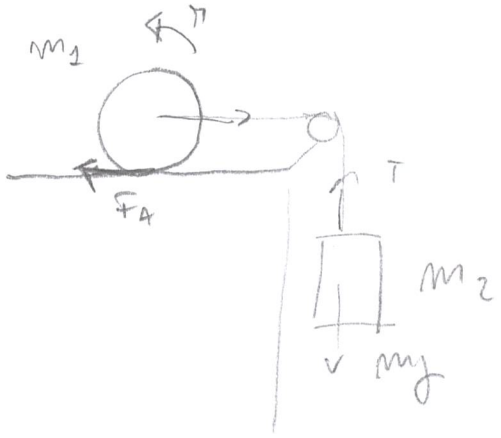
Es. 1



Equazioni del moto

$$\begin{cases} M - F_A R = I \frac{d\omega}{dt} = \frac{1}{2} m_1 R^2 \cdot \frac{a}{R} \\ F_a - T = m_1 a \\ T - m_2 g = m_2 a \end{cases}$$

Risolvendo:

$$F_a = (m_1 + m_2) a + m_2 g$$

$$M - \left[(m_1 + m_2) a + m_2 g \right] R = \frac{1}{2} m_1 a R$$

$$\frac{1}{2} m_1 a + (m_1 + m_2) a = -m_2 g + \frac{M}{R}$$

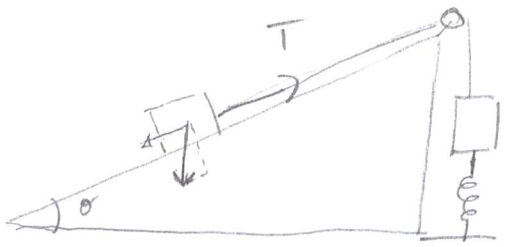
$$a = \frac{-m_2 g + \frac{M}{R}}{\frac{3}{2} m_1 + m_2} \approx 3{,}7 \quad m/s^2$$

$$F_a = (m_1 + m_2) a + m_2 g = 49{,}2 \ N$$

$$T = F_a - m_1 a = (m_1 + m_2) a + m_2 g - m_1 g = m_2 (a + g) = 27 \ N$$

$$F_A < \mu_s N = \mu_s m_1 g \quad \Rightarrow \quad \mu_s > \frac{F_A}{m_1 g} = 0{,}83$$

ES. 2



Equazioni del moto:

$$\begin{cases} m_1 g \sin\vartheta - T = 0 \\ T - m_2 g - k h_0 = 0 \end{cases}$$

$$-k h_0 - m_2 g - m_1 g \sin\vartheta = 0$$

$$h_0 = \frac{1}{k}\left(m_1 \sin\vartheta - m_2\right) g \simeq 7{,}35\,m.$$

$$\begin{cases} m_1 g \sin\vartheta - T = m_1 a \\ T - m_2 g - k h_1 = m_2 a \end{cases}$$

$$a = \frac{\left(m_1 \sin\vartheta - m_2\right) g - k h_1}{(m_1 + m_2)} = 1{,}8\ m/s^2$$

NB: H = altezza del corpo 1

$$m_1 g H_i = m_2 g H_F + \frac{1}{2} k h_F^2 + \frac{1}{2} m_1 V_F^2 + \frac{1}{2} m_2 V_F^2$$

$$m_1 g \left(H_i - H_F\right) = \frac{1}{2}\left(k h_F^2 + m_1 V_F^2 + m_2 V_F^2\right)$$

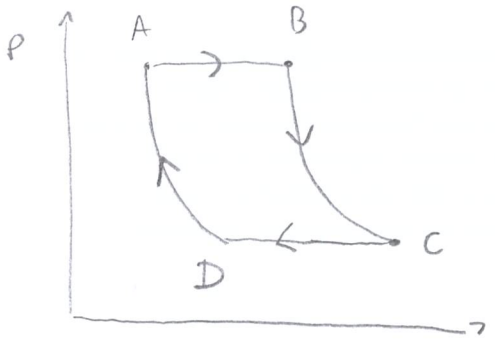
NB: $\left(H_i - H_F\right) = h_F \sin\vartheta$

$$m_1 g\, h_F \sin\vartheta = \frac{1}{2}\left(k h_F^2 + m_1 v_F^2 + m_2 v_F^2\right)$$

$$v_F^2 = \frac{2(m_1 \sin\vartheta - m_2) g h_F - k h_F^2}{m_1 + m_2}$$

$$v_F = \sqrt{\frac{2(m_1 \sin\vartheta - m_2) g h_F - k h_F^2}{m_1 + m_2}} \approx 3{,}10 \text{ m/s}$$

ES. 3



$V_B = 2 V_A$

$V_C = 2 V_B$

$P_A = 1 \cdot 10^6$ Pa

$V_A = 0,05$ m^3

$$T_A = \frac{P_A V_A}{nR} = 601,7 \text{ K}$$

$P_B = P_A$ $\quad V_B = 2V_A = 0,1$ m^3 $\quad T_B = \frac{P_B V_B}{nR} = 1203,4 \text{ K} = 2T_A$

$$P_C = \frac{nRT_C}{V_C} = \frac{nRT_B}{2V_B} = \frac{2nRT_A}{4V_A} = \frac{P_A}{2} = 5 \cdot 10^5 \text{ Pa}$$

$V_C = 2V_B = 4V_A = 0,2$ m^3

$T_C = T_B = 1203,4$ K

$$P_D = P_C = \frac{P_A}{2} = 5 \cdot 10^5 \text{ Pa}$$

$$V_D = \frac{nRT_D}{P_D} = 2V_A = 0,1 \text{ m}^3$$

$T_D = T_A = 601,7$ K.

- Calore e lavoro scambiato:

$$Q_{AB} = nc_p (T_B - T_A) = \frac{5}{2} R n T_A = 1,25 \cdot 10^5 \text{ J}$$

$$L_{AB} = P_A (V_B - V_A) = nRT_A = 5 \cdot 10^4 \text{ J}$$

$$Q_{BC} = L_{BC} = nRT_B \ln \frac{V_C}{V_B} = 2nRT_A \ln 2 = 6,9 \cdot 10^4 \text{ J}$$

$$Q_{CD} = nc_p (T_D - T_C) = -\frac{5}{2} R n T_A = -1,25 \cdot 10^5 \text{ J}$$

$$L_{CD} = P_C (V_D - V_C) = -nRT_A = -5 \cdot 10^4 \text{ J}$$

$$Q_{DA} = L_{DA} = nRT_D \ln \frac{V_A}{V_D} = -nRT_A \ln 2 = 3{,}46 \cdot 10^4 \text{ J}$$

$$L_{TOT} = nRT_A + 2nRT_A \ln 2 - nRT_A - nRT_A \ln 2 = nRT_A \ln 2 = 3{,}46 \cdot 10^4 \text{ J}$$

$$Q_{ASS} = \frac{5}{2} nRT_A + 2nRT_A \ln 2 = nRT_A \left(\frac{5}{2} + 2\ln 2\right) = 1{,}94 \cdot 10^5 \text{ J}$$

$$\eta = \frac{nRT_A \ln 2}{nRT_A\left(\frac{5}{2} + 2\ln 2\right)} = \frac{\ln 2}{\frac{5}{2} + 2\ln 2} = 0{,}18$$

La variazione di entropia dell'universo è $\neq 0$ solo lungo DA (irrev.)

$$\Delta S_{gas} = \int \left(\frac{dQ}{T}\right)_{rev} = nR \ln \frac{V_A}{V_D} = -nR \ln 2 = -57{,}6 \text{ J/K}$$

$$\Delta S_{sorg} = \frac{\Delta Q}{T_A} = \frac{-L}{T_A} = \frac{-P_A(V_A - V_D)}{T_A} = \frac{P_A V_A}{T_A} = nR = 83{,}1 \text{ J/K}$$

$$\Delta S_{TOT} = \Delta S_{gas} + \Delta S_{sorg} = +25{,}5 \text{ J/K}$$