change the characteristic indices of the system by integral shifts, and so they are also isomonodromic.

In the seminal paper [Sak01] H. Sakai suggested a unified classification scheme for both continuous and discrete Painlevé equations that is based on Algebraic Geometry. In this scheme, certain classes of discrete Painlevé equations are closely connected to the continuous one, but some equations are purely discrete. The isomonodromic approach is then a natural way of studying both types of discrete Painlevé equations in the same framework.

This motivated H. Sakai to pose the following question (*Problem A* in [Sak07]): *how to represent these new purely discrete equations in the isomonodromic framework*? This question was first answered by P. Boalch in [Boa09], where he identified the Fuchsian systems whose Schlesinger transformations have the required type. However, Boalch's approach was based on studying the symmetries of the corresponding Fuchsian systems and no explicit equations were written.

In [DST13] we addressed this issue and wrote explicit evolution equations for a discrete dynamical system given by rank-one elementary Schlesinger transformations and considered their reductions to difference Schlesinger equations. However, classes of difference Painlevé equations that have the largest symmetry groups, $E_7^{(1)}$ and $E_8^{(1)}$ in the Sakai's classification scheme [Sak01], correspond to Schlesinger transformations of Fuchsian systems with degenerate eigenvalues. In this talk we explain how to extend our evolution equations to higher-rank elementary Schlesinger transformations and show, as an example, how to obtain difference Painlevé equations with the symmetry group $E_7^{(1)}$ in this way. This talk is based on the recent preprint [DT14].

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Density layered fluids in $2D$ channels: Hamiltonian pictures and conserved quantities.

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We discuss aspects of the theory of incompressible stratified Euler fluids in $2D$ channels. In particular, our focus will be on conserved quantities, both for continuous and sharp (two-layer) stratifications. Following [1, 5] Hamiltonian pictures (both in the full 2D case and in the long wave 1D limit) will be discussed and specialized to our model(s) [2, 3].

In particular we shall show that in the case of sharp two layering, the long wave model may reduce, in the dispersionless limit, to the Airy system (that is, the dispersionless defocusing NLS system). This happens provided the so-called Boussinesq approximation, retaining stratification only in the buoyancy terms, is enforced as in [4]. The non-Boussinesq case can be treated as a deformation of such an integrable system.

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Scattering data computation for the Zakharov-Shabat system

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In this talk we present a numerical method to compute the scattering data for the Zakharov-Shabat system associated to the initial value problem for the nonlinear Schrödinger equation

$$\begin{cases} \mathbf{i}u_t + u_{xx} \pm 2|u|^2 u = 0, \quad x \in \mathbb{R}, \quad t > 0 \\ u(x,0) = u_0(x), \quad x \in \mathbb{R} \end{cases} \tag{1}$$

where $\mathbf{i}$ denotes the imaginary unit, $u = u(x,t)$ is the unknown potential, the subscripts x and t designate partial derivatives with respect to position and time, $u_0 \in L^1(\mathbb{R})$ is the initial potential and the $\pm$ sign depends on symmetry properties of u.

Taking into account that a numerical method to solve (1) by means of the Inverse Scattering Technique (IST) exists, knowing the starting data [2], this method allows us to implement the whole procedure of the IST by simply starting from the initial potential. We also note that the method has an independent interest in some engineering fields where the problem is to compute the scattering data [3].

The numerical method which, at our best knowledge, is the first numerical method proposed to compute all scattering data, is based on the following steps:

1. the computation of the so-called auxiliary functions by solving systems of structured Volterra integral equations on unbounded domains;
2. the approximation of the transmission matrices and then of the scattering matrix;
3. the approximation of Marchenko kernels by solving structured Volterra integral equations on unbounded domains;
4. the computation of bound states and norming constants by the identification of parameters of proper monomial-exponential sums [1].

Numerical tests which confirm the effectiveness of the method in the focusing and defocusing case will also illustrated.

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Some conservation laws for a generalized Fisher equation in cylindrical coordinates

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The Fisher equation which shows the spread of an advantageous gene into a population, was proposed for population dynamics in 1930. The analysis and study of the Fisher equation is used to model heat and reaction-diffusion problems applied to mathematical biology, physics, astrophysics, chemistry, genetics, bacterial growth problems as well as to the development and growth of solid tumours. For some special wave speeds the equation is shown to be of Painlevé type and the general solution for these wave speeds were found in ref [1]. Generalizations of the Fisher equation are needed to more accutarely model complex diffusion and reactions effects found in many biological systems. There are many models that use nonlinear dispersal to describe the tendency for diffusion to increase due to overcrowding [9]. The Fisher equation with the diffusive term generalized to yield a nonlinear diffusion equation with a reaction term. It is known that conservation laws play a significant role in the solution process of an equation or a system of differential equations. Although not all of the conservation laws of partial differential equations (PDEs) may have physical interpretation they are essential in studying the integrability of the PDEs. Moreover, the conservation laws are used for analysis, particularly, development of numerical schemes, study of properties such as bi-Hamiltonian structures and recursion operators, and reduction of partial differential equations. For variational problems, the Noether theorem can be used for the derivation of conservation laws. For non variational problems there are different methods for the construction of conservation laws In [2], Anco and Bluman gave a general treatment of a direct conservation law method for partial differential equations expressed in a standard Cauchy-Kovaleskaya form. In [7] a general theorem which does not require the existence of Lagrangians has been introduced. This theorem is based on the concept of adjoint equations for nonlinear equations. The concept of strictly self-adjoint equations has been generalized [3, 8]. In this work we study a generalization of the well known Fisher equation in cylindrical coordinates. We determine the subclasses of these equations which are weak self-adjoint and nonlinearly self-adjoint. By using a general theorem on conservation laws proved by Nail Ibragimov and the symmetry generators derived in [5], we find conservation laws for these partial differential equations without classical Lagrangians. We also derive some conservation laws by using the multipliers direct method of Anco and Bluman.

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Cluster Structures on Poisson-Lie Groups

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Cluster algebras are axiomatically defined commutative ring equipped with a distinguished set of generators (cluster variables) subdivided into overlapping subsets (clusters) of the same cardinality subject to some polynomial relations. They were initially introduced by S. Fomin and A. Zelevinsky in [3] to study total positivity and (dual) canonical bases in semisimple algebraic groups. Relations of cluster algebra type can be observed in many areas of mathematics (e.g., Plücker and Ptolemy relations, Stokes curves and wall-crossing phenomena, Feynman integrals, Somos sequences and Hirota equations). The rapid development of the cluster algebra theory revealed relations between cluster algebras and grassmannians, quiver representations, Teichmüller theory Poisson geometry, spectral networks, 3d gauge theories, and many other branches of mathematics and, of late, theoretical physics.

Of particular relevance to this workshop are recently established connections between cluster algebras and integrable systems - see [2, 4, 6, 9, 11, 12], to name just a few recent works on the subject. Underlying Hamiltonian structure for most of the integrable systems studied in these papers is naturally described in the context of Poisson-Lie groups. The goal of this talk is to report on the status of the project devoted to investigation of cluster structures on simple complex Lie groups compatible (in the sense of [5]) with Poisson-Lie structures arising from Belavin-Drinfeld classification [1].

To provide more details, let us consider a Lie group $\mathcal{G}$ equipped with a Poisson bracket $\{\cdot,\cdot\}$. $\mathcal{G}$ is called a *Poisson–Lie group* if the multiplication map

$$\mathcal{G}\times\mathcal{G}\ni(x,y)\mapsto xy\in\mathcal{G}$$

is Poisson. The tangent Lie algebra $\mathfrak{g}$ of a Poisson-Lie group $\mathcal{G}$ has a natural structure of a *Lie bialgebra*. We are interested in the case when $\mathcal{G}$ be a simple complex Lie group and its tangent Lie bialgebra is *factorizable*.

A factorizable Lie bialgebra structure on a complex simple Lie algebra can be described in terms of a classical R-matrix, $r\in\mathfrak{g}\otimes\mathfrak{g}$, a solution of the *classical Yang-Baxter equation* which satisfy an additional condition that $r+r^{21}$ is an element of $\mathfrak{g}\otimes\mathfrak{g}$ that defines an invariant nondegenerate inner product on $\mathfrak{g}$. (Here r^{12} is obtained from $\mathbf{r}$ by switching factors on tensor products.) Classical R-matrices were classified, up to an automorphism, by Belavin and Drinfeld in [1]. Let $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}$, Φ be the root system associated with $\mathfrak{g}$, Φ^+ be the set of positive roots, and $\Delta\subset\Phi^+$ be the set of positive simple roots. The Killing form on $\mathfrak{g}$ is denoted by $\langle\ ,\ \rangle$. A *Belavin-Drinfeld (BD) triple* $T=(\Gamma_1,\Gamma_2,\gamma)$ consists of two subsets Γ_1,Γ_2 of Δ and an isometry $\gamma:\Gamma_1\to\Gamma_2$ nilpotent in the following sense: for every $\alpha\in\Gamma_1$ there exists $m\in\mathbb{N}$ such that $\gamma^j(\alpha)\in\Gamma_1$ for $j=0,\ldots,m-1$, but $\gamma^m(\alpha)\notin\Gamma_1$. To each T there corresponds a set $\mathcal{R}_T$ of classical R-matrices that we call the *BD class* corresponding to T. Two R-matrices in $\mathcal{R}_T$ the same BD class differ by an element from $\mathfrak{h}\otimes\mathfrak{h}$ satisfying a linear relation specified by T. We denote by $\{\cdot,\cdot\}_r$ the Poisson-Lie bracket associated with $r\in\mathcal{R}_T$.

Given a BD triple T for $\mathcal{G}$, define the torus $\mathcal{H}_T=\exp\mathfrak{h}_T\subset\mathcal{G}$.

In [7] we conjectured that there exists a classification of regular cluster structures on $\mathcal{G}$ that is completely parallel to the Belavin-Drinfeld classification.

Conjecture. *Let $\mathcal{G}$ be a simple complex Lie group. For any BD triple $T=(\Gamma_1,\Gamma_2,\gamma)$ there exists a cluster structure $(\mathcal{C}_T,\varphi_T)$ on $\mathcal{G}$ such that*

(i) *the number of stable variables is $2k_T$, and the corresponding extended exchange matrix has a full rank;*

(ii) *$(\mathcal{C}_T,\varphi_T)$ is regular, and the corresponding upper cluster algebra $\overline{\mathcal{A}}_{\mathbb{C}}(\mathcal{C}_T)$ is naturally isomorphic to $\mathcal{O}(\mathcal{G})$;*

(iii) *the global toric action of $(\mathbb{C}^*)^{2k_T}$ on $\mathbb{C}(\mathcal{G})$ is generated by the action of $\mathcal{H}_T\times\mathcal{H}_T$ on $\mathcal{G}$ given by $(H_1,H_2)(X)=H_1XH_2$;*

(iv) *for any $r\in\mathcal{R}_T$, $\{\cdot,\cdot\}_r$ is compatible with $\mathcal{C}_T$;*

(v) *a Poisson–Lie bracket on $\mathcal{G}$ is compatible with $\mathcal{C}_T$ only if it is a scalar multiple $\{\cdot,\cdot\}_r$ for some $r\in\mathcal{R}_T$.*

The BD data is said to be *trivial* if $\Gamma_1 = \Gamma_2 = \varnothing$. In this case, $\mathcal{H}_T = \mathcal{H}$ is the Cartan subgroup in $\mathcal{G}$. The resulting Poisson bracket is called *the standard Poisson–Lie structure* on $\mathcal{G}$. The Conjecture 1 in this case was verified in [7]. In SL_n, the Belavin-Drinfeld data of maximal size ($\Gamma_1 = \{\alpha_2, \dots, \alpha_{n-1}\}$, $\Gamma_2 = \{\alpha_1, \dots, \alpha_{n-2}\}$, $\gamma(\alpha_i) = \alpha_{i-1}$) gives rise to the *Cremmer-Gervais Poisson structure*. The strategy we recently employed in [8, 10] to prove the conjecture above in the Cremmer-Gervais case together with an intuition gained by computer-aided verification of the conjecture for any BD data in $SL_n, n \leq 5$ allow us to construct an initial cluster and an initial exchange quiver all BD triples in SL_n.

I will use the Cremmer-Gervais case to outline the key features of our approach and also explain how the construction can be extended to finding cluster structures in the dual Poisson-Lie group and in the Drinfeld double.

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Vortex and Dipole Solitons in $\mathcal{PT}$-Symmetric Lattices with Positive and Negative Defects

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The nonlinear Schrödinger (NLS) equation containing both cubic and quintic terms describes many physical situations and arises in particular in optics [1]. In nonlinear optics, the cubic-quintic nonlinear Schrödinger (CQNLS) equation describes the propagation of an electromagnetic wave in photorefractive materials. The cubic-quintic nonlinearity is due to an intrinsic nonlinear resonance in the material, which brings along strong two-photon absorption [2].

In this talk, the existence and stability of vortex and dipole solitons in cubic-quintic media on parity-time ($\mathcal{PT}$) symmetric lattices with positive and negative defects will be investigated. The governing equation for the physical model that has been used in this study is the CQNLS equation with a $\mathcal{PT}$-symmetric potential in (2+1)D space:

$$iu_z + u_{xx} + u_{yy} + \alpha|u|^2u + \beta|u|^4u + V_{PT} \cdot u = 0 \tag{1}$$

where $u(x, y, z)$ corresponds to the complex-valued, slowly varying amplitude of the field in the xy-plane, propagating in the z direction; $u_{xx} + u_{yy}$ corresponds to diffraction; $V_{PT}(x, y)$ is the external $\mathcal{PT}$-symmetric potential; α and β are coefficients of the cubic and quintic nonlinearities, respectively. α and β can be positive or negative, indicating that the nonlinear optical process is self-focusing or self-defocusing, respectively. Thus, in this work, we consider four different media, namely self-focusing cubic, self-defocusing quintic media ($\alpha = 1, \beta = -1$); self-focusing cubic, self-focusing quintic media ($\alpha = \beta = 1$); self-defocusing cubic, self-defocusing quintic media ($\alpha = \beta = -1$) and self-defocusing cubic, self-focusing quintic media ($\alpha = -1, \beta = 1$).

Solutions to equation (1) are obtained numerically by spectral methods [3]. The investigated potentials satisfy the necessary condition for $\mathcal{PT}$-symmetry $V_{PT}(x, y) = {V_{PT}}^*(-x, -y)$ [4] and are of the following form:

$$V_{PT}(x, y) = V_0\left|2\cos(kx) + 2\cos(ky) + e^{i\theta(x,y)}\right|^2 + iW_0\left[\sin(2x) + \sin(2y)\right] \tag{2}$$

where V_0 and W_0 represent the depths of the real and imaginary parts of the potential, respectively and $\theta(x, y)$ is a phase function given by

$$\theta(x, y) = \arctan\left(\frac{y}{x} - \frac{\pi}{kx}\right) \pm \arctan\left(\frac{y}{x} + \frac{\pi}{kx}\right) \tag{3}$$

which engenders the positive and negative defect [5].

During my talk, I will show the numerical existence of vortex and dipole solitons on $\mathcal{PT}$-symmetric lattices for varying potential depths and defects. Next, I will investigate the linear and nonlinear stability properties of the lattice solitons and discuss the effect of different defects on the soliton stability.

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Delay Painlevé equations

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Differential-delay equations, which relate derivatives and shifts of a function with respect to a single independent variable, arise in many applications. Over the last few decades, several examples of differential delay equations have been discovered that appear to be of Painlevé-type. In particular, some such equations arise as reductions of integrable differential-difference equations and are the compatibility conditions for related linear problems. Others have been discovered using singularity confinement methods.

This talk will present methods for detecting such equations based on the value distribution of meromorphic solutions. As is the case with difference equations, we show that within the classes of equations studied the complexity of meromorphic solutions of the differential-difference Painlevé equations is lower (as measured by growth in the sense of Nevanlinna) than meromorphic solutions of other equations. This property naturally forces a kind of singularity confinement on solutions.

Special solutions of delay Painlevé equations will also be studied. Differential-delay generalisations of QRT mappings will be presented. Many of the known discrete Painlevé equations were first discovered as deformations of QRT mappings.

Some of the results presented are from joint work with Bjorn Berntson (UCL) and some are joint with Risto Korhonen (University of Eastern Finland).

Several nonlocal extensions of the nonlinear Schrödinger type equation

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In 2013, Prof. Ablowitz and Musslimani [1] have introduced a nonlocal extension of the nonlinear Schrodinger equation (NLS), i.e., $iq_t(x,t) - q_{xx}(x,t) + 2q(x,t)q^*(-x,t)q(x,t) = 0$, which is called nonlocal NLS equation, by setting a nonlocal reduction $q(x,t) = r^*(-x,t)$ from the second flow of the AKNS system. In this talk, we shall provide several nonlocal extensions of the nonlinear Schrödinger type equation from Lie algebra splittings and automorphisms [2]. Moreover, rational forms of the smooth solution for the nonlocal NLS are given explicitly [3]. For example, the first-order rational solution of the nonlocal NLS is plotted in Figure 1 [3].

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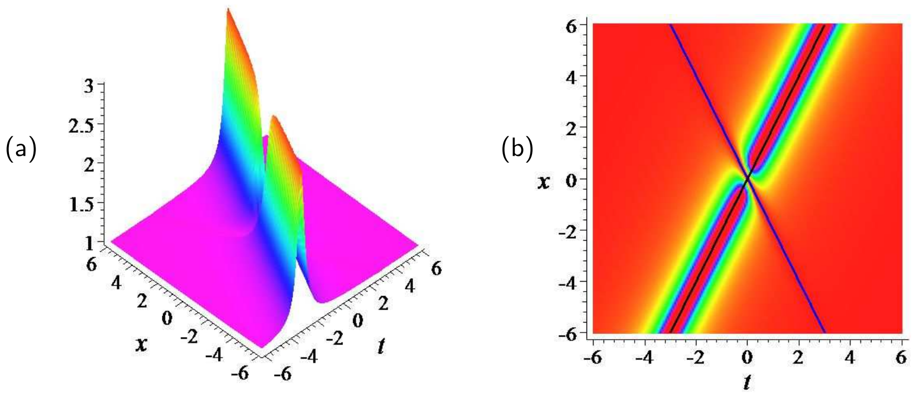

Figure 1: First-order rational solution $q_1^{[1]}$of the nonlocal NLS equation. (a) Profile of $|q_1^{[1]}|$ on (x,t)-plane. (b) Density plot of (a), the black line is $x = 2t$, the blue line is $x = -2t$.

The symmetry approach for higher Lagrangian systems

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In this talk we will explain how the symmetry approach of Shabat et al. [1] can be applied to study the integrability of Lagrangian systems of higher order with Lagrangian

$$\mathcal{L} = \mathcal{L}(x, u, u_x, u_t, u_{xx}) \tag{1}$$

where $u = u(x,t)$, $u_x = \frac{\partial u}{\partial x}$, etc. This family of systems includes relevant examples such as

$$\begin{aligned}
\mathcal{L} &= \frac{1}{2}\left(u_t + u_{xx} - u_x^2\right)^2 && \text{generalised NLS}\\
\mathcal{L} &= \frac{1}{2}\left(u_t + u_{xx}\right)^2 + \frac{1}{2}u_x^3 && \text{Boussinesq}\\
\mathcal{L} &= \frac{\left(u_t + u_{xx} - \frac{1}{2}R'(u)\right)^2}{4\left(u_x^2 - R(u)\right)} + \frac{1}{12}R''(u) && \text{Landau} - \text{Lifshitz}
\end{aligned}$$

The Euler equations $\frac{\delta \mathcal{L}}{\delta u} = 0$ of systems with Lagrangian (1) are PDEs of the form

$$u_{tt} = F(x, u, u_x, u_t, u_{xx}, u_{xt}, u_{xxx}, u_{xxxx}) \tag{2}$$

which are not in evolutionary form. This implies that the symmetry approach in its original formulation cannot be applied directly to this problem. But the extension of the approach given in [2] can be adapted to treat equations of the form (2), and further specialise it to the Lagrangian case.

In this talk we will explain the basics of the method and the appropriate adaptations to study the integrability of the mentioned Lagrangian systems. A summary of results will be shown in an accompanying poster.

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Riemann waves in inhomogeneous hydrodynamic-type systems

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The integrability properties of multidimensional dispersionless systems have been put into light by several methods in the recent years, amongst others Lax representations and inverse scattering transform for vector fields [2] on one hand and hydrodynamic reductions and Painlevé reductions [1] on the other hand. Related inhomogeneous hydrodynamic systems of the Gibbons-Tsarev type appear to possess similar features, especially Lax representations as was shown in [3]. In this contribution, we investigate the construction of Riemann-invariant solutions for systems with inhomogeneous part and the role of these solutions in indicating integrability. In particular, we study the symmetries of generalised systems of the Gibbons-Tsarev type and compare with the integrability conditions for their associated multidimensional homogeneous systems.

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Resolution of supersymmetric sigma models and constant curvature surfaces

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The study of exact solutions of integrable models is a subject of great interest in the mathematics and physics communities. In particular, the bosonic integrable CP^{N-1} sigma model has found applications in physics, biology and mathematics. For this model, a classification of such solutions [1] is complete. For a general Grassmannian $G(M, N)$ $(M > 1)$ bosonic sigma model, such a classification is not complete and has been the object of extensive research works (see, for example, [2, 3, 4]).

The supersymmetric (SUSY) generalization of this problem has lead to some results in the case of the SUSY CP^{N-1} sigma model [5, 6, 7, 8, 9]. Recently, the correspondence with the constant curvature surfaces in the Lie algebra $su(n)$ and a generalized Veronese curve has been established [10]. Some assumptions have been made such as supersymmetric translational invariance. A general construction was still missing and we are investigating a new way of constructing general such solutions. Some hints are given to extend our analysis to general SUSY Grassmannian models.

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On spinors, strings, integrable models and decomposed Yang-Mills theory

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Based on my work with Prof Jiang and Prof Niemi:
This paper deals with various interrelations between strings and surfaces in three-dimensional ambient space, two-dimensional integrable models, and two-dimensional and four-dimensional decomposed $SU(2)$ Yang-Mills theories. Initially, a spinor version of the Frenet equation is introduced in order to describe the differential geometry of static three-dimensional stringlike structures. Then its relation to the structure of the $su_(2)$ Lie algebra valued Maurer-Cartan one-form is presented, while by introducing time evolution of the string a Lax pair is obtained, as an integrability condition. In addition, it is shown how the Lax pair of the integrable nonlinear Schrödinger equation becomes embedded into the Lax pair of the time extended spinor Frenet equation, and it is described how a spinor-based projection operator formalism can be used to construct the conserved quantities, in the case of the nonlinear Schrödinger equation. Then the Lax pair structure of the time extended spinor Frenet equation is related to properties of flat connections in a two-dimensional decomposed $SU(2)$ Yang-Mills theory. In addition, the connection between the decomposed Yang-Mills and the Gauss-Codazzi equation that describes surfaces in three-dimensional ambient space is presented. In that context the relation between isothermic surfaces and integrable models is discussed. Finally, the utility of the Cartan approach to differential geometry is considered. In particular, the similarities between the Cartan formalism and the structure of both two-dimensional and four-dimensional decomposed $SU(2)$ Yang-Mills theories are discussed, while the description of two-dimensional integrable models as embedded structures in the four-dimensional decomposed $SU(2)$ Yang-Mills theory are presented.

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Dirichlet to Neumann map for 1-d Cubic NLS

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Initial-boundary value problems for 1-dimensional 'completely integrable' equations can be solved via an extension of the inverse scattering method, which is due to Fokas and his collaborators. An interesting feature of this method is that it requires more data than needed for a well-posed problem. In the case of cubic NLS, knowledge of the Dirichet data suffices to make the problem well-posed but the Fokas method also requires knowledge of some Neumann data. In this talk, we report on recent work with D. Antonopoulou, where we provide a rigorous study of Dirichlet to Neumann map for a large class of decaying Dirichlet data. We show that the Neumann data are also sufficiently decaying and hence, the Fokas method can be justified.

Peakon–antipeakon solutions of the Novikov equation

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In recent years, there has been considerable interest in peakon solutions to partial differential equations such as Camassa–Holm [1, 2, 3] and other related equations. In this talk, we are interested in the Novikov equation [4, 5], which, like its Camassa–Holm relative, is also of the form $u_t - u_{xxt} = f(u, u_x, u_{xx}, u_{xxx})$, though the right hand side here contains cubic nonlinearities.

The talk is based on an upcoming article, which covers peakon–antipeakon solutions of the Novikov equation on the basis of known solution formulas [6] from the pure peakon case. A priori, these formulas are valid only for some interval of time and only for some initial values. The aim of the article is to study the Novikov multipeakon solution formulas in detail, to overcome these problems.

We find that the formulas for locations and heights of the peakons are valid for all times, at least in an ODE sense. Also, we suggest a procedure of how to deal with multipeakons where the initial conditions are such that the usual spectral data are not well-defined as residues of single poles of a Weyl function.

In particular we cover the interaction between one peakon and one antipeakon, revealing some unexpected properties. For example, with complex spectral data, the solution is shown to be periodic, except for a translation, with an infinite number of collisions between the peakon and the antipeakon.

Also, plotting solution formulas for larger number of peakons shows that there are similarities to the phenomenon called "waltzing peakons" [7].

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Bispectrality and Duality of Integrable Systems

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Two classical integrable systems are dual when their "action-angle maps" are inverses. At the quantum level, this is nicely represented by the bispectrality of the Hamiltonian operators (i.e. the operators share an eigenfunction, but with the role of the spatial and spectral parameters reversed). Strangely, bispectrality also arises as a consequence of duality at the classical level, in the form of bispectral Lax operators for an associated soliton equation. This talk will review numerous examples of this bispectrality-duality correspondence, focusing especially on the bispectral representation of the self-duality of the Calogero-Moser system and it generalizations.

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Localized Solutions of the Linear and Nonlinear Wave Equations

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Following the tradition in the nano and picosecond optics, the basic theoretical studies continue to investigate the processes of linear propagation of shorter, femtosecond and atosecond laser pulses through the corresponding envelope equation in paraxial spatio-temporal approximation. I will present mathematical arguments and proof, that paraxial optics is not valid for large band attosecond and phase-modulated femtosecond pulses. In air, due to the small dispersion, the wave equation as well as the $3D + 1$ non-paraxial amplitude equation describe more accurate the pulse dynamics. In my presentation new exact localized solutions of the linear wave and non-paraxial amplitude equations will be presented. The solutions describe the real diffraction of the laser pulses without Fresnel or Fraunhofer approximation to be used [1]. They discover one new law of diffraction of the localized optical waves - with initial enlarging of their spectra the Fraunhofer zone become closer and closer to the source. The analytical results are compared with the diffraction experiments of atosecond pulses and numerical investigations. Thus, in the nonlinear theory one important question appear: How broad spectrally must be the initial pulse to be in the regime of the Fraunhofer diffraction and, in addition, to has sufficient power for a non-linear mode of propagation. In this nonlinear regime we solve the corresponding nonlinear scalar and vector equations and obtain Lorentz type solitary waves [2]. Progress in the nonlinear wave optics encouraged us to examine the task, set in the 30s of the 20th century by Euler, Kockel and Heisenberg [3] of nonlinear polarization of vacuum. We investigate wave dynamics in nonlinear vacuum in the frame of a system of nonlinear wave vector equations. The corresponding solitary solution admits shock wave dynamics in 3D+1 space [4].

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$1+1$-dimensional Yang-Mills equations and mass as quasiclassical correction to action

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Two-dimensional Yang-Mills models in a pseudo-euclidean space are considered from a point of view of a class of nonlinear Klein-Gordon-Fock equations. Underlying ideas for this investigation were taken from works of Baseyan [1], Corrigan [2] and Nahm [3].

This paper is a direct development of author's results [4] in which one dimensional model immersed in SU(2) Yang-Mills theory was studied in the context of Nahm model. The author's main result [4] is a demonstration of existence and evaluation of nonzero quantum correction to action against classical zero mass as a consequence of the proposed model. The one-dimensional Yang-Mills-Nahm models were considered from algebrogeometric points of view. A quasiclassical quantization of the models is based on path integral construction and its zeta function representation in terms of a Green function diagonal for an auxiliary heat equation with an elliptic potential. The Green function diagonal and, hence, the generalized zeta function and its derivative are expressed via solutions of Drach equation [5] and, alternatively, by means of Its-Matveev [6] formalism in terms of Riemann theta-functions. The weak point of the description is namely the one-dimensionality of the reduction that provoke ambiguity of the interpretation of the correction as the mass.

The task of this work is the derivation and solution of the field equations for a class of the two dimensional models. The result of the reduction of the basic Yang-Mills equations and the corresponding Lagrangian is similar to the one-dimensional one: we obtain $1{+}1$ ϕ^4 (Ginzburg-Landau) model equations with the zero mass term and coefficients that depend on algebraic closure of an matrix anzatz for the gauge fields that fix the model. The stationary and directed waves are thought as quasiperiodic solutions of the model equations that are expressed in terms of elliptic functions. Its quantization is performed by means of quasiclassical Feynmann-Maslov integral, which evaluation and quantum corrections to action is based on the mentioned technique of the generalized zeta-function renormalization in terms of the nonlinear Drach equation. It is derived for the Green function diagonal (within the the heat kernel formalism) and gives polynomial solutions in elliptic variables.

An alternative approach based on Baker-Akhiezer functions for Kadomtsev-Petviashvili equation is formulated. The quantum corrections to action of the model are evaluated. The fields from the class of elliptic functions are properly studied. Extra variables of arbitrary dimensions are accounted for the model applications of the solutions in elementary particles physics. For a model, which field is represented via elliptic (lemniscate) integral by construction, Yang-Mills field mass is defined as the quantum correction, in the quasiclassical approximation it is evaluated via hyperelliptic integral.

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Thermodynamics of macroscopic systems with inverse-cube forces

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A system consisting of an arbitrary number of particles of equal masses interacting via an arbitrary potential of homogeneity degree -2 and confined by an isotropic harmonic potential has the property of sustaining undamped isochronous compressional oscillations, as has been shown earlier. In this paper, we review this finding. We also discuss the concept of thermodynamic equilibrium for such systems. We require a generalization of the usual concept of equilibrium ensemble, for which macroscopic compressional oscillations arise. It turns out that these oscillations are adiabatic, and that correspondingly, the temperature varies when the size of the system does (in the specific case stated above, this dependence is one of inverse proportionality). It is also shown that some of these results extend to the quantal case.

Third-order superintegrable systems with potentials satisfying nonlinear equations

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The conditions for superintegrable systems in two-dimensional Euclidean space admitting separation of variables in an orthogonal coordinate system and a functionally independent third-order integral are studied.

As it is well known, there are 4 in-equivalent choices of orthogonal coordinates for which the Helmholtz equation (i.e. the equation for eigenvectors of the Hamiltonian with vanishing

potential) admits separation of variables. Namely, Cartesian, polar, parabolic and elliptic coordinates. Any integrable system with second-order integrals is separable in (at least) one of these coordinate systems. We consider systems that are superintegrable, i.e. allow an additional independent integral, which is supposed to be of third order in momenta. The investigation of third-order superintegrable systems that admit separation of variables in Cartesian [1] and polar coordinates [4] lead to the discovery of new superintegrable "nonlinear" potentials, i.e. potentials involving solutions of non-linear ODEs, including Painlevé transcendents. These further discoveries, along with recalling that the defining equations for a superintegrable potential are non-linear, might lead one to believe that such "nonlinear" potentials are ubiquitous. However, our work [2] shows that only systems that separate in subgroup type coordinates, Cartesian or polar, admit potentials that can be described in terms of nonlinear special functions. As in the parabolic case [3], systems separating in elliptic coordinates are shown to have potentials with only non-movable singularities.

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Symmetry Properties of the discretization procedures of the Liouville equation

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The Liouville equation is well known to be linearizable by a point transformation. It has an infinite dimensional Lie point symmetry algebra isomorphic to a direct sum of two Virasoro algebras. We show that it is not possible to discretize the equation keeping the entire symmetry algebra as point symmetries. We do however construct a difference system approximating the Liouville equation that is invariant under the maximal finite subalgebra $SL_x(2,\mathbb{R}) \otimes SL_y(2,\mathbb{R})$. The invariant scheme is an explicit one and provides a much better approximation of exact solutions than comparable standard (non invariant) schemes. Comparisons with the generalized symmetry invariant scheme and integrable discrete scheme are discussed [1].

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Multiple rogue waves and extremal rogue waves in $1{+}1$ and $2{+}1$ integrable systems related with AKNS and KP hierarchies

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Since the appearance of articles [1, 2] and the Ph.D thesis [3] in 2010-2011, the concept of multi-rogue waves (MRW) solutions, first introduced in our works for the focusing NLS equation and the KP-I equation, soon became a well accepted paradigm which was extended to many other integrable models. The aforementioned works were further developed in [4].

In these works the parametrisation, allowing to consider the higher Peregrine breathers as the simple reduction of the MRW solutions, was first found for the rank 2 solutions, and later extended to arbitrary ranks. There is a deep difference between the genuine Peregrine breather (P_1-breather) , which is an isolated solution, and P_n breather which belongs to the $2n-2$ parametric family of solutions of the NLS equation. For sufficiently small values of parameters, MRW solutions have a shape, very close to that of P_n-breather, (or rank n Peregrine breather), as we called it in [4] . In $1+1$ case for generic values of parameters the shape of magnitude contains $n(n+1)/2$ peaks [2, 3] with the heights close to that of P_1-breather. When one or many of the parameters are big enough, one can observe various approximately symmetric configurations. For small values of parameters ,- the number of peaks and their heights of might be different. For instance, for P_n breather, the extreme number of maxima $n(n+1)-1$ is attended, and, there is one central maximum of the height $2n+1$, and $n(n+1)-2$ of much smaller maxima in (x,t) plane [3, 4]. Surprisingly enough, the number of minima of the absolute value of MRW solutions is $n(n+1)$. It remains the same for all values of the parameters.

These properties of MRW solutions of the NLS equation are responsible for the behavior of rogue waves solutions of the KP-I equation, induced by the NLS-KP correspondence. The KP-I equation has, for any rank n, the solutions , containing $n(n+1)/2$ separated "long living" rogue waves. For large times, these waves have almost the same magnitude and are moving with asymptotically constant velocities, progressively diverging when $|t|\to\infty$. For some finite interval of time, they concentrate inside the small domain, in which their movement accelerates while approaching to a minimal distances from each over, in general not entering in the full collision. Under the special choice of initial data, the confluence (full collision) of these rogue waves takes place at the single point of the space-time (x,y,t), leading to appearance of "extremal" , very short living, rogue wave, generated by the P_n breather solution of the NLS equation via NLS-KP correspondence. This will be illustrated by the movies, describing the different scenario of the rogue waves collisions, (see also [4]).

We also explain [5] how the same wronskian formulas for the MRW solutions of the NLS equation under an appropriate re-parametrisation allow to describe the the MRW solutions for the commuting flows of AKNS hierarchy which yields many equations relevant to nonlinear optics.

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Darboux transformations for Lax operators associated with Kac-Moody algebras

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We consider Lax operators for two–dimensional "periodic" Toda type systems corresponding to classical series of Kac-Moody algebras and $G_2^{(1)}$ [1]. For these Lax operators we construct systematically elementary Darboux transformations and integrable differential-difference systems (Bäcklund transformations). Bianchi permutability of Bäcklund transformations, or, more precisely, the commutativity conditions for the Darboux maps leads to to a system of integrable partial difference equations. Thus, with every classical Kac-Moody Lie algebra and $G_2^{(1)}$ we associate an integrable Toda type system, a pair of differential-difference systems and a partial difference system. These differential-difference systems represents Bäcklund transformations for the Toda type system and serves as non-local symmetries for the partial-difference system of equations.

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Route to thermalization in the α-Fermi-Pasta-Ulam system

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We study the original α-Fermi-Pasta-Ulam (FPU) system with $N = 16, 32$ and 64 masses connected by a nonlinear quadratic spring. Our approach is based on resonant wave-wave interaction theory, i.e. we assume that, in the weakly nonlinear regime (the one in which Fermi was originally interested), the large time dynamics is ruled by exact resonances. After a detailed analysis of the α-FPU equation of motion, we find that the first non trivial resonances correspond to six-wave interactions. Those are precisely the interactions responsible for the thermalization of the energy in the spectrum. We predict that for small amplitude random waves the time scale of such interactions is extremely large and it is of the order of $1/\epsilon^8$, where ϵ is the small parameter in the system. The wave-wave interaction theory is not based on any threshold: equipartition is predicted for arbitrary small nonlinearity. Our results are supported by extensive numerical simulations. A key role in our finding is played by the *Umklapp* (flip over) resonant interactions, typical of discrete systems. The thermodynamic limit is also briefly discussed. Details can be found in [1].

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Topological effects on momentum and vorticity evolution in stratified fluids

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In this talk I will consider a two-fluid system in a channel. Both fluids are supposed to be inviscid, incompressible, and homogeneous. I will show that the topological properties of the fluids domains affect total horizontal momentum and vorticity evolution. In the first part I will treat the 2-dimensional case and I will suppose that the density of the upper fluid limits to zero [1]. In the second part I will deal with the total vorticity evolution in the 3-dimensional case.

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A classification of 4D consistent maps

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It is nowadays a well-established fact that integrability of 2D discrete equations can be identified with their 3D consistency [1]. Our aim is to turn our attention to integrability of 3D discrete systems, now understood as 4D consistency. The most striking feature is that the number of integrable systems drops dramatically with the growth of dimension: only half a dozen of discrete 3D systems with the property of 4D consistency are known. All of them are of a geometric origin.

Our investigation is devoted to 3D maps $\Phi : (x_1, x_2, x_3) \mapsto (\widetilde{x}_1, \widetilde{x}_2, \widetilde{x}_3)$, where each component $\widetilde{x}_k$ is defined as a formal series

$$\widetilde{x}_k = x_k + \sum_{i=2}^{\infty} A_k^{(i)}(x_1, x_2, x_3), \qquad k = 1, 2, 3. \tag{1}$$

Here $A_k^{(i)}$ is a homogeneous polynomial of (x_1,x_2,x_3) of degree i. After setting $(x_1,x_2,x_3) = (x_{23},x_{31},x_{12})$ we combinatorially assign the quantities x_{jk} to the three faces of a 3D cube parallel to the coordinate plane jk, and the quantities $\widetilde{x}_{jk} = T_i x_{jk}$ to the three opposite faces (T_i stands for the unit shift in the i-th coordinate direction). Thus, we consider a 3D system with fields assigned to elementary squares, given by the formulas

$$(T_3x_{12}, T_2x_{13}, T_1x_{23}) = \Phi\left(x_{12}, x_{13}, x_{23}\right). \tag{2}$$

The *4D consistency* of (2) can be formulated as follows. Consider the initial value problem with the data $x_{ij} = x_{ji}$, $i,j = 1,2,3,4$, prescribed at six squares adjacent to one common vertex of a 4D cube. Then the application of (2) to the four 3D cubes adjacent to this vertex allows one to determine all $T_k x_{ij}$. At the second stage, the map is applied to the other four 3-faces of the 4D cube, with the result being all $T_m(T_k x_{ij})$ computed in two different ways (since each of the corresponding squares is shared by two 3-faces). Now, 4D consistency of the map means that $T_m(T_k x_{ij}) = T_k(T_m x_{ij})$ for any permutation i,j,k,m of $1,2,3,4$ and for arbitrary initial data.

The only known 3D map of type (2) which is 4D consistent is the *symmetric discrete Darboux system*

$$T_k x_{ij} = \frac{x_{ij} + x_{ik}x_{kj}}{\sqrt{1-x_{ik}^2}\sqrt{1-x_{kj}^2}}. \tag{3}$$

After defining the group of admissible transformations for the classification we prove that (3) is the unique 4D consistent map of type (2). In particular, in such a case, the formal series (1) are convergent. For the map (3), whose 4D consistency can be explained by means of spherical geometry [2], one can also prove its Arnold-Liouville integrability and its solvability.

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Cauchy–Jost function and hierarchies of integrable equations

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The Cauchy–Jost function $F(x,\lambda,\mu)$, i.e., a primite of the product of the dual Jost solution and Jost solution itself, with spectral parameters λ and μ correspondingly, is well know to appear naturally under binary Darboux transformation, [1]. In [2] this function was called the Cauchy–Baker–Akhiezer one and its properties where discussed. Here we study properties of this function in detail. In particular, we show that under assumption of dependence on infinite number of times $x = (x_1, x_2, \ldots)$, this function obeys equation $\sum_{k=0}^{\infty} \frac{\partial_{x_k}}{z^{k+1}} F(x,\lambda,\mu) = -F(x,\lambda,z)F(x,z,\mu)$ in terms of asymptotic series with respect to complex λ, μ and , z. We prove that this equation, in analogy to [3] generates the whole hierarchy of integrable equations. This property is illustrated by examples of the KP and DSII hierarchies. We also prove that the Darboux transformed Cauchy–Jost function obeys linear equation on the background of the original one.

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Dynamics of rolling and sliding axially symmetric rigid bodies: Jellett's egg (JE), Tippe top (TT), rolling and sliding disc (sRD). Asymptotic solutions and numerical sampling.

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Equations of motion for purely rolling axially symmetric rigid bodies have 4 degrees of freedom and are completely integrable (Routh, Chaplygin). When sliding is allowed there are 2 additional degrees of freedom for motion of centre of mass, equations are dissipative and integrability is lost. Analysis of this strongly nonlinear dynamical systems is difficult and progress is limited. The main tools in study of global dynamics is the monotonously decreasing energy function, theorems on stability of asymtotic solutions and the use of LaSalle type theorems. An additional usefull property is that the underlying purely rolling problem is integrable and this can be used for studying dynamics with sliding as in the case of the Tippe top where it helped to explained oscillatory behaviour of inverting solutions of TT [1, 2].

Equations for the Jellett´s egg (an axially symmetric ellipsoid with halv axis a, b) are interesting as the parameters a, b may be deformed to derive equations of other rigid bodies such as the Tippe top and the sliding and rolling Disc (sRD). This is used for understanding the common structure of all these equations and enables study of how features of dynamics change when the parameters are deformed. The use of the JE equations makes possible a uniform analysis of bifurcation diagrams of asymptotic solutions for JE, TT, sRD and are a basis for understanding of what asymptotically happens to these rolling and sliding bodies. The asymptotic solutions provide also a usefull framework for numerical sampling of solutions to get an idea of what happens at different initial condition regimes.

The present understanding of inversion of the Tippe top is a result of several papers [3, 4, 5] which studied stability of asymptotic solutions, as a function of physical parameters and of the initial conditions. It is presently well understood that the sliding friction is responsible for inversion and that the inversion can take place only for the values of parameters $1-\alpha<\gamma=I_1/I_3<1+\alpha$ where $0<\alpha<1$ measures the eccentricity of the center of mass and I_1, I_3 are the main moments of inertia. But these results **do not** prove that the Tippe top has to invert, do not specify the range of initial conditions when TT is inverting and do not explain dynamical behaviour of inverting solutions. To address the last question we have introduced [1, 2] a new method of studying dynamics that is based on deformation of integrals of motion of an integrable sub-case when the TT is rolling without sliding. This approach leads to one (nonintegrable) *Main Equation for the Tippe Top* that describes behavior of the inclination angle $\theta(t)$ between the symmetry axis $\hat{\mathbf{3}}$ and the vertical axis $\hat{\mathbf{z}}$. This equation helps to prove that during the inversion the symmetry axis nutates fast within a narrow band that is moving from the nbhd of the north pole to the nbhd of the south pole of the unit sphere.

Numerical analysis of solutions requires a clever way of testing initial conditions so that small number of simulations provides understanding of what happens at different choices of initial conditions and how resilient for perturbations is the inverting behaviour of the Tippe top. Our study [6] shows that initiation of inversion requires reaching a treshold value of the angular velocity $\dot{\varphi}$ and synchronisation of other variables.

Equations for rolling and simultaniously sliding Disc (sRD) seem to be studied very little as they are not integrable [7]. In this case it is the asymptotic solutions [9] that provide a framework for understanding general solutions as they have constant energy and attract other solutions. I will discuss also suitable way of sampling different regimes of initial conditions to obtain some understanding of global features of dynamical behaviour.

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Stefan-Type Moving Boundary Problems for the Harry Dym Equation and its Reciprocals

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Moving boundary problems of Stefan-type have their physical origin in the analysis of the melting of solids and the freezing of liquids ([1, 2]). The classical Stefan problem involves a linear heat conduction equation, but involves a nonlinear condition on the boundary separating the phases. Nonlinear Burgers-Stefan problems have been investigated via an integral representation with origin in work of Calogero *et al* (see e.g. [3, 4] and literature cited therein). On the other hand, reciprocal-type transformations may be used to obtain parametric solution of Stefan problems involving the nonlinear melting processes in a range of simple metals [5].

However, whereas there has been and continues to be extensive research on Stefan problems for linear and linked C-integrable equations, the literature on moving boundary problems for S-integrable equations is sparse indeed. Nonetheless, one intriguing solitonic connection was made in [6] where, in an investigation of the Saffman-Taylor problem with surface tension, a one-parameter class of solutions was isolated in a description of the motion of an interface between a viscous and non-viscous two-dimensional fluid: this class was shown to be linked to the well-known Harry-Dym equation of soliton theory. Here, classes of novel moving boundary problems are discussed both for the Harry Dym equation and the Korteweg-de Vries singularity manifold equation to which it is linked via a reciprocal transformation. The boundary conditions adopted involve prescribed density and flux on the moving surface, as in classical Stefan problems. A symmetry reduction allows exact solution of privileged infinite sequences of such nonlinear moving boundary problems in terms either of Yablonski-Vorob'ev polynomials or classical Airy function solutions of Painlevé II. These solutions are generated by the iterated action of its Bäcklund transformation. The latter procedure has previously been applied to isolate infinite sequences exactly solvable steady state boundary value problems arising out of the Nernst-Planck system descriptive of two-ion electro-diffusion [7]. The iterative action in that electolytic setting is associated with quantized fluxes of the ionic species [8].

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Discrete and Continuous Nonlocal NLS Equation

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In the first part, we study the existence and bifurcation results for quasi periodic traveling waves of discrete nonlinear Schrödinger equations with nonlocal interactions and with polynomial type potentials. We employ variational ana topological methods to prove the existence of traveling waves in nonlocal DNLS lattice. Next, we examine the combined effects of cubic and quintic terms of the long range type in the dynamics of a double well potential (nonlocal NLS). While in the case of cubic and quintic interactions of the same kind (e.g. both attractive or both repulsive), only a symmetry breaking bifurcation can be identified, a remarkable effect that emerges e.g. in the setting of repulsive cubic but attractive quintic interactions is a "symmetry restoring" bifurcation. Namely, in addition to the supercritical pitchfork that leads to a spontaneous symmetry breaking of the anti-symmetric state, there is a subcritical pitchfork that eventually reunites the asymmetric daughter branch with the anti-symmetric parent one. The relevant bifurcations, the stability of the branches and their dynamical implications are examined both in the reduced (ODE) and in the full (PDE) setting. The model is argued to be of physical relevance, especially so in the context of optical thermal media.

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Automorphic Lie Algebras and Root System Cohomology

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The method of reduction groups [Mik81, LM05] leads to the following mathematical problem setting, which we describe here without getting all the technicalities right: We start with a finite group G acting on $\mathbb{CP}^1$ (thereby restricting the group to $\mathbb{Z}/N, \mathbb{D}_N, \mathbb{T}, \mathbb{O}, \mathbb{Y}$) and with an irreducible representation space V. On V we assume the linear action of a Lie algebra $\mathfrak{g}$ (as in $\mathfrak{sl}(V)$). This induces an action of G on $\mathfrak{g}$ by conjugation. We then have an action on $\mathfrak{g} \otimes \mathcal{M}(\mathbb{CP}^1)$ and call $(\mathfrak{g} \otimes \mathcal{M}(\mathbb{CP}^1))^G$ an Automorphic Lie Algebra (ALiA), where $\mathcal{M}$ stands for meromorphic.

The computation and classification of ALiAs can be done by hand calculation in the case of $\mathbb{D}_N$ [KLS14], and when the representation of $\mathbb{CP}^1$ and $V = \mathbb{C}^2$ are the same [LS10], but in general this seems unfeasible. We have developed a FORM program [KUVV13], calling on GAP [GAP08] and Singular [GP08], to compute the invariant matrices in the case of $\mathfrak{g} = \mathfrak{sl}, \mathfrak{so}, \mathfrak{sp}$. For $\mathfrak{g} = \mathfrak{sl}$ we have computed the Chevalley normal form of the ALiAs based on all the irreducible representations with poles at an exceptional group orbit.

The ALiAs are Lie algebras over the ring of polynomials in the modular invariant $\mathfrak{k}[\mathbb{I}]$, where $\mathfrak{k}$ is the splitting field of the group G. Not working over the field $\mathbb{C}$ complicates the analysis, specifically finding the Chevalley normal form, but when all is said and done, one is rewarded with results that are as simple as possible and can be partially checked by predictions made on the basis of the action of the generators of G on the irreducible representations [Kni14], more specifically using the codimensions of their invariant subspaces.

This talk will give an explicit description of the results obtained so far. They indicate that if two ALiAs can be isomorphic as Lie Algebras, based on the (co)dimension counts, they are.

It will then go on to sketch a cohomology theory for root systems that can be used in the classification of Lie algebras depending polynomially on parameters. This gives us explicit criteria to test whether a given Chevalley normal form does indeed define a Lie algebra and how to find a model for this Lie algebra by integrating a differential two form.

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Integrable dispersionless PDEs in multidimensions: rigorous aspects of the Cauchy problem, wave breaking and exact implicit solutions

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We first review the formal aspects of the theory of integrable dispersionless PDEs (including, as distinguished examples, the dispersionless Kadomtsev - Petviashvili, the heavenly and the Boyer-Finley equations) arising as commutation condition of multidimensional vector fields, obtained in collaboration with S. V. Manakov: the IST formalism for solving the Cauchy problem, the construction of the longtime behavior of solutions and of exact implicit solutions, and the analytical aspects of multidimensional wave breaking [1] - [5]. We also present some recent results on the rigorous aspects of such a theory, obtained in collaboration with P. Grinevich and D. Wu [6].

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Banach space geometry and construction of solutions by limiting processes

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In the 90ies the study of countable superpositions of solitons was initiated by Gesztesy and al. Their approach was via a step by step analysis of the ISM. In our talk we will explain how the original method can be replaced by Banach space geometry. We will start from operator formulas for integrable systems and recall then the necessary background from functional analysis. In the main part we will combine these ingredients in order to extend the initial results to superposition of more complicated solutions (formations, weakly bound groups) and of matrix solutions.

R. Boll consistent around the cube systems and their linearizability

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Consistency around the cube has proved to be one of the most useful concept in studying discrete, multilinear, integrable, nonlinear systems defined on a quad-graph, soon becoming a definition itself of integrability. An algorithmic procedure in fact provides a (true) Lax pair and Bäcklund transformations for any consistent system. A first classification of these equations was presented in [1], later extended in [2]. A complete classification (under the additional hypothesis of the tetrahedron property) has been finally obtained in [3]-[4].

When one assume the more general context of [3]-[4], where a priori different equations live on the faces of the consistency cube, the strictly equivalence between existence of a (non trivial)

Lax pair, Bäcklund transformations and consistency has been somehow criticized in [5]. The notion of *weak Lax pair* was thus introduced.

In this seminar we show how this critics can be fully reabsorbed, once the consistent systems of Boll are properly extended on the lattice. After all the independent equations inside an equivalence class have been identified, an algebraic entropy test reveals their *linearizability* ($C-$integrability). We construct the Lax representation, whose weakness obviously reflects the linearizability property. We also provide explicit examples of the linearization procedure.

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Exchange algebras, spontaneous symmetry breaking and Poisson structures for differential and difference operators

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We discuss Poisson structures in the space of solutions of linear difference and differential equations on the line. Natural covariance properties with respect to gauge transformations and the to the action of the differential/difference Galois groups make these structures very rigid and allow to fix it almost uniquely. This construction leads to a new class of classical r-matrices and to a peculiar symmetry breaking which endows differential Galois groups with a Poisson structure. The main results are exposed in paper [1].

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Free and Driven Solitonic Spin Wave "Bullet" Mode Excited by Pure Spin Current

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It is known that self-localized solitonic spin wave excitations - spin wave "bullets"- play an important role in the magnetization dynamics of magnetic nano-structures driven by spin-polarized electric current [1, 2]. The equation of motion for the magnetization, where positive magnetic damping caused by the spin-electron interaction and negative magnetic damping caused by spin-transfer torque effect are taken into account, can be reduced to a Ginzburg-Landau equation [1, 3] that has a stable solitonic solution in a two-dimensional case. This stable solution correctly describes the self-localized spin wave "bullet" mode observed in experiment [2], and practically coincides with the corresponding two-dimensional solution of the nonlinear Schrodinger equation [1].

Here we demonstrate that a similar self-localized spin wave "bullet" mode can be excited by a pure spin current. This pure spin current is caused by the spin Hall effect in a layered structure consisting of a thin layer of platinum (Pt) placed in contact with a layer of a ferromagnetic metal (Permalloy (Py)). The spin current flowing into Py is perpendicular to the direction of the charge current in Pt. It compensates the magnetic damping in Py and excites a localized non-propagating spin wave "bullet" mode of a microwave frequency in a Py layer [4]. The experiment was made by the group of Professor S. Demokritov at the Muenster University, Germany, and the generation of single-mode coherent auto-oscillations was demonstrated in a device that combines local injection of a pure spin current with enhanced spin-wave radiation losses. Counter-intuitively, radiation losses lead to the suppression of the nonlinear processes that prevent auto-oscillation by redistributing the energy of the spin current between the different spin wave modes. Thus, the spatial localization of the spin current enables excitation of a particular standing auto-oscillation mode - a solitonic spin wave "bullet".

We also report a study of the effects of external driving microwave signals on an auto-oscillator where the self-localized solitonic spin wave "bullet" mode is excited [5]. Our results show that such a nonlinear auto-oscillator can be efficiently synchronized by the application of a microwave signal at approximately twice the frequency of the auto-oscillation, which opens additional possibilities for the development of novel spintronic devices. We find that the synchronization exhibits a threshold determined by the magnetic fluctuations pumped above their thermal level by the spin current, and is significantly influenced by the nonlinear self-localized nature of the auto-oscillatory mode. These findings suggest a new route for the implementation of nano-scale microwave sources in the next generation of integrated nano-electronics.

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Dispersionful Version of WDVV Associativity Equations

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The Witten-Dijkgraaf-Verlinde-Verlinde (WDVV) equations arise as the conditions of associativity of an algebra in an N dimensional space. These equations are fully integrable for any N. Furthermore, the compatibility conditions for the WDVV equation can be written as a hydrodynamic type system of PDEs, which possesses a bi-Hamiltonian structure. This structure allows us to construct members of the positive and negative parts of the hierarchy. Exploiting a special transformation, together with this hierarchy, we are able to construct dispersionfull version of the above equations.

Young diagrams associated with the tropical periodic Toda lattice

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The Toda lattice is one of the most famous integrable systems in classical mechanics. Recently, one of its variations is attracting attentions in the interplay of integrable systems and tropical geometry. We call this system the tropical periodic Toda lattice (trop p-Toda) [1, 2]. Its evolution equation was known as the ultradiscretization/tropicalization of the discrete periodic Toda lattice equation (in a non-standard notation which I used in [3])

$$\bar{a}_{2n-1} + \bar{a}_{2n} = a_{2n} + a_{2n+1}, \quad \bar{a}_{2n}\bar{a}_{2n+1} = a_{2n+1}a_{2n+2}, \tag{1}$$

where $a_n = a_n^t, \bar{a}_n = a_n^{t+1}$ are dependent variables depending on discrete spatial $n \in \mathbf{Z}_{2N}$ and temporal $t \in \mathbf{Z}$ coordinates. In [4], Inoue and Takenawa studied trop p-Toda and clarified its iso-level set structure under a condition (on its conserved quantities) called *generic*. However, without this condition we have so far no suitable description of the connected components of the iso-level set, which have different sizes according to their internal symmetries.

Recently, I studied trop p-Toda and proved that its associated 'Young diagrams' (YDs) given by two different definitions are identical [3]. From one of the definitions one immediately sees that the common YD is preserved under the time evolution. I believe that this identification of the YDs is the first step in clarifying the iso-level set structure of this dynamical system in *general* cases, i. e. not restricted to *generic* cases.

The two definitions of the YDs are as follows. The first is related to the Lax representation of the discrete periodic Toda lattice. It leads to its k-th conserved quantity expressed as a sum of products of k dependent variables

$$h_k = \sum_{\substack{1 \le i_1 \triangleleft i_2 \triangleleft \cdots \triangleleft i_k \le 2N \\ (i_1, i_k) \neq (1, 2N)}} a_{i_1} a_{i_2} \ldots a_{i_k}, \tag{2}$$

where $i \triangleleft j \Leftrightarrow i + 1 < j$, i. e. whose indices are in nearest neighbor exclusion condition. Then the corresponding conserved quantity of trop p-Toda

$$H_k = \min_{\substack{1 \le i_1 \triangleleft i_2 \triangleleft \cdots \triangleleft i_k \le 2N \\ (i_1, i_k) \neq (1, 2N)}} \left(A_{i_1} + A_{i_2} + \cdots + A_{i_k}\right), \tag{3}$$

is defined as its tropicalization. I show that the above condition on the indices leads to the weak convexity condition $H_k + H_{k+2} \ge 2H_{k+1}$, that enables us to represent them as a YD with horizontal edges of lengths $H_{N+1-i} - H_{N-i} (1 \leq i \leq N)$. The second is related to a generalization of the Kerov-Kirillov-Reshetikhin bijection in combinatorics of Bethe ansats, more precisely one of its variations in sl_2 case. In fact, it is a real continuous analogue of the bijection reflecting the real-valuedness of the above lengths.

As a special case the trop p-Toda reduces to an integrable (soliton) cellular automaton known as the periodic box-ball system. In this case the YD represents the content of solitons in the system, and the generic condition is requiring no two solitons have a common amplitude. I note that in this case the iso-level set structure has been clarified without the generic condition [5], where the Kerov-Kirillov-Reshetikhin bijection played an important role in describing the action-angle variables of the system.

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Stability analysis of equilibria for certain generalized free rigid body dynamics

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This talk is based on the collaborations [12, 13] with Tudor Ratiu.
The free rigid body dynamics is one of the solvable problems in theoretical mechanics. Its complete integrability and the stability properties of the equilibria are well known. By the influence of the rapid development of the studies on infinite dimensional integrable systems, such as Korteweg-de Vries equation, the free rigid body dynamics has been generalized, first, to higher dimensional rotation group $SO(n)$ and, then, to arbitrary semi-simple Lie groups by Mishchenko and Fomenko [9, 10].

The goal of these researches is to show the complete integrability of the generalized free rigid body dynamics. However, it is very natural to investigate the stability of the equilibria from the viewpoint of dynamical systems theory. For the original free rigid body dynamics, the rotations around the long and the short axes of the inertia tensor are stable, while those around the middle axis are unstable. For the above generalized free rigid body dynamics, it is rather recent that the stability and analysis for the equilibria are performed. in the case of the $SO(4)$ free rigid body dynamics, the stability of a certain class of equilibria has been studied by Fehér-Marshall

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[7] and the complete stability analysis was carried out by Birtea-Cașu-Ratiu-Turhan [1]. For general $SO(n)$, the stability of a special family of equilibria was analysed by Spiegler [14] and, more recently, by Izosimov [8], who gives the complete stability analysis for generic isolated equilibria on the basis of the methods by Bolsinov-Oshemkov [6]. (See [5] for more sophisticated treatment.)

An important feature of the above generalized free rigid body dynamics is their bi-Hamiltonian structure [3, 4, 11]. Bolsinov and Oshemkov [6] developed a systematic method of dealing with the complete integrability and the non-degeneracy of the equilibria for Hamiltonian systems which admit bi-Hamiltonian structures. Once the equilibria are non-degenerate, we can show that the Birkhoff normal form of the Hamiltonian is obtained through convergent canonical transformation. From this fact, we can deduce the nonlinear (Lyapunov) stability of non-degenerate linear stable equilibria.

In this talk, analysed are the stability of the isolated equilibria for the Mishchenko-Fomenko generalized free rigid body on the real Lie groups whose Lie algebras are normal or compact real forms of some complex semi-simple Lie algebras. As the main results, it is shown that all the equilibria of the Mishchenko-Fomenko free rigid body dynamics on normal real form of complex semi-simple Lie algebras are hyperbolic and that all those on compact real form of complex semi-simple Lie algebras are elliptic. These are sharp contrasts compared to the ordinary free rigid body dynamics on $SO(3)$.

As applications, the stability of the isolated equilibria are analysed for a natural generalization of the free rigid body dynamics on the unitary group $U(n)$ and also for the Bloch-Iserles systems defined on the space of the symmetric matrices [2]. For the former systems, all the generic isolated equilibria are elliptic, whereas, for the latter systems, all the generic isolated equilibria are hyperbolic.

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Haantjes manifolds and integrable systems

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A general theory of finite-dimensional integrable systems is proposed, based on the geometry of Haantjes tensors [1]. Inspired by the very recent definition of Haantjes manifolds [2], we introduce the class of symplectic-Haantjes manifolds (or $\omega\mathcal{H}$ manifold) and the notion of Lenard-Haantjes chains, as a generalization of the famous Lenard-Magri chains. Then, we prove that, under mild assumptions, the existence of a Haantjes structure is equivalent to the Liouville-Arnold integrability of each Hamiltonian system belonging to a Lenard-Haantjes chain. Furthermore, we propose an approach to the separation of variables, related to the geometry of Haantjes manifolds. A special class of coordinates, called Darboux-Haantjes coordinates [3], will be constructed from the Haantjes structure associated with an integrable systems. They enable the additive separation of variables of the Hamilton-Jacobi equation. Finally, we present some applications of our approach to the study of some relevant systems introduced by F. Calogero.

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Closed form solution of the Heisenberg equation

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Let us consider the Heisenberg ferromagnet equation (HF)

$$\vec{m}_t = -\vec{m} \wedge \vec{m}_{xx}, \tag{1}$$

where $\vec{m}(z,t) \in \mathcal{R}^3$ is a vector function satisfying $\vec{m}(z,t) \to \vec{e}_3$ as $z \to \pm\infty$ and $\|\vec{m}(z,t)\| = \|\vec{e}_3\|$.

A formula for soliton solutions of the HF equation is obtained via the IST. In the reflectionless case the kernel $F(x+y;t)$ of the relevant Marchenko integral equation can be written in a factorized form by using a "suitable" triplet of constant matrices (A,B,C) and the matrix exponential. Since the kernel is separable in x and y, the corresponding Marchenko integral equation is explicitly solved using linear algebra, which yields exact solutions to the HF equation. The concise solution formula presented yields exact soliton solutions for special choices of the matrix triplet (A,B,C).

It is also well known (see [1, 2]) that the HF equation can be written as

$$M_t = \frac{1}{2i}[M, M_{xx}] \tag{2}$$

where $\vec{s} = (\sigma_1, \sigma_2, \sigma_3)$ (σ_i denotes the Pauli matrices) and $M = \vec{m} \cdot \vec{s}$. Furthermore, we have $M = M^\dagger$, $M^2 = I_2$ and $\mathrm{Tr}(M) = 0$.

It is possible to prove the existence of a one-to-one correspondence between hermitian solutions $M(x,t)$ of (2) satisfying

$$M(x,t)^2 = I_2$$

and the functions $V(x,t)$ with values in $SU(2)$ such that

$$M(x,t) = V(x,t)^{-1}\sigma_3 V(x,t). \tag{3}$$

Here $V(x,t)$ is the Jost solution obtained developing the IST for the nonlinear Schrödinger equation.

We use (3) to get explicit solutions of (2). The solutions found in this way are not, in general, soliton solutions. Some interesting example will be discussed.

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Quantum Calogero-Moser systems: a view from infinity

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The classical Calogero-Moser problems are known to have several quantum integrable versions, which are non-symmetric (so-called deformed quantum Calogero-Moser systems). The importance of the deformed Calogero-Moser systems became clear after the discovery of their deep relations with the theory of generalised discriminants, with the theory of simple Lie superalgebras as well as of an intriguing link with the theory of logarithmic Frobenius structures. Their integrability is not obvious at all and initially was proved by lengthy calculations.

I will explain how the integrability of the deformed Calogero-Moser systems can be easily "seen from infinity" by developing the corresponding Dunkl operator technique for the infinite number of particles. As a corollary a quantum Lax matrix and a simple construction of the integrals for the deformed Calogero-Moser systems will be presented.

The talk is based on the recent paper with A.N. Sergeev [1].

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The inverse scattering transform for the focusing nonlinear Schrödinger equation with a one-sided non-zero boundary condition

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We present the inverse scattering transform as a tool to solve the initial-value problem for the focusing nonlinear Schrödinger equation with one-sided non-zero boundary value $q_r(t) \equiv A_r e^{-2iA_r^2 t + i\theta_r}$, $A_r \geq 0$, $0 \leq \theta_r < 2\pi$, as $x \to +\infty$. The direct problem is shown to be well-defined for solutions $q(x,t)$ to the focusing nonlinear Schrödinger equation such that $[q(x,t) - q_r(t)\vartheta(x)] \in L^{1,1}(\mathbb{R})$ [$\vartheta(x)$ denotes the Heaviside function] with respect to $x \in \mathbb{R}$ for all $t \geq 0$, for which analyticity properties of eigenfunctions and scattering data are established. The inverse scattering problem is formulated both via (left and right) Marchenko integral equations and as a Riemann-Hilbert problem on a single sheet of the scattering variables $\lambda_r = \sqrt{k^2 + A_r^2}$, where k is the usual complex scattering parameter in the inverse scattering transform. Unlike the case of fully asymmetric boundary conditions [2] and similarly to the same-amplitude case dealt with in [1], the direct and inverse problems are also formulated in terms of a suitable uniformization variable that maps the two-sheeted Riemann surface for k into a single copy of the complex plane. The time evolution of the scattering coefficients is then derived, showing that, unlike the case of solutions with the same amplitude as $x \to \pm\infty$, here both reflection and transmission coefficients have a nontrivial (although explicit) time dependence. These results will be instrumental for the investigation of the long-time asymptotic behavior of physically relevant solutions to the focusing nonlinear Schrödinger equation with nontrivial boundary conditions, either via the nonlinear steepest descent method on the Riemann-Hilbert problem, or via matched asymptotic expansions on the Marchenko integral equations.

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General Nth order integrals of the motion in classical and quantum mechanics

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The general form of an integral of motion that is a polynomial of order N in the momenta is presented for a Hamiltonian system in two-dimensional Euclidean space. The classical and the quantum cases are treated separately, emphasizing both the similarities and the differences between the two. The main application will be to study Nth order superintegrable systems that allow separation of variables in the Hamilton-Jacobi and Schrodinger equations, respectively. So far the study of superintegrable systems with third and fourth order integrals has lead to quantum superintegrable systems with "exotic" potentials expressed in terms of Painlevé transcendents or solutions of 4th order ODEs with the Painlevé property. Higher values of N should lead to families of higher order ODEs and the corresponding higher Painlevé functions. This is joint work with Sarah Post.

The Inverse Spectral Transform for the Dunajski hierarchy and some of its reductions, I: Cauchy problem and longtime behavior of solutions

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In this research we apply the formal Inverse Spectral Transform for integrable dispersionless PDEs arising from the commutation condition of pairs of one-parameter families of vector fields, recently developed by S. V. Manakov and one of the authors, to one distinguished class of equations, the so-called Dunajski hierarchy. We concentrate, for concreteness, i) on the system of PDEs characterizing a general anti-self-dual conformal structure in neutral signature, ii) on its first commuting flow, and iii) on some of their basic and novel reductions. We formally solve their Cauchy problem and we use it to construct the longtime behavior of solutions, showing, in particular, that unlike the case of soliton PDEs, different dispersionless PDEs belonging to the same hierarchy of commuting flows evolve in time in very different ways, exhibiting either a smooth dynamics or a gradient catastrophe at finite time.

Non-periodic one-gap potential in quantum mechanics

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The spectral theory of the Schrodinger operator on the whole line is not developed properly so far. The existing theory is constituted completely for two classes of potentials: for periodic one and for the fast decaying in two directions. Little is known besides these two limiting cases. There are also known some classes of random potentials with pure discrete spectrum (Anderson localization). The other known non-periodic bounded potentials are n-zone potentials expressed in terms of θ-functions on Jacobians of hyperelliptic algebraic curves. They are quasiperiodic and have pure continuous spectrum.

We address the following question. Can one construct non-periodic (and non-quasiperiodic) bounded potentials such that the Schrodinger operator has pure continuous N-gap spectrum similar to the spectrum of periodic potentials? Answer is positive. We have constructed a broad class of such non-periodic potentials.

In this talk we discuss one-gap reflectionless potentials. The spectrum consists of one conductivity zone in the negative energy half-line and on the whole positive half-line. The potential is characterized by two positive Holder - α continuous functions (dressing functions) defined onside the conductivity zone. All these potentials are limits of N-solitonic solutions at $N \to \infty$. The spectrum in the general case is double-generated. Corresponding wave functions have simple analytic structure (two symmetric cuts) on the k-plane. Wave functions on these cuts obey some Riemann-Hilbert problem which is equivalent to a system of singular integral equations. These equations are uniquely resolved and can be efficiently solved numerically. The procedure of "dressing" is abundant: different dressings can generate the same potentials. The class of dressing functions leading to construction of periodic potentials is described explicitly.

The results of this work are supported by massive numerical calculations.

New mechanism for mass generation: Coupled linear wave equation and Sine-Gordon equation in (1+2) and (1+3) dimensions

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Coupling of the linear wave equation and the Sine-Gordon equation in (1+2) and (1+3) dimensions (SGE) offers a new mechanism for mass generation.

The SGE has single- and multi-front solutions, which are obtained through a Hirota algorithm (the extension of kink solutions in (1+1) dimensions). The multi-front solutions are divided into two unconnected subsets. One subset contains solutions, in which at least some clusters of fronts propagate at velocities that are equal to, or exceed the speed of light (c = 1). Some of the solutions may propagate rigidly at velocities that are equal to or exceed c. The solutions in this cluster may have a two-dimensional structure, or a three-dimensional structure (branes). The other subset contains solutions, which propagate rigidly at velocities that are lower than c. These latter solutions have a two-dimensional structure.

A functional of the solution of SGE, which naturally arises from the equation, vanishes identically when computed for any single-front solution. When applied to a slower-than-light multi-front solution, it generates positive definite structures that are localized around front junctions. These structures propagate together with the solution at its velocity (lower than c) and emulate spatially extended, free, massive relativistic particles. The profile of these structures (their "mass density", if interpreted as emulating particles) obeys the linear wave equation, to which a driving term generated from a slower-than-light, multi-front solution of the SGE is added. In itself, the wave equation generates solutions that represent massless particles. The SGE driving term enables the wave equation to admit a solution that is just the spatially localized structure. This result can be also formulated through the expansion in powers of a small coupling coefficient of the Euler-Lagrange equations of a Lagrangian system.

Two important characteristics of the multi-front solutions of the SGE play a crucial role in obtaining these results: First, that the parameters in the Hirota algorithm are viewed as tachyonic momentum vectors in Minkowski space, and second, that the solutions are invariant under Lorentz transformations.

2 Mini-workshops

2.1 MW1: From linear to nonlinear ODEs: Darboux transformations and exceptional orthogonal polynomials

Coordinator: David Gomez-Ullate Oteiza (Universidad Complutense de Madrid, Spain)

Review Lecture

An overview of exceptional orthogonal polynomials

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Exceptional orthogonal polynomials are dense families of orthogonal polynomials that satisfy a Sturm-Liouville problem. They differ from classical polynomials in that their degree sequence contains a finite number of gaps. Darboux transformations are intimately connected with the derivation of such families, and so is the notion of bispectrality and other tools that appear in the theory of integrable systems. In mathematical physics, these functions allow to write exact solutions to rational extensions of classical quantum potentials, which have trivial monodromy. From the point of view of special functions and orthogonal polynomials, they are polynomial systems formed by solutions to Fuchsian equations that belong to the Heine-Stieltjes class.

In this introductory talk, we will review their construction and main mathematical properties.

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Rational extensions of the trigonometric Darboux-Pöschl-Teller potential based on para-Jacobi polynomials

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The possibility for the Jacobi equation to admit in some cases general solutions that are polynomials has been recently highlighted by Calogero and Yi, who termed them para-Jacobi polynomials. Such polynomials are used here to build seed functions of a Darboux-Bäcklund transformation for the trigonometric Darboux-Pöschl-Teller potential. As a result, one-step regular rational extensions of the latter depending both on an integer index n and on a continuously varying parameter λ are constructed. For each value of n, the eigenstates of these extended potentials are associated with a novel family of λ-dependent polynomials, which are orthogonal on $(-1, 1)$.

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Zeros of exceptional Hermite polynomials

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Exceptional orthogonal polynomials were introduced by Gomez-Ullate, Kamran and Milson as polynomial eigenfunctions of second order differential equations with the remarkable property that some degrees are missing, i.e., there is not a polynomial for every degree. However, they do constitute a complete orthogonal system with respect to a weight function that is typically a rational modification of a classical (Hermite, Laguerre, Jacobi) weight function. For the case of exceptional Hermite polynomials these weights take the form $W(x)^{-2}e^{-x^2}$ where $W(x)$ is a Wronskian determinant constructed out of a finite number of Hermite polynomials. The cases of interest are when W has no zeros on the real line. It is known that in those cases most of the zeros of the exceptional Hermite polynomials are real. Our new result is that they asymptotically distribute themselves in the same way as the zeros of usual Hermite polynomials, and in addition, that the non-real zeros tend to the zeros of W as the degree tends to infinity.

This is joint work with Robert Milson (Dalhousie University).

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On the hypergeometric expressions of the exceptional Jacobi polynomials

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First we briefly review how to construct the exceptional orthogonal polynomials from the classical orthogonal polynomials. Then we discuss the hypergeometric expressions of the type 1 and type 2 exceptional Jacobi polynomials from the point view of the Darboux transformation. The q-analogue cases are also explored in detail.

Exceptional Orthogonal Polynomials and the Darboux transformation

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Exceptional orthogonal polynomials, so named because they span a non-standard polynomial flag, are defined as polynomial eigenfunctions of Sturm-Liouville problems. By allowing for the possibility that the resulting sequence of polynomial degrees admits a number of gaps, we extend the classical families of Hermite, Laguerre and Jacobi. A recent conjecture posits that every family of exceptional orthogonal polynomials is a multi-step Darboux transformation of a classical family. We will discuss the state of the conjecture and related questions.

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2.2 MW2: Representation theory, special functions and Painlevé equations

Coordinators: Nalini Joshi (University of Sydney, Australia) and Marta Mazzocco (Loughborough University, UK)

Review Lecture

Complex Painlevé Dynamics

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I will outline the study of complex dynamics of solutions of the continuous and discrete Painlevé equations in their space of initial values [4] in an asymptotic limit. This talk focusses on a geometric approach to describing their asymptotic properties. I will review results obtained so far for the Painlevé equations [1, 2, 3], before explaining extensions of this approach to discrete Painlevé equations. In particular, I will focus on finding properties of transcendental solutions that cannot be expressed in terms of earlier known functions.

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Cluster algebras and Stokes Phenomena

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It is well known that the Painelvé differential equations describe monodromy preserving deformations of an auxiliary linear systems of first order ODEs. In the case of the sixth Painlevé equation, such auxiliary linear system is a Fuchsian system with four simple poles, and the monodromy data associated to it belong to a two dimensional manifold that can be identified with the character variety of a Riemann sphere with 4 holes.

In the case of the other Painlevé equations the auxiliary linear system has non-simple poles and exhibits Stokes phenomenon. In this talk we answer the open problem of associating a generalised character variety to such systems. We will show that the Stokes phenomenon corresponds to the fact that some infinite directions need to be attached to the holes in the Riemann surface. We call this class of Riemann surfaces "Riemann surfaces with cusped boundaries". We show that these Riemann surfaces admit a complete lamination made of arcs which start and finish at cusps. We fully characterise the Poisson algebra satisfied by these arcs and show it to be the cluster algebra Poisson algebra.

Quantum character varieties, classical Painlevé equations and Liouville theory

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The Riemann-Hilbert correspondence is a map between the moduli space of flat logarithmic connections on punctured Riemann surfaces and the moduli space of representations of the corresponding fundamental groups. The simplest nontrivial example involves $SL(2, C)$-connections on the 4-punctured Riemann sphere. Monodromy preserving deformation of such connections is described by the Painlevé VI equation, and the appropriate space of monodromy data (classical character variety) consists of conjugacy classes of triples of $SL(2, C)$-matrices. Solving Painlevé VI amounts to constructing an explicit inverse of the Riemann-Hilbert map. I will explain how this problem can be solved using theoretical physics tools, namely conformal blocks of the Liouville theory and combinatorial formulas for instanton partition functions of supersymmetric gauge theories.

Liouville conformal blocks are matrix elements of products of chiral vertex operators intertwining irreducible representations of the Virasoro algebra. Conformal blocks containing level 2 degenerate insertions solve a quantum version of the isomonodromic Riemann-Hilbert problem: the relevant 2×2 monodromy matrices are operator-valued and involve translations of the intermediate momenta. When the Virasoro central charge is equal to 1, quantum monodromy contains a commutative subalgebra. Its action on spaces of degenerate conformal blocks may be diagonalized by Fourier transform which produces an ordinary $SL(2, C)$-valued monodromy. We thereby obtain an explicit solution of the *classical* Riemann-Hilbert problem on the sphere with an arbitrary number of punctures and the associated tau function (general solution of the Garnier system) as Fourier transforms of $c = 1$ Liouville conformal blocks.

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Geometric introduction to discrete Painlevé equations

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The theory of discrete Painlevé equations has made great progress in the last two decades. In this talk I will try to give a pedagogical introduction on this fascinating subject focusing on its geometric aspects [1, 2]. We will also touch upon some recent developments in Lax formulations, special solutions, quantization etc.

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2.3 MW3: Inverse scattering transform and Riemann-Hilbert problems

Coordinator: Gino Biondini (State University of New York at Buffalo, USA)

REVIEW LECTURE

Inverse scattering transform and Riemann-Hilbert problems: perspectives and open problems

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The inverse scattering transform (IST), introduced in 1967 by Gardner, Green, Kruskal and Miura to solve the initial-value problem for the Korteweg-deVries equation, continues to be a subject of intense study. Current areas of research include (but are not limited to) problems with non-trivial boundary conditions, boundary-value problems, asymptotics and semiclassical limits, multi-component systems and multi-dimensional problems. Considerable effort is also devoted to the study of Riemann-Hilbert problems, which in addition to being a key tool in the IST, have found applications to areas such as orthogonal polynomials and random matrix theory.

After a general introduction to the IST, I will give an overview of some current research directions and open problems. The last part of the talk will focus on a specific application of the IST: the study of the nonlinear stage of modulational instability (MI). I will first show how MI manifests itself within the context of the IST for the focusing nonlinear Schrodinger (NLS) equation. Then I will show how the nonlinear stage of MI can be characterized by computing the long-time asymptotics of solutions of NLS with initial conditions that are a small perturbation of a constant background.

Singular Limits for Integrable Equations

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Various singular limits (long-time behavior, small-dispersion or semiclassical limits) for linear constant-coefficient equations can be analyzed with great precision through the combination of Fourier or Green's function integral representations of solutions with classical methods of asymptotic expansions for integrals. For nonlinear integrable equations, the role of an explicit integral representation of solutions is instead played by a Riemann-Hilbert problem of analytic function theory. There is an analogue of the steepest descent method for the asymptotic expansion of integrals that applies to Riemann-Hilbert problems, originally invented by Percy Deift and Xin Zhou. The Deift-Zhou method allows nonlinear integrable problems to be analyzed in similar limits as classical methods allow for linear problems, with similar precision. Naturally new phenomena appear in the nonlinear setting.

This talk will review some of these ideas and then present some details in the context of recent work joint with R. Buckingham on critical phenomena that appear in solutions of the sine-Gordon equation in the semiclassical limit.

Large-Time Asymptotics for Completely Integrable PDE's in Two Space Dimensions

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This talk focusses on long-time asymptotics for the Davey-Stewartson II (DS II) equation, a completely integrable PDE in two space dimensions which describes the amplitude envelope of weakly nonlinear surface waves. In the defocussing case, we can use inverse scattering methods to compute the leading asymptotic behavior in terms of an associated linear problem. In the focussing case we can show spectral instability of soliton solutions, extending results of Gady'lshin and Kiselev.

Our results draw on the $\overline{\partial}$-method developed by Ablowitz-Fokas and Beals-Coifrman in the 1980's. As we will discuss, the $\overline{\partial}$-method is a two-dimensional counterpart of the Riemann-Hilbert method which has been applied with great success to the study of large-parameter asymptotics in one dimension. The long-term goal of the research to be discussed is to develop analogues techniques for the $\overline{\partial}$-problem in two dimensions.

In both the Riemann-Hilbert problem and $\overline{\partial}$-problems for completely integrable systems, space and time enter as parameters in an oscillatory phase. Computing large-time asymptotics of solutions requires a nonlinear analogue of the method of stationary phase. The Davey-Stewartson II equation provides the simplest example in two-dimensions of what such a method might look like. We will discuss prospects for attacking other two-dimensional dispersive equations, such as the KP II equation, by an extension of the methods used here.

Explicit Solutions to Integrable Evolution Equations

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A review is presented for constructing certain explicit solutions to integrable evolution equations. The solutions are expressed as formulas in a compact form in terms of a constant matrix triplet. The construction is based on solving the associated Marchenko integral equations explicitly by representing their kernels in terms of the constant matrix triplet, using matrix exponentials, and exploiting the separability of those kernels. The method is illustrated with some examples. The review is based on joint work with F. Demontis and C. van der Mee from University of Cagliari.

2.4 MW4: Rogue waves in integrable and non-integrable models

Coordinators: Miguel Onorato (Università di Torino, Italy) and Sara Lombardo (Northumbria University, UK)

REVIEW LECTURE

Rogue waves and soliton theory

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Rogue waves appear in different physical contexts such as fluid dynamics and optics. In addition integrable wave equations seem to reasonably model rogue waves by predicting their properties and behaviors. We give an introduction to the constructive methods of integrability as applied to models of interest, In particular we discuss the coupled nonlinear Schrödinger equations, the equations describing the resonant interaction of 3 waves and the two coupled equations which model the grating effect in optical Kerr media.

Hydrodynamics of Exact Nonlinear Schrödinger Equation Solutions - Theory and Experiments

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The dynamics of water waves in finite as well as in infinite water depth conditions can be approximated by the nonlinear Schrödinger equation (NLSE) [1, 2]. The particularity of the NLSE is it integrability [3]. In fact, it admits a family of stationary and pulsating solutions, also referred to as breathers solutions [4, 5, 6]. The latter describe different stages of the modulation instability process, therefore, the dynamics of extreme waves [7, 8]. The observation of breather solutions in different nonlinear dispersive media attracted the scientific interest recently [9, 10, 11]. Here, laboratory experiments, conducted in different wave facilities, on stationary and pulsating NLSE solutions are presented [12, 13]. Observed characteristic properties, related to NLSE localizations are highlighted. Furthermore, a range of applications [14] as well as model limitations [15] are emphasized and discussed in detail.

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A mechanism for rogue wave formation in deep water

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It is common for mid-ocean storm waves to reach seven meters in height, and in extreme conditions such waves can reach heights of fifteen meters. However, for centuries maritime folklore told of the existence of vastly more massive waves that could appear without warning in mid-ocean, against the prevailing current and wave direction, and often in perfectly clear weather. These waves are called rogue waves. A rogue wave is a highly localized phenomenon both in space and duration,most frequently occurring far out at sea. Historically oceanographers have discounted these reports as tall tales, i.e. the embellished stories of mariners with too much time at sea. But in the last years scientists have discovered strong evidence indicating that such massive rogue waves do exist and while the phenomenon has become the subject of recent scientific study, their origin still remains a mystery of the deep.

In this talk, a rogue wave formation mechanism is proposed within the framework of a coupled nonlinear Schrödinger (CNLS) system corresponding to the interaction of two waves propagating in oblique directions in deep water. A *rogue condition* is introduced that links the angle of interaction with the group velocities of these waves: different angles of interaction can result in a major enhancement of rogue events in both numbers and amplitude. For a range of interacting directions it is found that the CNLS system exhibits significantly more extreme wave amplitude events than its scalar counterpart. Furthermore, the rogue events of the coupled system are found to be well approximated by hyperbolic secant functions; they are vectorial soliton-type solutions of the CNLS system, typically not considered to be integrable. Overall, our results indicate that crossing states provide an important mechanism for the generation of rogue water wave events.

Baseband Modulation Instability as the Origin of Rogue Waves

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Extreme wave events, also referred to as rogue waves, are mostly known as oceanic phenomena responsible for a large number of maritime disasters [1, 2]. These waves have height and steepness much greater than expected from the sea average state [3]: not only appear in oceans, but also in the atmosphere, in optics, in plasmas, in superfluids, in Bose-Einstein condensates and as capillary waves. The common features and differences among freak wave manifestations in their different contexts is a subject of intense discussion [4].

A formal mathematical description of a rogue wave is provided by the so-called Peregrine soliton [5]. This solitary wave is a solution of the scalar Nonlinear Schrödinger Equation (NLSE) with the property of being localized in both coordinates. For several systems the standard focusing NLSE turns out to be an oversimplified description: this fact pushes the research to move beyond this model. Rogue-wave families have been recently found as solutions of the vector NLSE (VNLSE) [6], the three-wave resonant interaction equations [7], the coupled Hirota equations, and the long-wave-short-wave resonance [8].

As far as rogue waves excitation is concerned, it is generally recognized that modulation instability (MI) is among the several mechanisms which may lead to rogue wave excitation. Nevertheless, the conditions under which MI may produce an extreme wave event are not fully understood. A rogue wave may be the result of MI, but conversely not every kind of MI necessarily leads to rogue-wave generation [6].

Here we will show that the condition for the existence of rogue wave solutions in different nonlinear wave models, which are commonly used both in optics and hydrodynamics, coincides with the condition of baseband MI. Namely, rogue waves exist if and only if the MI spectrum contains the zero-frequency perturbation as a limiting case. We shall consider here a generalized NLSE equation (the Fokas-Lenells equation), the defocusing VNLSE and the long-wave-short-wave resonance. As we shall see, in the baseband-MI regime multiple rogue waves can be excited. Conversely, in the pass-band regime, MI only leads to the birth of nonlinear oscillations.

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Nonlinear interaction between Rogue waves

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In the experiments on multi-filamentation two basic trends are observed. The first one is connected with reduction of the number of filaments as a function of the distance [1]. The second trend is observed recently in [2, 3] as mergers between tree or four filaments in one (Rogue) wave. We propose a nonlinear vector model, where the role of cross-phase modulation (CPM) and degenerate four-photon parametric mixing (FPPM) to these processes is investigated. The system non-paraxial equations of the amplitudes of two-component electrical field at one carrying frequency has the form

$$-2i\frac{k_0}{v_{gr}}\frac{\partial A_i}{\partial t} = \Delta_\perp A_i - \frac{\beta+1}{v_{gr}}\left(\frac{\partial^2 A_i}{\partial t^2} - 2v_{gr}\frac{\partial^2 A_i}{\partial t \partial z}\right) - \beta\frac{\partial^2 A_i}{\partial z^2}$$
$$+k_0^2\tilde{n}_2\left[\left(A_i^2 + A_j^2\right)A_i \exp\left(2ik_0\left(z-\left(v_{ph}-v_{gr}\right)t\right)\right) + \left(|A_i|^2 + \frac{2}{3}|A_j|^2\right)A_i + \frac{1}{3}A_i^* A_j^2\right] \qquad (1)$$

where $\{i,j\} = \{x,y\}, i \neq j$, $\tilde{n}_2 = \frac{3}{8}n_2$ is the nonlinear refractive index, v_{gr} and v_{ph} are the group and phase velocities correspondingly, $\beta = k_0 v_{gr}^2 k''$ and k'' is the group velocity dispersion. Each of the pulses admits $\vec{x}$ and $\vec{y}$ components: $\vec{A}_k = A_{k,x}\vec{x} + A_{k,y}\vec{y},\ k = 1,2$. The collinear interaction between the optical pulses is investigated numerically in the following two cases: I) Fig. 1a) - the initial phase difference between the pulses is not equal to zero ($\Delta\varphi = \pi/4$). In this case, by FPPM, an intensive exchange of energy is observed. Thus, the reduction of the filaments is a result of the parametric processes; II) Fig. 1.b) - the initial phase difference between the pulses is equal to zero ($\Delta\varphi = 0$). An attraction and merging due to CPM mechanism are clearly seen. We perform additional numerical analysis with three, four and five pulses. These investigations give us confidence to claim, that the observed in [1-3] processes are results of nonlinear interactions due to FPPM and CPM mechanisms.

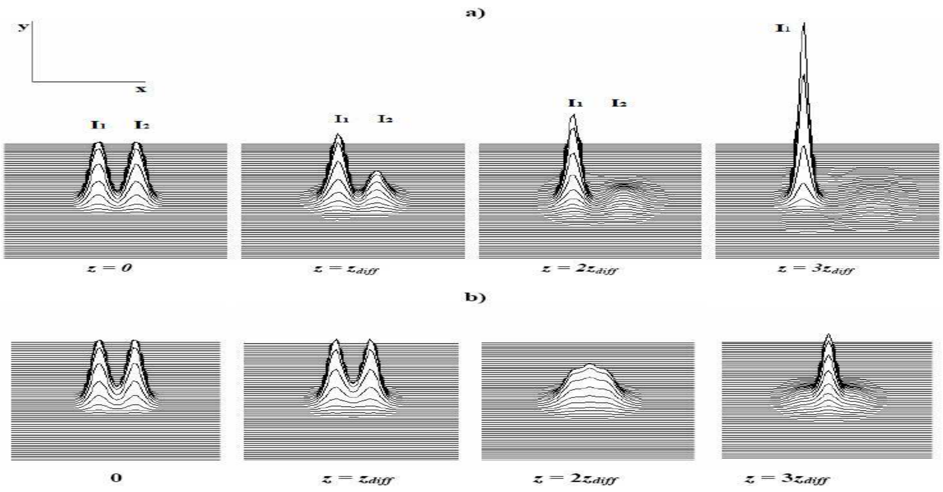

Figure 2: *a*) Energy exchange between two collinear filaments $\vec{A}_1$ and $\vec{A}_2$ with initial phase difference $\varphi = \pi/4$ governed by the system of equations (1). Due to degenerated FPPM process one of the filaments is amplified while the other filament enters in linear mode and vanishes. b) Attraction between two collinear filaments when the initial phase difference is $\varphi = 0$. We observe a merging between the pulses due to CPM.

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