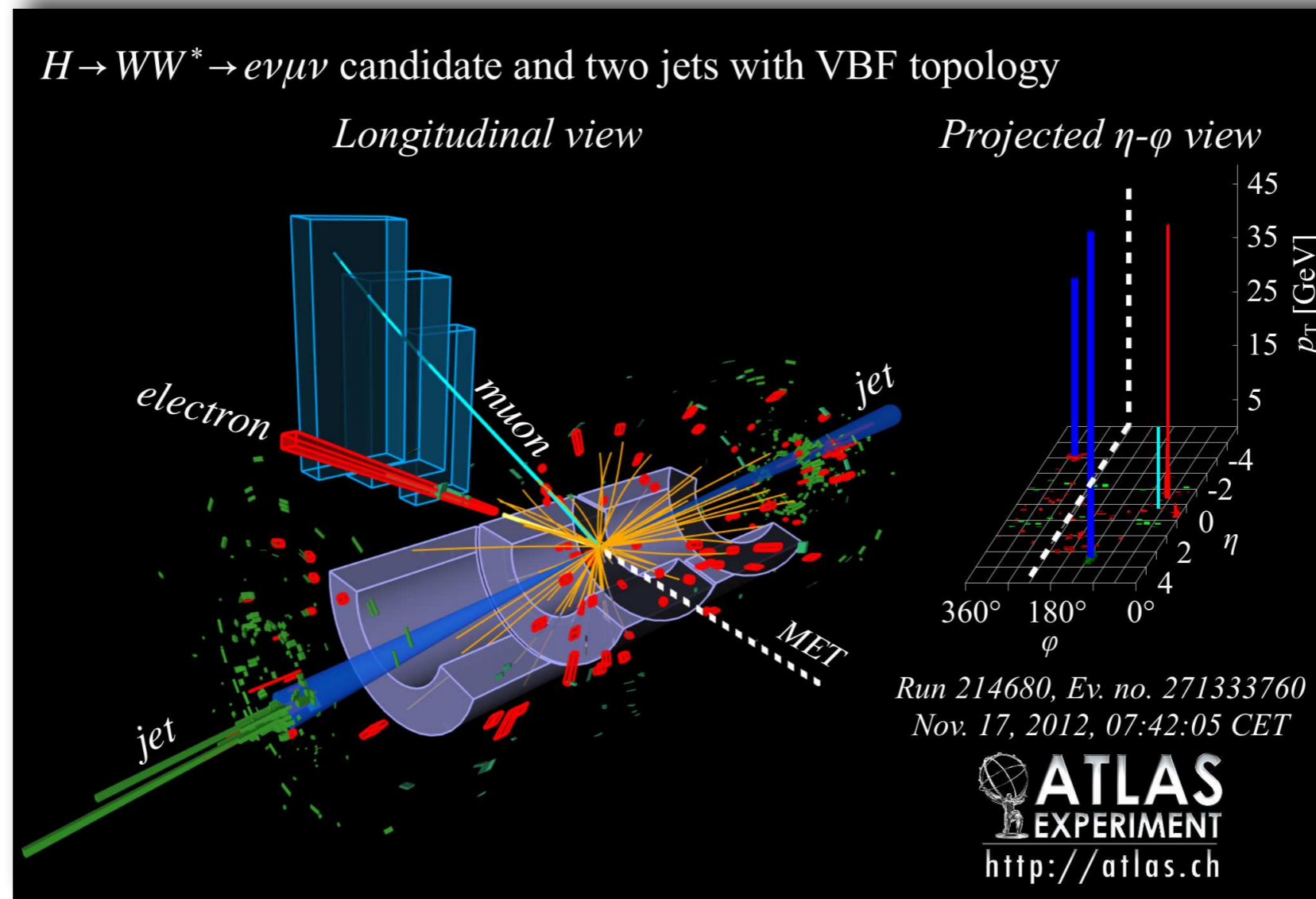


Monte Carlo Generators at colliders

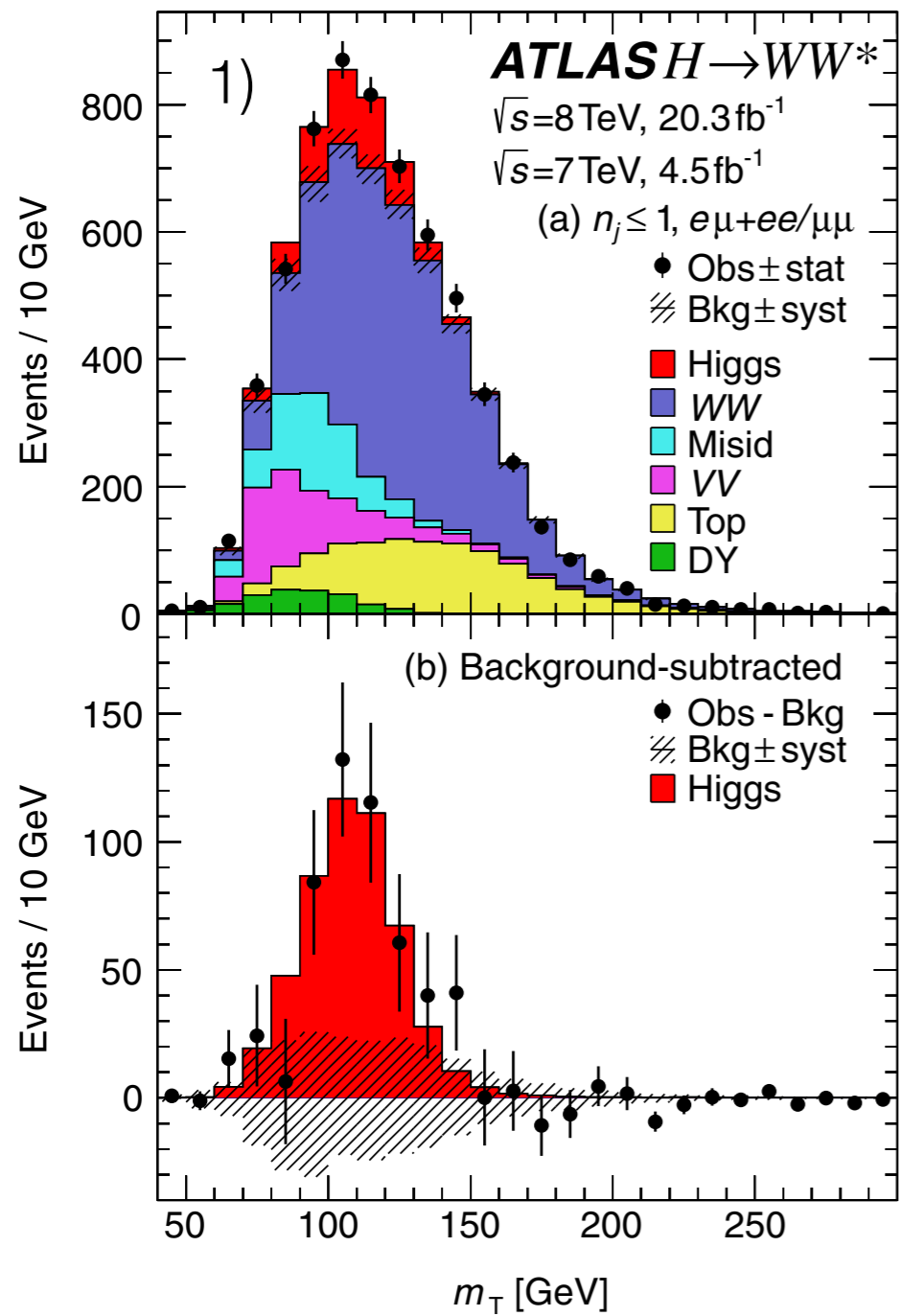
High energy physics simulation



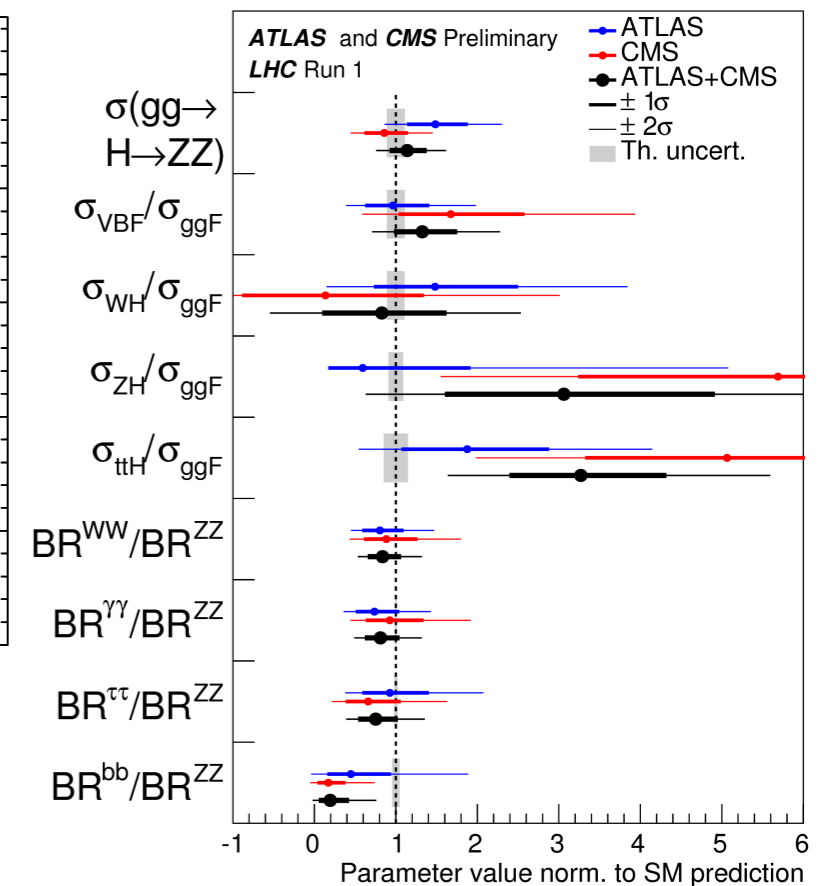
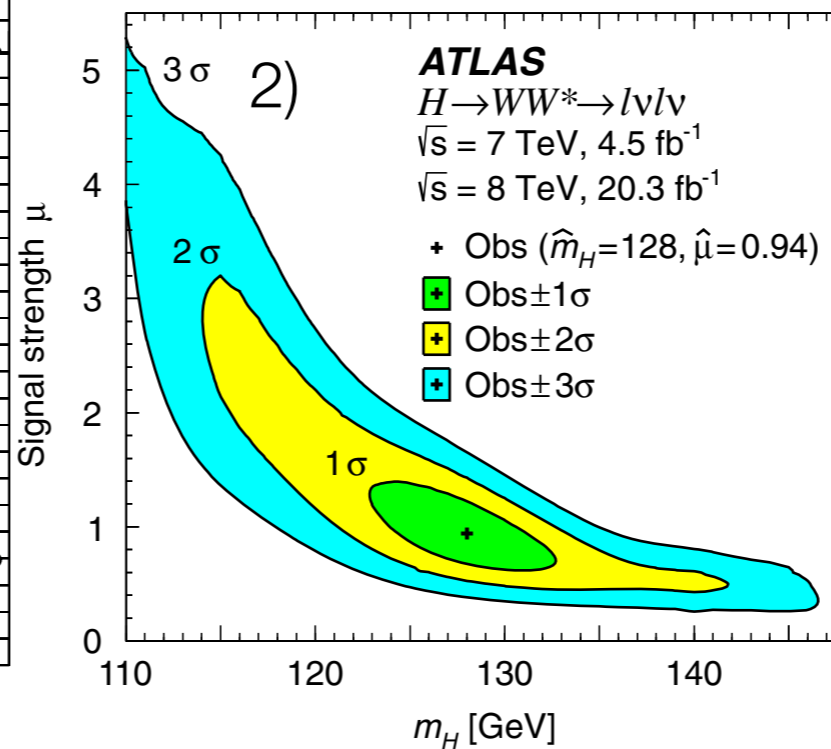
Acknowledgements

These slides have been prepared using heavily the material prepared by Christian Schultz Coulon and clarifying few points plus updating to recent developments in proton-proton physics.

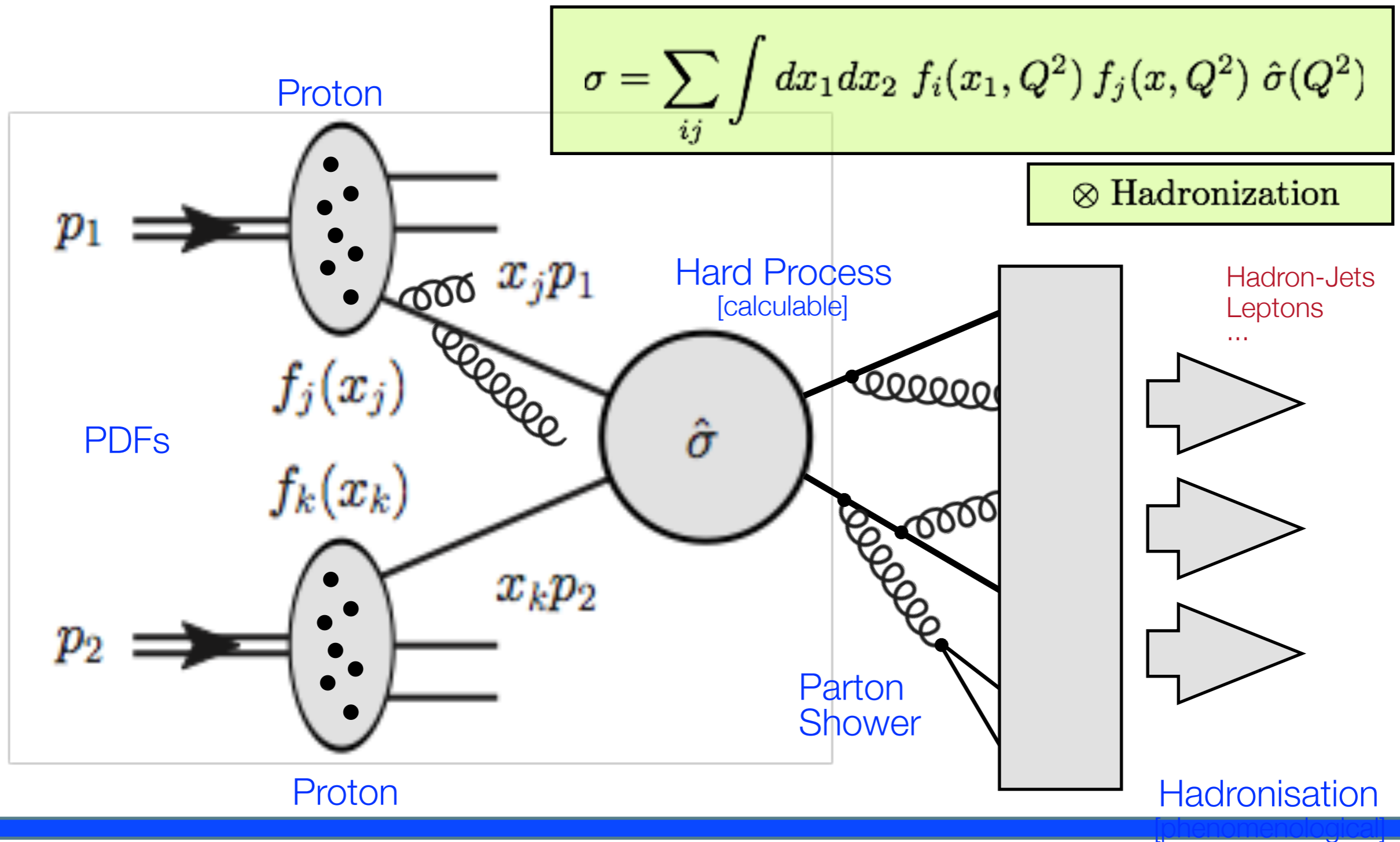
Why MC simulation?



- 1) to extract an interesting signal we need to subtract the expectation from known processes;
- 2) signal needs also to be modelled in order to compute detection efficiency and estimate production cross sections and couplings



The simulation chain



MC simulations in particle physics

How Monte Carlo simulation works

- Numerical process generation based on **random numbers**
- **Very powerful** method in particle physics

Event generation programs:

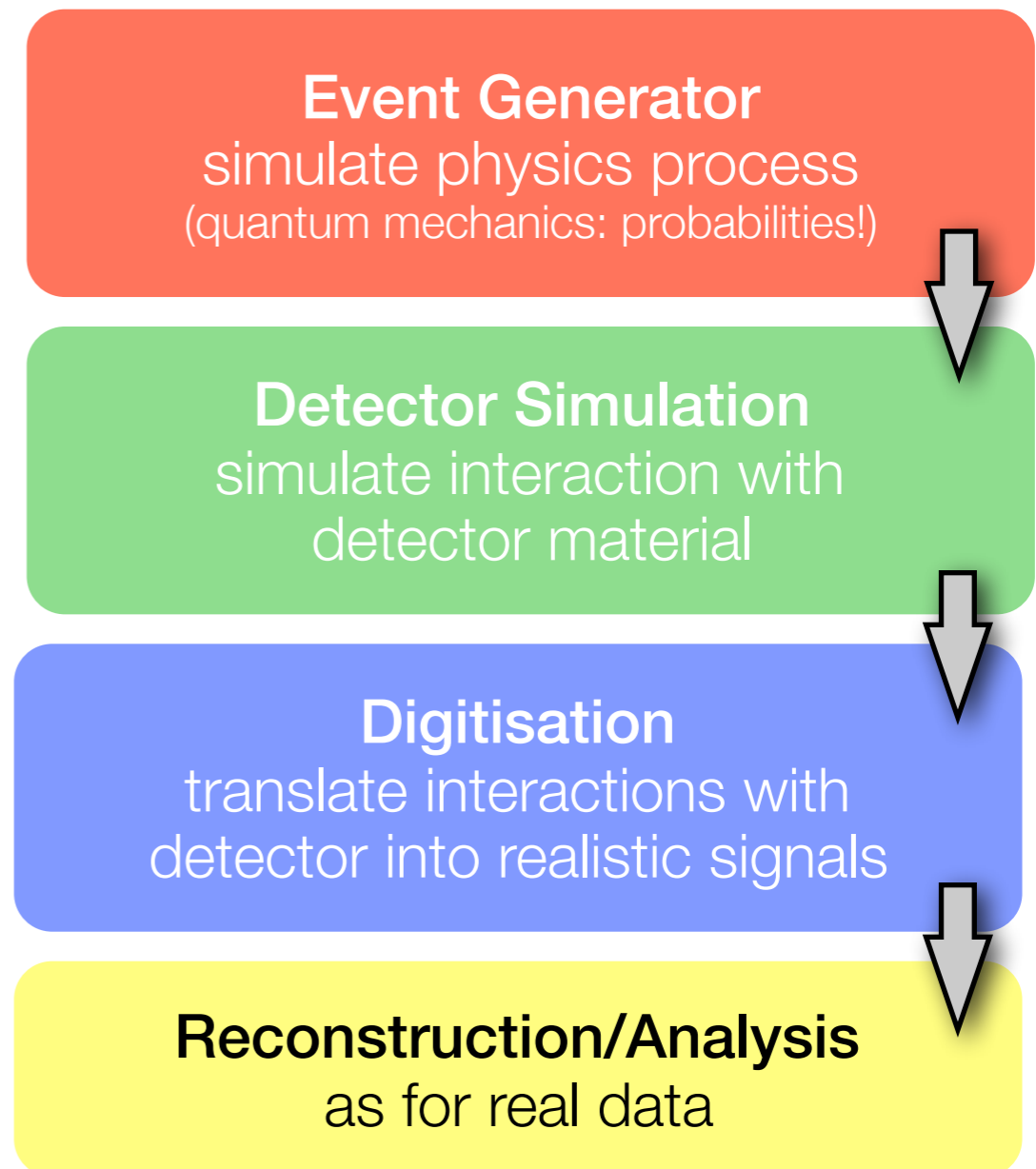
Pythia6, Pythia8, Herwig, Herwig7, Sherpa ...

**Hard partonic subprocess +
fragmentation and hadronisation ...**

Detector simulation:

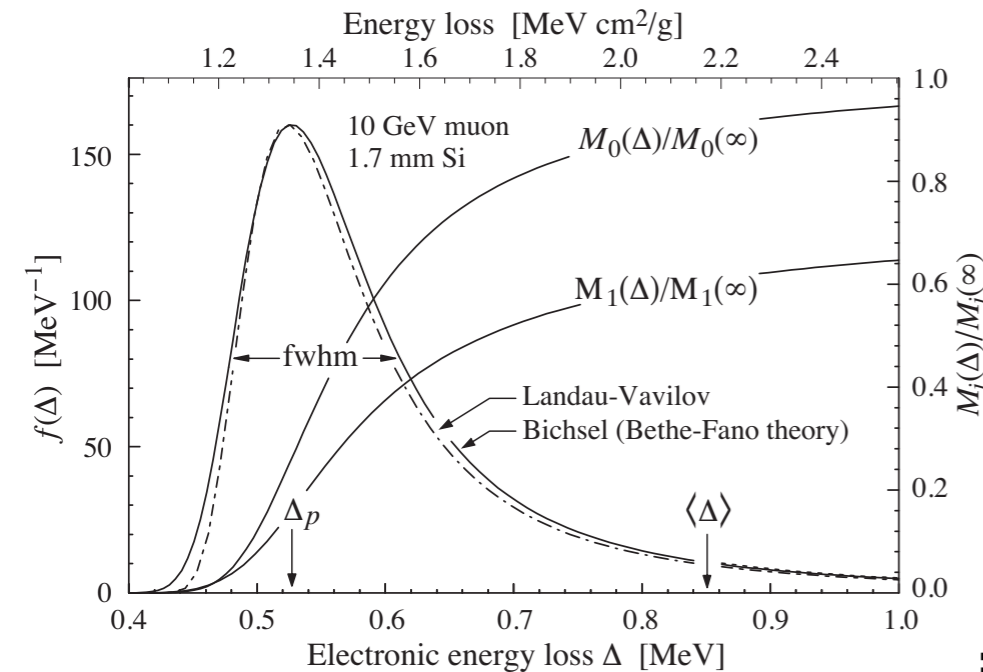
**Geant4
Fluka low energy hadron interactions...**

**interaction & response
of all produced particles ...**



Baseline of the simulation process

Typically, we need to generate a continuous variable following some distribution
 i.e. energy loss of a particle in a given material segment;
 angle of a photon in the h reference frame for the $h \rightarrow \gamma\gamma$ decay



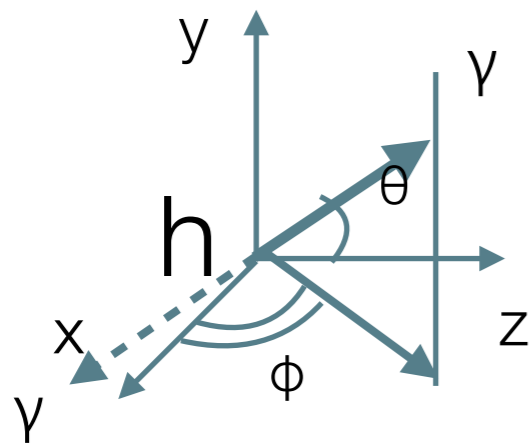
$$dP = f(x, ..)dx$$

↳ distribution formula
 probability to get an x_0 value between x and $x+dx$

if we want to simulate flat angular distributions, we can start from the azimuthal and polar angles

$$dP = f(\theta, \phi)d\theta d\phi = \text{sen}\theta d\theta d\phi$$

flat distribution in ϕ
 non flat in θ



Distribution function transformation properties

- 1) software libraries provide basic functions to produce flat distributed random numbers in the interval $[0,1]$ (ex. root TRandom3 class), they are typically fast and accurate uniform random numbers generators;
- 2) starting from uniform distributed random numbers, it is possible to generate numbers following any distribution using different techniques

$$dP_x = f(x)dx \quad y = g(x)$$
$$x \in [x_a, x_b]$$

$g(x)$ is a monotonic function of x
How “ y ” distributes in $[g(x_a), g(x_b)]$?

$$dP_y = h(y)dy = h(y)g'(x)dx$$

Because y is a monotonic function of x the probability to have y between $g(x)$ and $g(x+dx)$ is equal to the probability to have x between x and $x+dx$

$$h(y)g'(x) = f(x) \Rightarrow h(y) = \frac{f(x)}{g'(x)} = \frac{f(g^{-1}(y))}{g'(g^{-1}(y))}$$

Ex.: range map

$$[0, 1] \rightarrow [a, b] \quad y = (b - a)x + a$$

$$f(x) = 1 \quad g'(x) = b - a \quad h(y) = \frac{1}{b - a}$$

uniform

y is uniformly distributed in $[a,b]$

Distribution function transformation properties

Ex. 2: integration method:

$$y = g(x) = \frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' \quad g'(x) = \frac{f(x)}{\int_a^b f(x')dx'}$$

$$h(y) = \frac{f(x)}{g'(x)} = \frac{f[g^{-1}(y)]}{f[g^{-1}(y)]} \cdot \int_a^b f(x')dx' = \int_a^b f(x')dx'$$

y is uniformly distributed:

- 1) generate y flat in $[f_{\min}, f_{\max}]$;
- 2) compute $x = g^{-1}(y)$, x will be distributed in $g^{-1}(f_{\min}), g^{-1}(f_{\max})$

Finding $g^{-1}(y)$ is equivalent to solve the equation:

$$\frac{1}{\int_a^b f(x')dx'} \int_a^x f(x')dx' = y$$

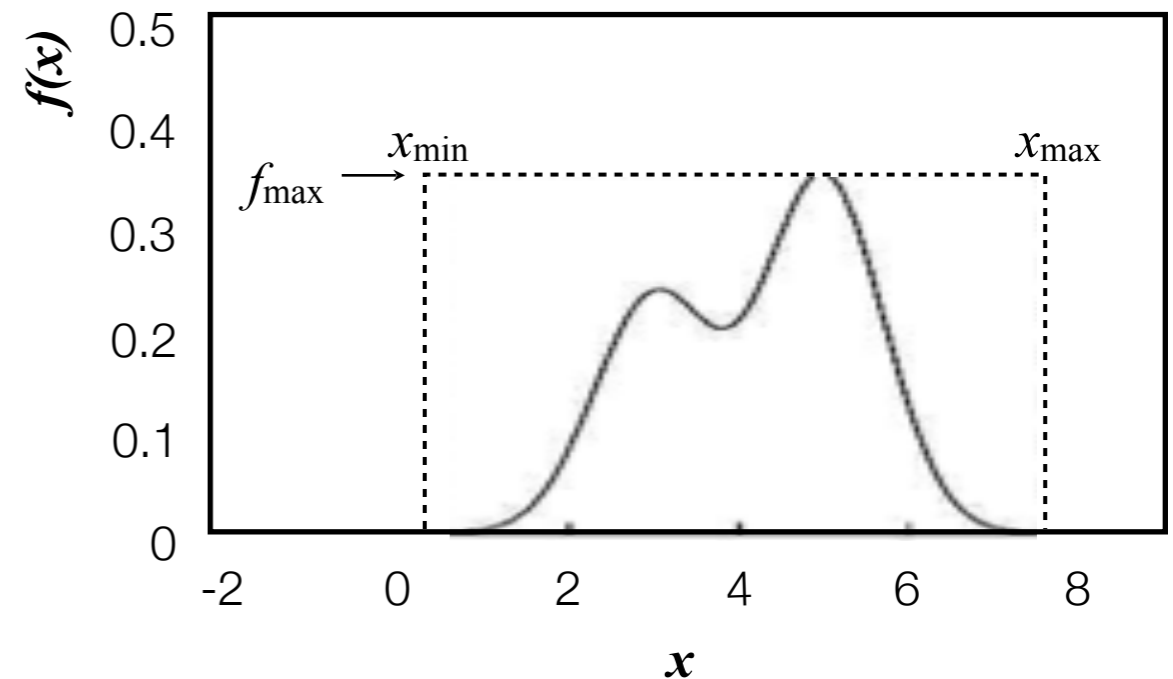
Hit or miss method.

- 1) generate x flat in x_{\min}, x_{\max}
- 2) generate y flat in $0, f_{\max}$
- 3) if $y < f(x)$ accept the event, otherwise ignore it

for a given x in $x, x+dx$ the fraction of accepted events is proportional to $f(x)dx \rightarrow dPx = f(x)dx$

1) advantages:

- can be used for all functions, even non continuous ...
- can be extended to N-dimension (generate x_1, x_2, \dots, x_n), y accept if $y < f(x_1, x_2, \dots, x_n)$



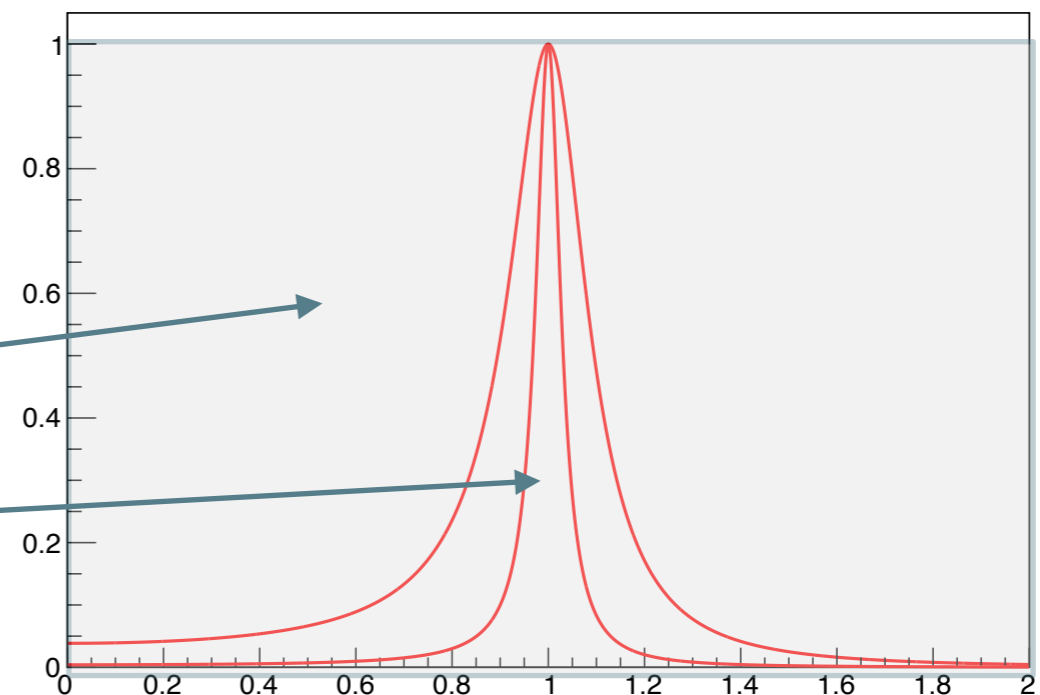
2) disadvantages

- can be extremely slow

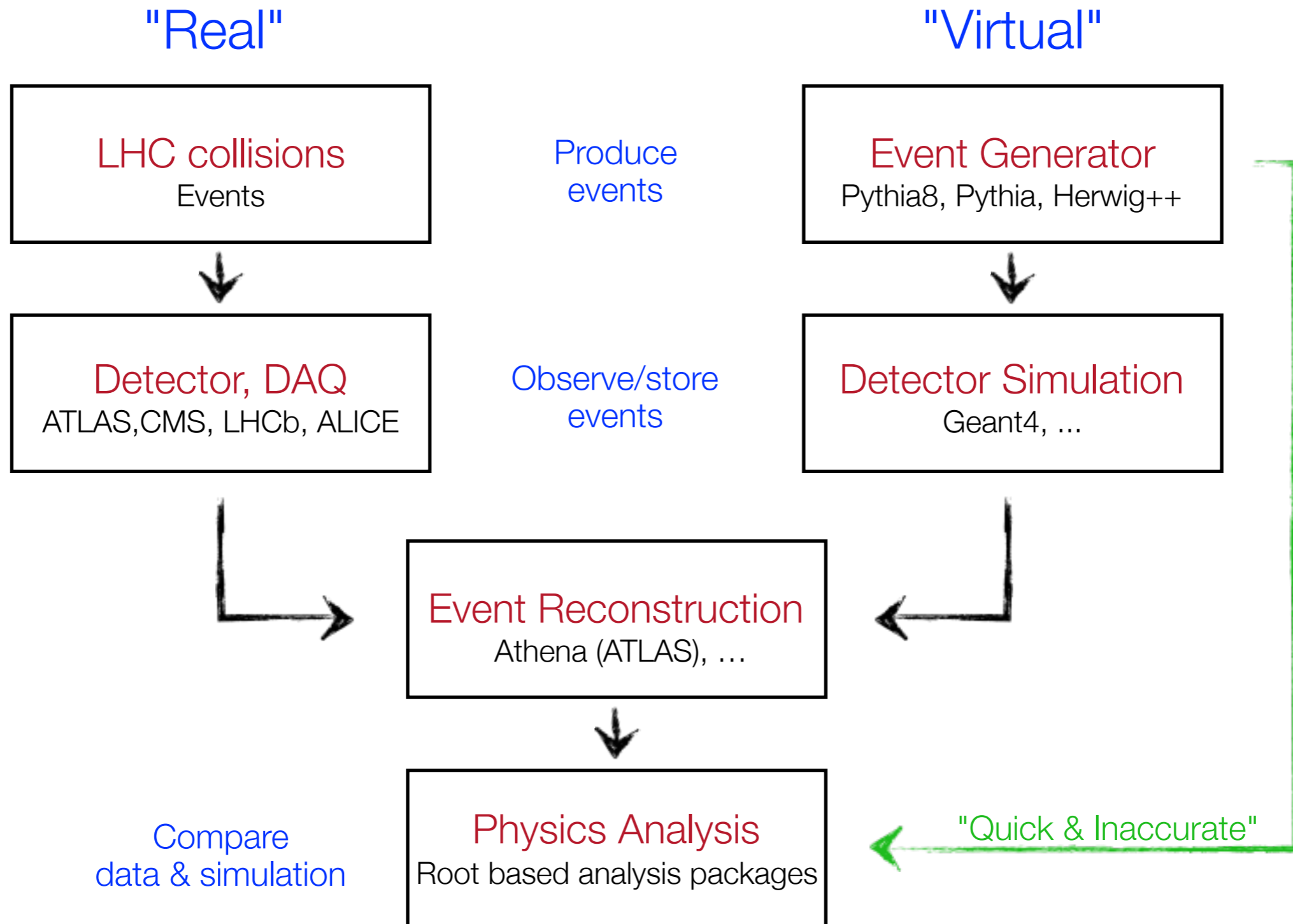
points generated uniformly in the square

points accepted only below the curve

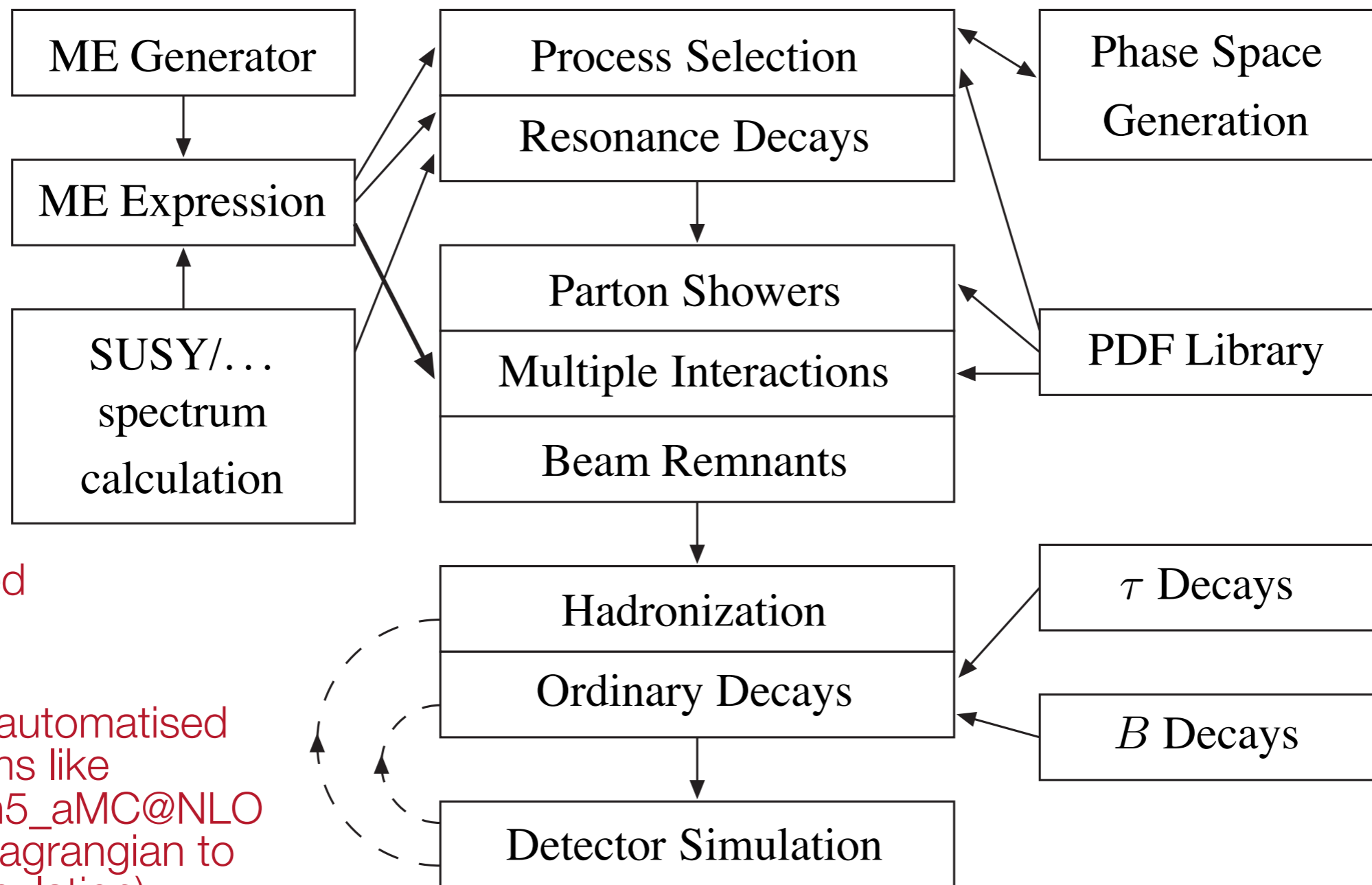
MC generators implement “smart” generation techniques to increase efficiencies



Comparison between real and simulated events



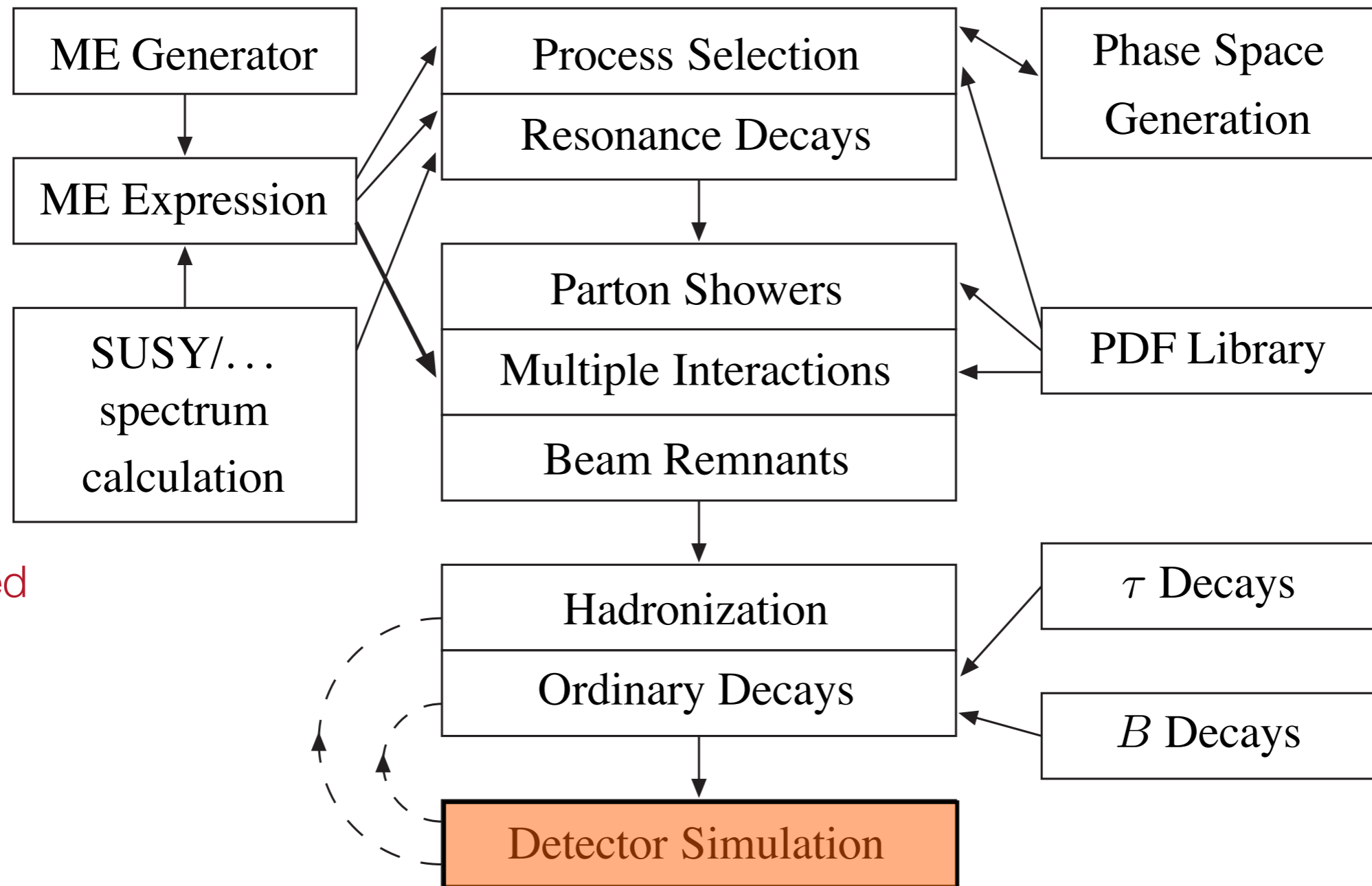
Simulation elements



Use specialised programs

Now fully automatised in programs like Madgraph5_aMC@NLO (from the lagrangian to the full simulation)

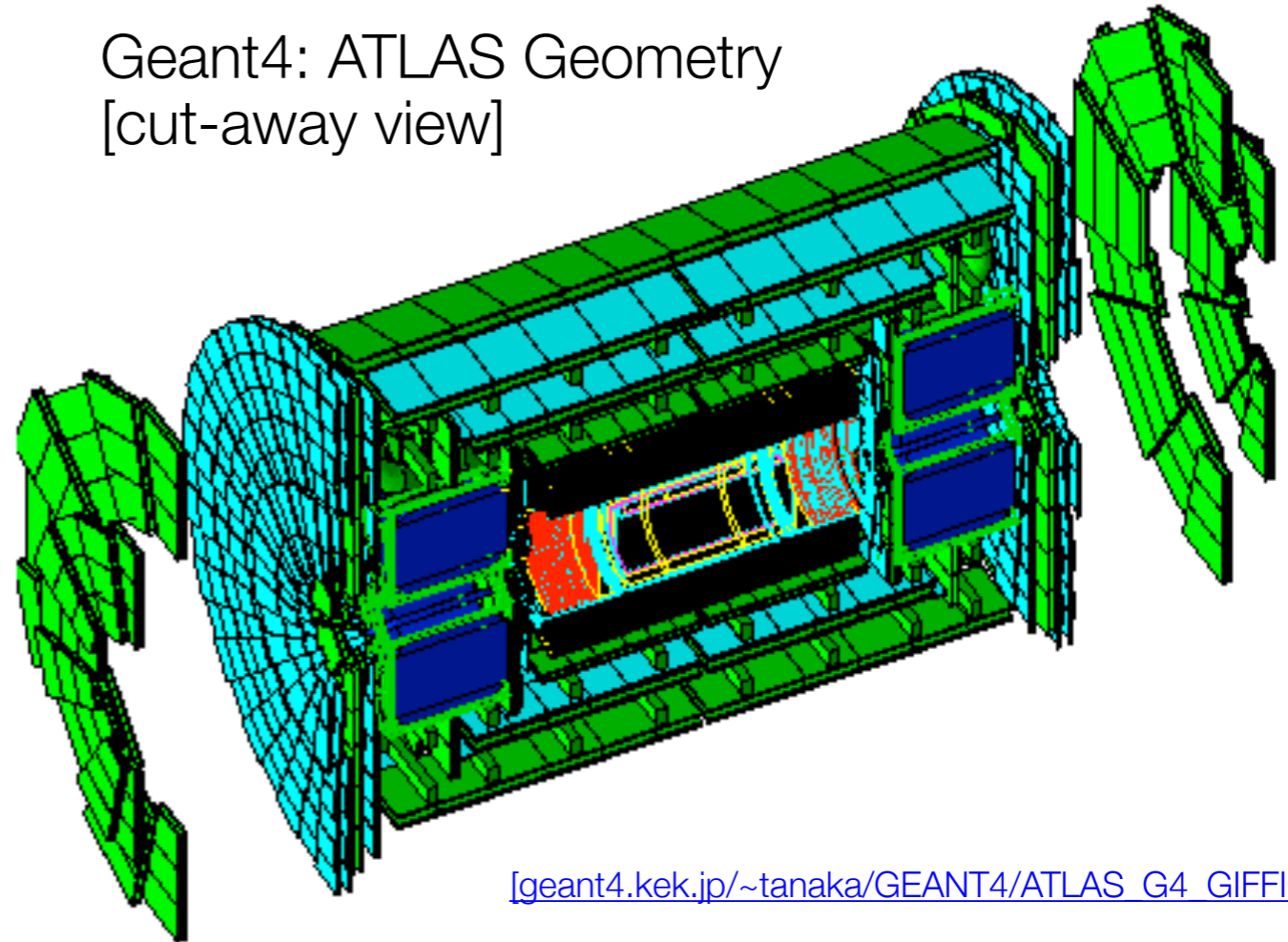
Simulation elements



Use specialised programs

GEANT Geometry And Tracking

Geant4: ATLAS Geometry
[cut-away view]



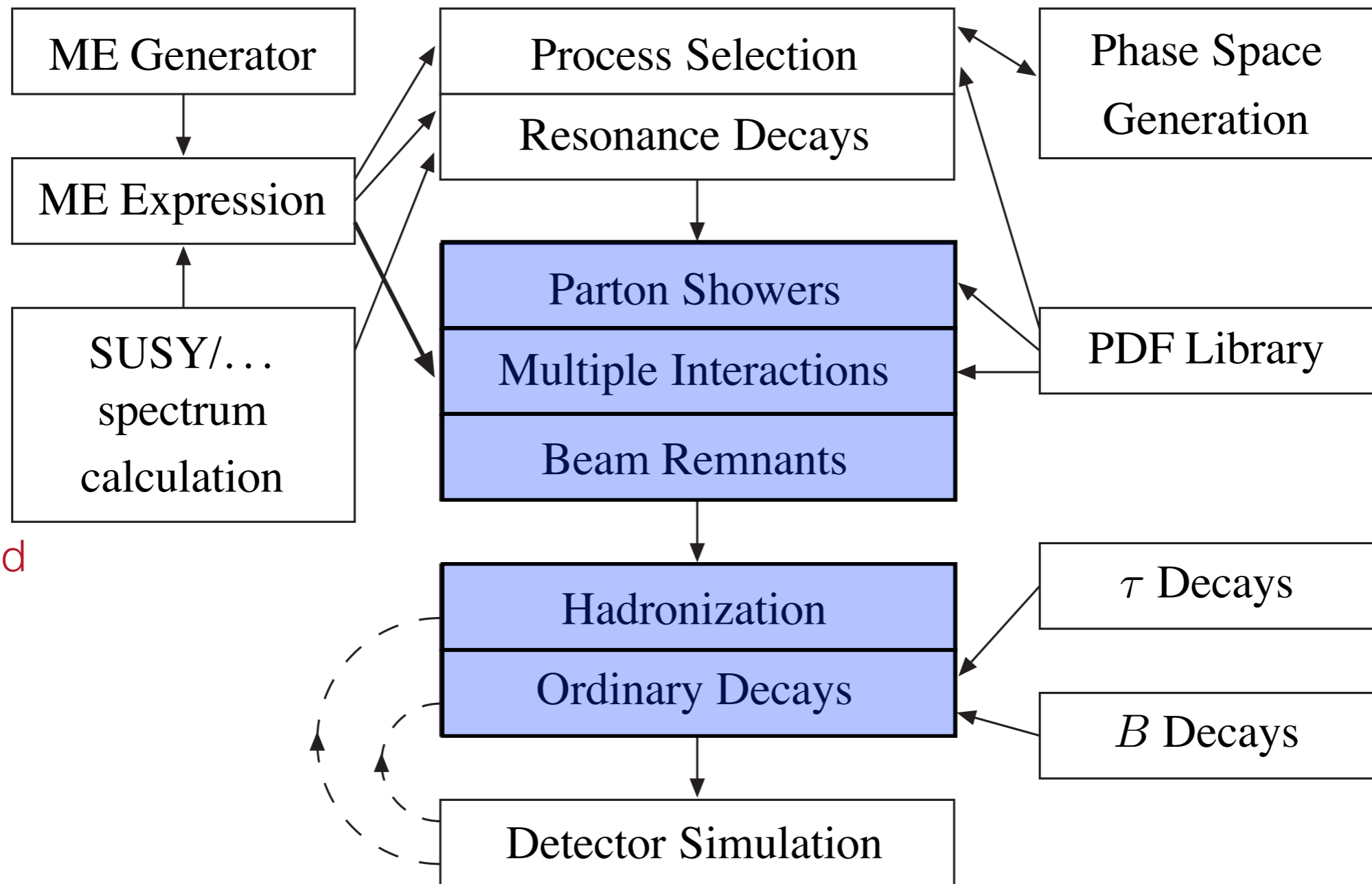
[\[geant4.kek.jp/~tanaka/GEANT4/ATLAS_G4_GIFFIG/\]](http://geant4.kek.jp/~tanaka/GEANT4/ATLAS_G4_GIFFIG/)

Detailed description of detector **geometry**
[sensitive & insensitive volumes]

Tracking of all particles through detector material ...

→ **Detector response**

Developed at CERN since 1974 (FORTRAN)
[Today: Geant4; programmed in C++]



Use specialised programs

Strong interactions:

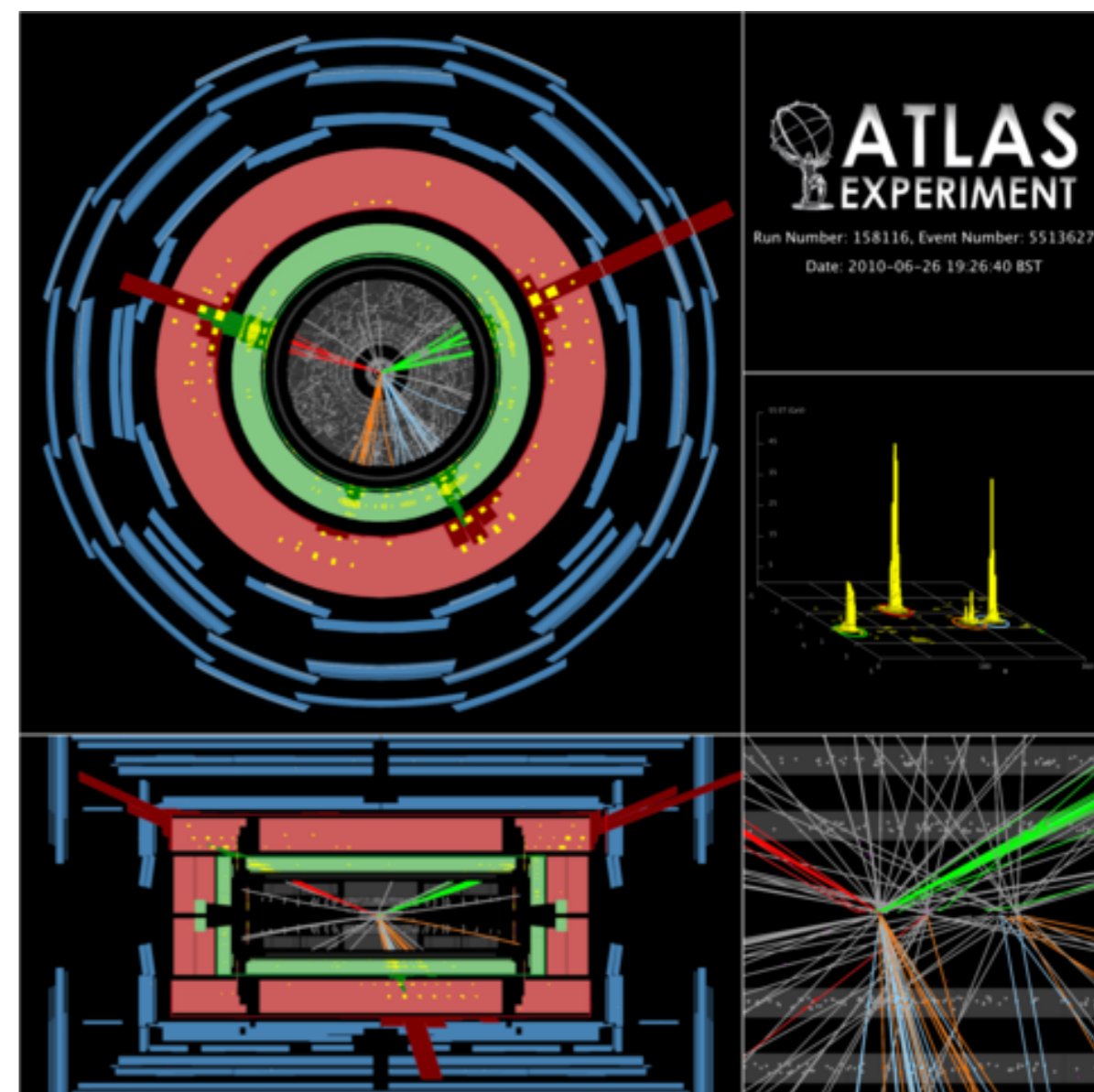
No free Quarks

Expect jets

i.e. bundles of particles at high energies
[hadron p_T range limited w.r.t. initial parton]

First observation of jets
in e^+e^- collisions @ $E_{CMS} > 6$ GeV
[SPEAR, SLAC, 1975]

Later also observed in
hadron-hadron collisions
[e.g. @ CERN ISR]



An event with 4 jets @ LHC

Goal: **Infer parton properties from jet properties**
[need to calculate and/or model fragmentation & hadronisation process]



Pure **matrix element (ME)** simulation:

MC integration of cross section & PDFs, no hadronisation
(recall: cross section = $|\text{matrix element}|^2 \otimes \text{phase space}$)

Useful for theoretical studies, no exclusive events generated

[Example: MCFM (<http://mcfm.fnal.gov>); many LHC processes up to NLO,
HNNLO (<http://theory.fi.infn.it/grazzini/codes.html>) Higgs production at NNLO]

Event generators:

Combination of ME and parton showers ...

Typical: generator for leading order ME
combined with leading log (LL) parton shower MC (see later)

Exclusive events → useful for experimentalists ...

Parton showers

A realistic simulation needs many particles in the final state, it is quite difficult (sometimes impossible) to compute a $pp(2) \rightarrow \text{many particles}$ process

$$(2 \rightarrow n) = \dots$$

$$\dots = (2 \rightarrow 2) \oplus \text{ISR} \oplus \text{FSR}$$

FSR: Final state radiation

$Q^2 \sim p_{\text{quark in}}^2 \sim m^2 > 0$ decreasing

[time-like shower]

$$p_{\text{quark in}} = p_{\text{quark out}} + p_{\text{gluon}}$$

ISR: Initial state radiation

$Q^2 \sim p_{\text{quark out}}^2 \sim -m^2 > 0$ increasing

[space-like shower]

$$p_{\text{quark out}} = p_{\text{quark in}} - p_{\text{gluon}}$$

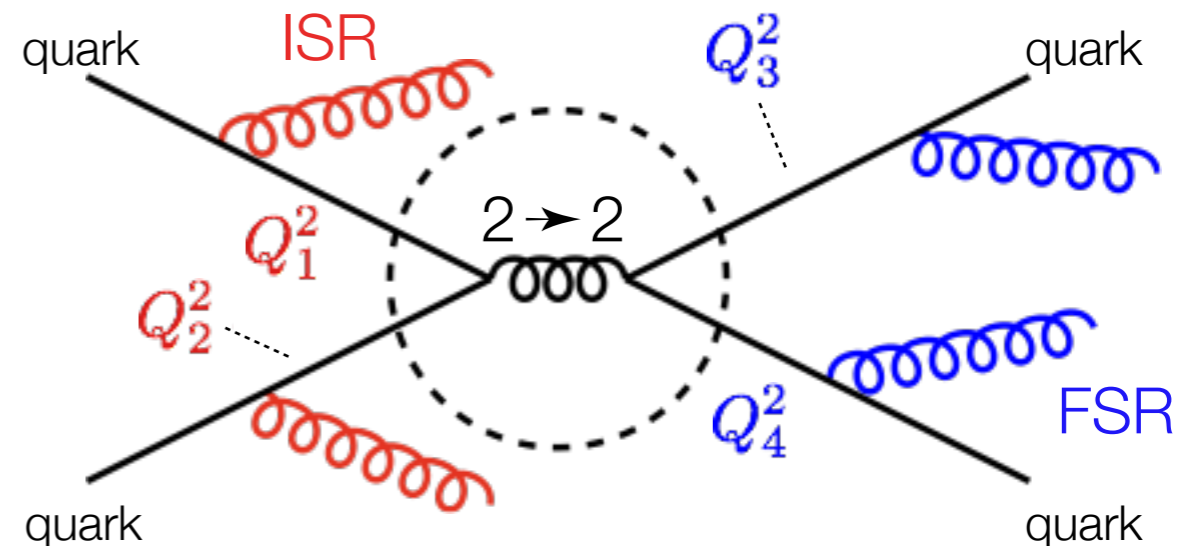
Hard process $[2 \rightarrow 2]$:

$$\sigma = \iiint dx_1 dx_2 d\hat{t} f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

Calculable

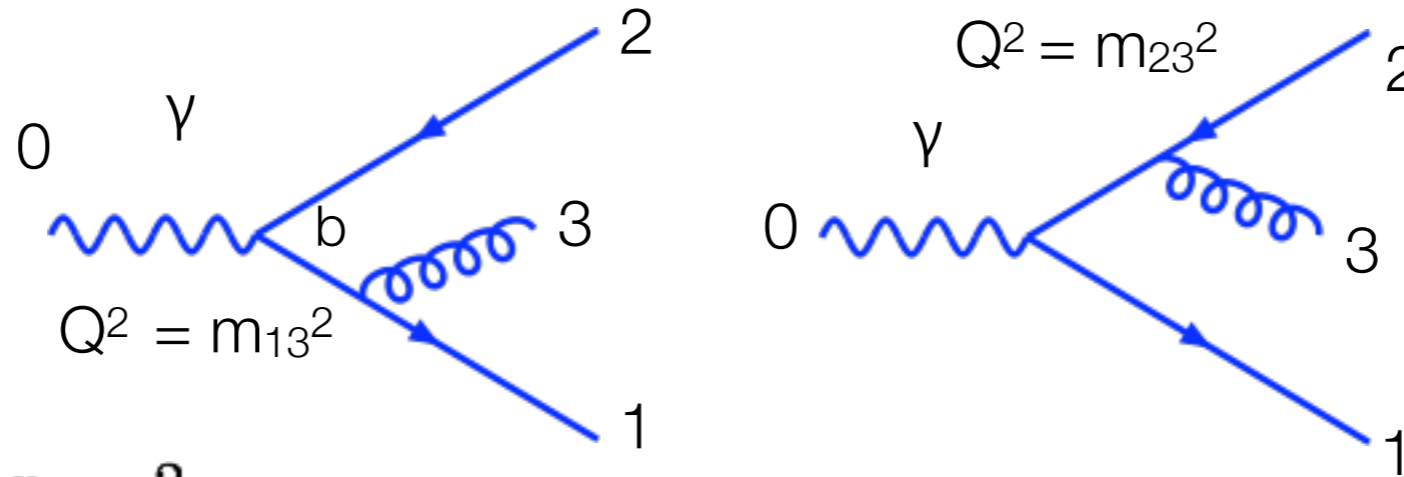
Shower evolution:

Viewed as probabilistic process, which occurs with unit total probability; cross section not directly affected; only indirectly via changed event shape.



Parton showers

$e^+e^- \rightarrow qqg$



$$x_i = \frac{2E_i}{E_{\text{cm}}} \quad x_1 + x_2 + x_3 = 2$$

Cross Section:
$$\frac{d\sigma_{\text{qqg}}}{dx_1 dx_2} = \frac{4}{3} \frac{\alpha_s}{2\pi} \cdot \sigma_0 \cdot \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Cross section has large contributions for $x_1, x_2 \rightarrow 1$

[$m_q = 0$; see e.g. Halzen/Martin]

from pt balance $1 - x_2 = \frac{m_{13}^2}{E_{\text{cm}}^2} = \frac{Q^2}{E_{\text{cm}}^2} \quad m_{13}^2 \sim 2E_1 E_2 (1 - \cos\theta) \quad x_2 \rightarrow 1 \Rightarrow m_{13}^2 \rightarrow 0 \Rightarrow \theta \rightarrow 0$ collinear limit

$$dx_2 = -\frac{dQ^2}{E_{\text{cm}}^2}$$

Rewrite for $x_2 \rightarrow 1$:
[qg collinear limit]

$$x_1 \approx z \quad dx_1 \approx dz$$

$$x_3 \approx 1 - z$$

$$E_q = E_1 = zE_b \quad E_g = E_3 = (1-z)E_b$$

$$d\mathcal{P} = \frac{d\sigma_{\text{qqg}}}{\sigma_0} = \frac{4}{3} \frac{\alpha_s}{2\pi} \cdot \frac{dx_2}{(1-x_2)} \cdot \frac{x_1^2 + x_2^2}{(1-x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \cdot \frac{dQ^2}{Q^2} \cdot \frac{4}{3} \left[\frac{1+z^2}{1-z} \right] dz$$

$z \rightarrow 1 \Rightarrow E_g \rightarrow 0$ soft divergence

$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz$$

Splitting probability determined by splitting functions $P_{q \rightarrow qg}$

Analogous splitting functions used in PDF evolution

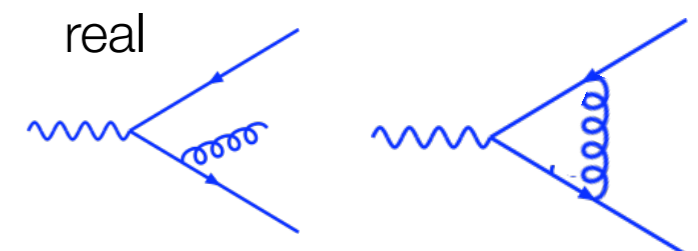
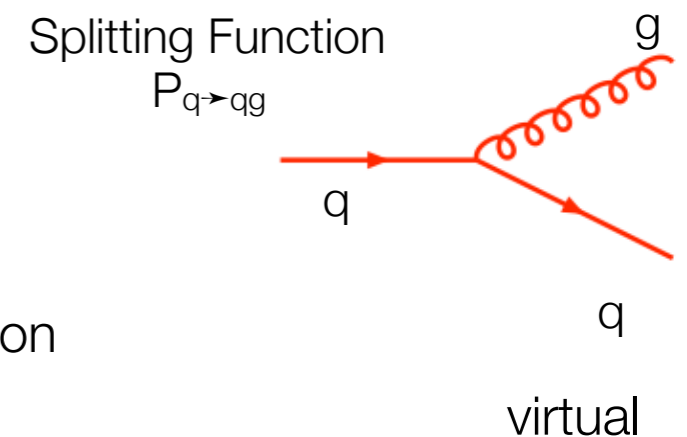
$$P_{q \rightarrow qg} = \frac{4}{3} \frac{1+z^2}{1-z}$$

$$P_{g \rightarrow gg} = 3 \frac{(1-z(1-z))^2}{z(1-z)}$$

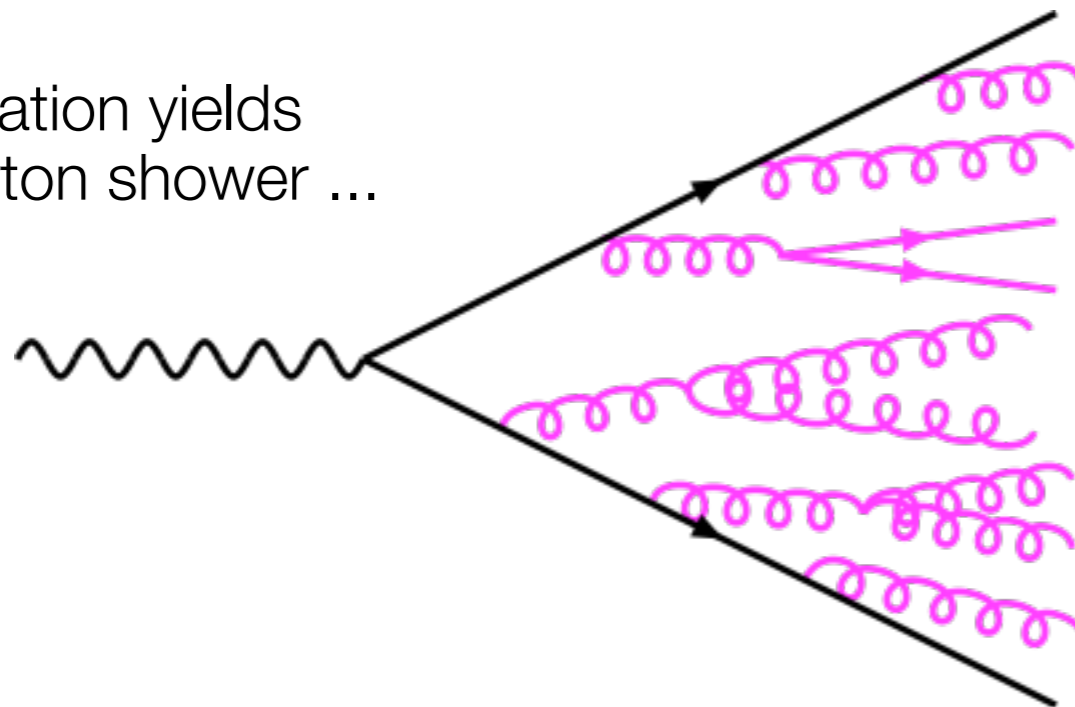
$$P_{g \rightarrow q\bar{q}} = \frac{n_f}{2} (z^2 + (1-z)^2)$$

z : fractional momentum of radiated parton
 n_f : number of quark flavours

In NLO calculations soft and collinear divergencies cancelled by virtual contributions: they persist in LO calculations.



Iteration yields parton shower ...



Need soft/collinear cut-offs to avoid non-perturbative regions ... [divergencies!]

Details model-dependent

e.g. $Q > m_0 = \min(m_{ij}) \approx 1 \text{ GeV}$,
 $z_{\min}(E, Q) < z < z_{\max}(E, Q)$ or
 $p_{\perp} > p_{\perp \min} \approx 0.5 \text{ GeV}$

Parton shower evolution 1

Conservation of total probability:

$$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$$

Time evolution:

$$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1) \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$$

$$\begin{aligned} \mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\ &= \lim_{n \rightarrow \infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})) \\ &= \exp \left(- \lim_{n \rightarrow \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \\ &= \exp \left(- \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \end{aligned}$$

$e^{-x} \approx 1 - x$
 [Taylor]

$$\rightarrow d\mathcal{P}_{\text{first}}(T) = d\mathcal{P}_{\text{something}}(T) \exp \left(- \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right)$$

Parton shower evolution 2

Instead of evolving to later and later times
need to evolve to smaller and smaller Q^2 ...

[Heisenberg: $Q \sim 1/t$]

Sudakov
Form Factor

$$d\mathcal{P}_{a \rightarrow bc} = \underbrace{\frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz}_{\text{Probability to radiated with virtuality } Q^2} \exp \left(\underbrace{- \sum_{b,c} \int_{Q^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz'}_{\text{No radiation for higher virtualities i.e. for } Q^2 \dots Q_{\max}^2} \right)$$

Probability to radiated
with virtuality Q^2

No radiation for higher
virtualities i.e. for $Q^2 \dots Q_{\max}^2$

Note that $\sum_{b,c} \iint d\mathcal{P}_{a \rightarrow bc} = 1 \dots$

[Convenient for Monte Carlo]

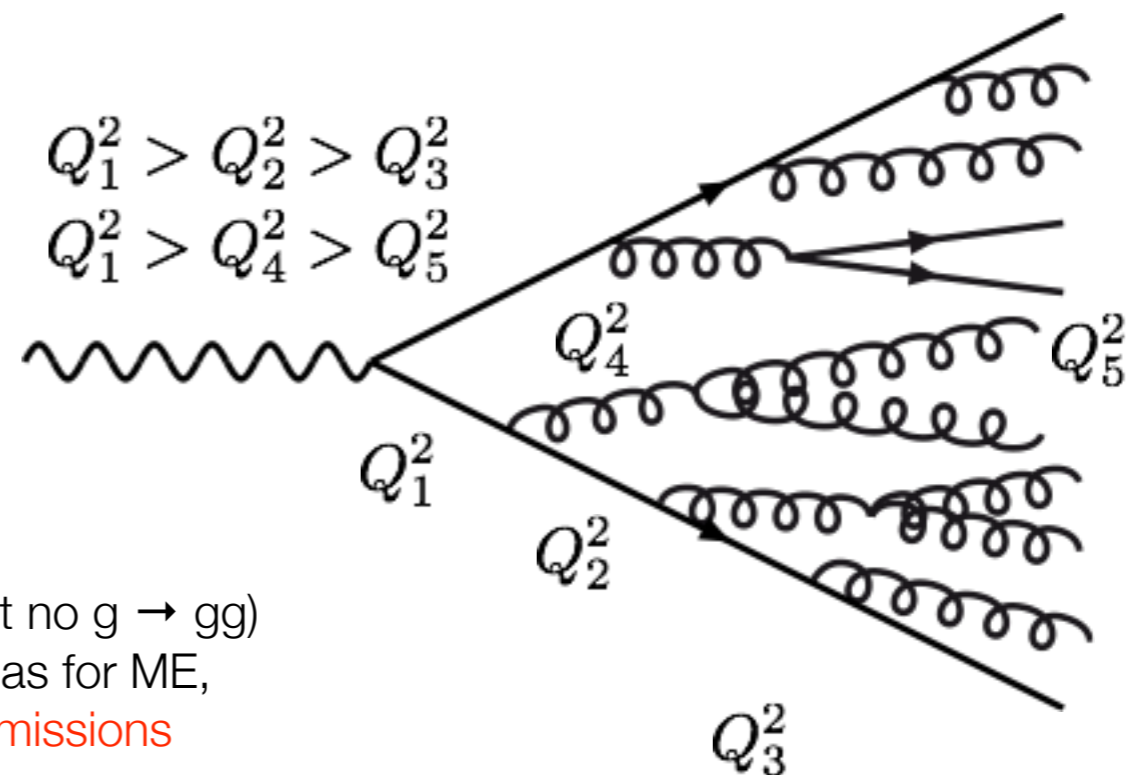
Sudakov form factor ...

... provides “time” ordering of shower ...
[lower $Q^2 \Leftrightarrow$ longer times]

... regulates singularity for first emission ...

But in the limit of repeated soft emissions $q \rightarrow qg$ (but no $g \rightarrow gg$)
one obtains the same inclusive Q emission spectrum as for ME,

i.e. **divergent ME spectrum \Leftrightarrow infinite number of PS emissions**



Sudakov picture of parton showers

Basic algorithm: Markov chain

[each step requires only knowledge of the previous step]

- (i) Start with virtuality Q_1 and momentum fraction x_1
- (ii) Generate target virtuality Q_2 with random number R_T uniform distributed in $[0,1]$

Probability to not have $Q_x > Q_2$

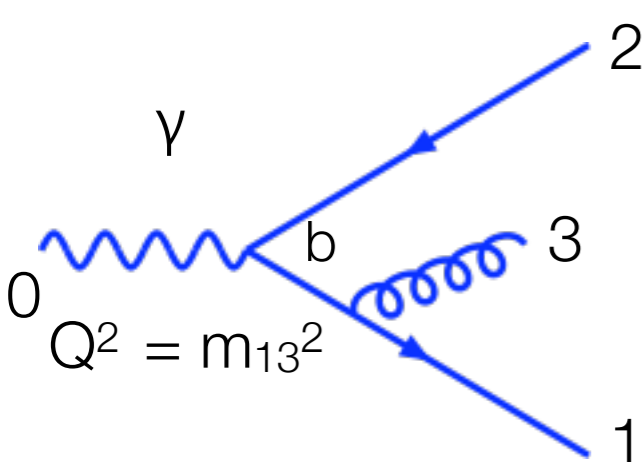
using:

$$\Delta(Q_i^2) = \exp \left(- \sum_{b,c} \int_{Q_i^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz' \right) \text{ solve the equation for } Q_2 \quad R_t = \frac{\Delta(Q_2^2)}{\Delta(Q_1^2)}$$

[probability to evolve from t_1 to t_2 without radiation]

- (iii) Q_2 known (x_2 known), need to compute $x_1 \sim z$

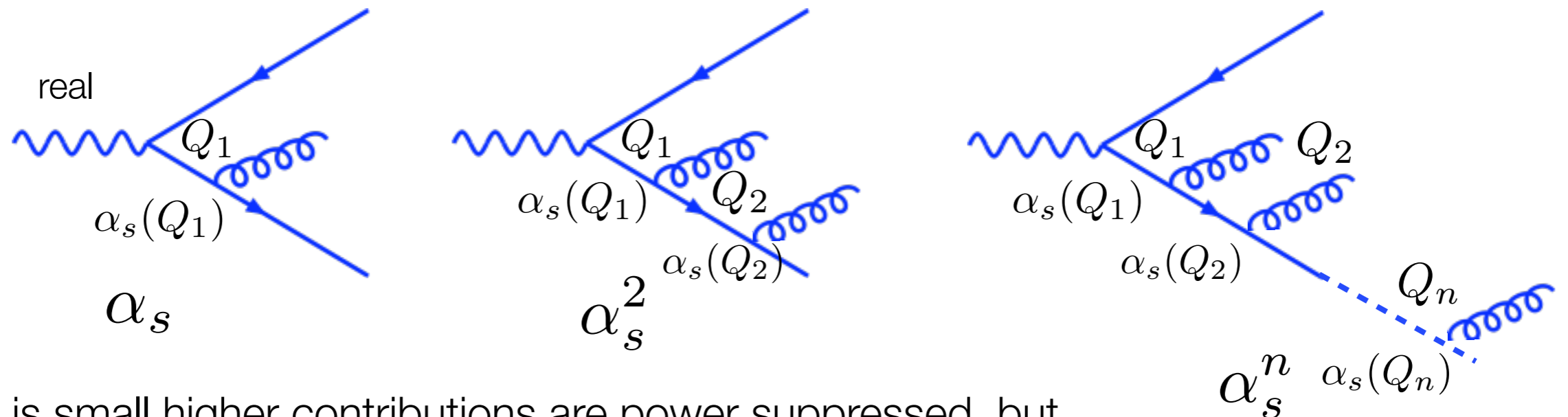
$$P_{q \rightarrow qg} = \frac{4}{3} \frac{1+z^2}{1-z} \quad R_z = \frac{\int_0^z P(z') dz'}{\int_0^1 P(z') dz'} \quad \text{flat distributed} \quad R_z \in [0, 1]$$



- (iv) Generate random azimuthal angle Φ flat distributed

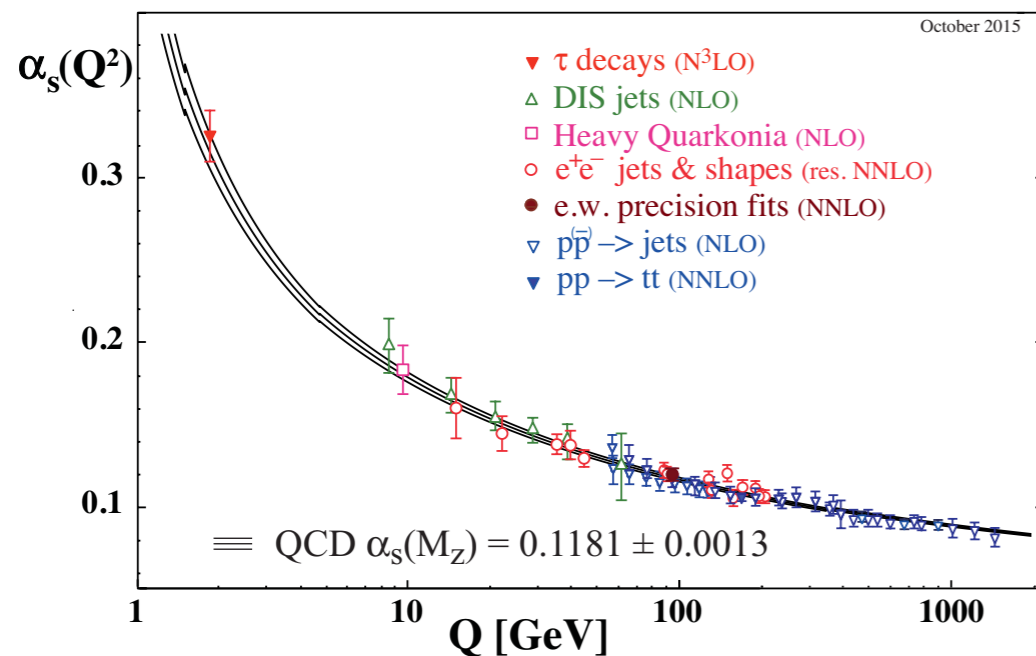
Process ends when partons are below threshold (p_T, Q)

Parton shower and logarithmic resummation



If α_s is small higher contributions are power suppressed, but...

α_s increases at small Q^2



$$\alpha_s(Q_n) \sim \alpha_s(Q_1) \ln(Q_1/Q_n)$$

$$\alpha_s(Q_1) + \alpha_s(Q_1)\alpha_s(Q_2) + \dots + \alpha_s(Q_1) \cdot \dots \cdot \alpha_s(Q_n)$$

$$\sim [\alpha_s(Q_1) \ln(Q_1)]^2 \sim [\alpha_s(Q_1) \ln(Q_1)]^n$$

if $\alpha_s(Q_1) \ln(Q_1)$

is large, the expansion is broken, PS allows to sum up all the large contributions [Leading Log resummation]

Parton shower ordering

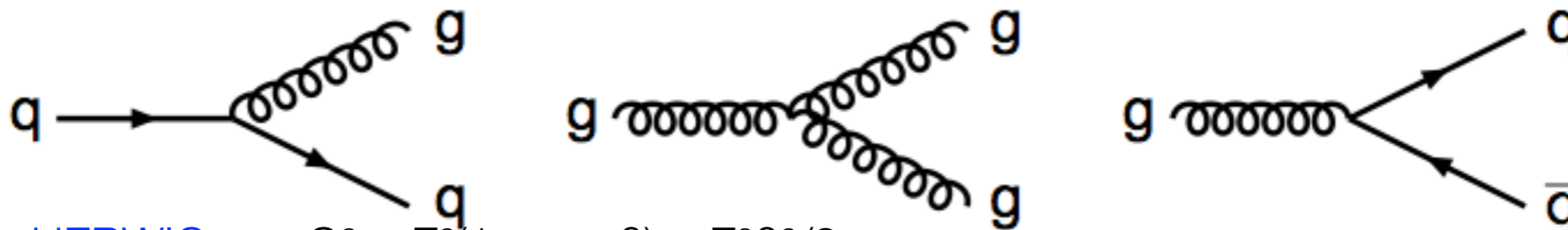
$$d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) dz \exp \left(- \sum_{b,c} \int_{Q^2}^{Q_{\max}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') dz' \right)$$

In the splitting function appears only dQ^2/Q^2 , therefore if $P = f(z)Q^2$ $dP/P = dQ^2/Q^2$

Three main approaches to showering in use:

$p_{\perp}^2 \approx z(1-z)m^2$ p_{\perp} ordered showers $E^2\theta^2 \approx m^2/(z(1-z))$ angular ordered showers

Two are based on the standard shower language of $a \rightarrow bc$ successive branchings:

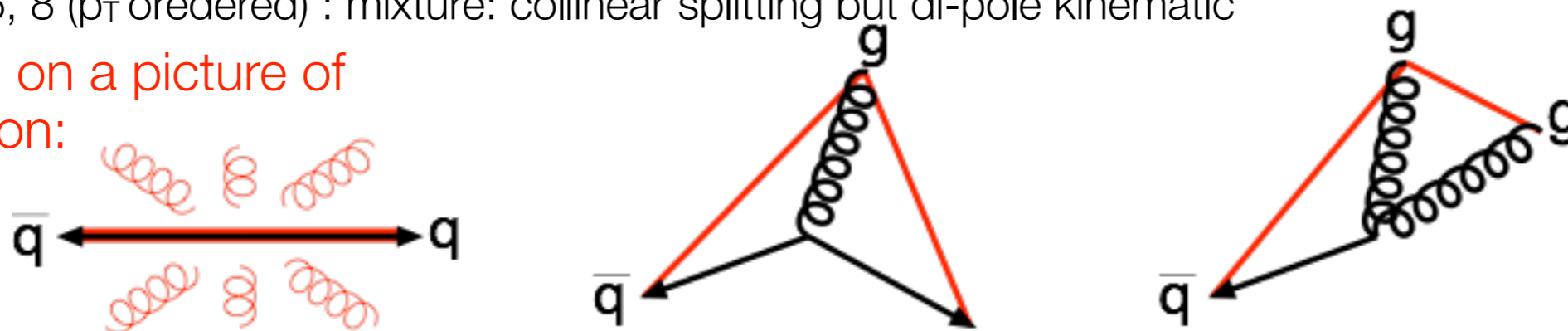


HERWIG, HERWIG++ : $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$

PYTHIA, 8 (basic) : $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

PYTHIA6, 8 (p_{\perp} ordered) : mixture: collinear splitting but di-pole kinematic

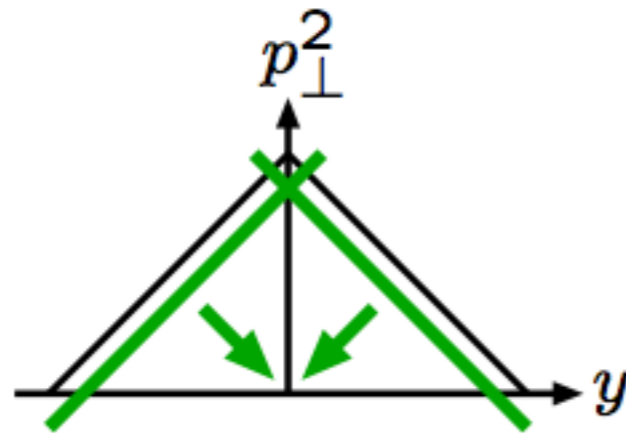
One is based on a picture of dipole emission:



Ariadne : $Q^2 = p_{\perp}^2$; FSR mainly, ISR is primitive ...

consider the full recoil and not only the branching

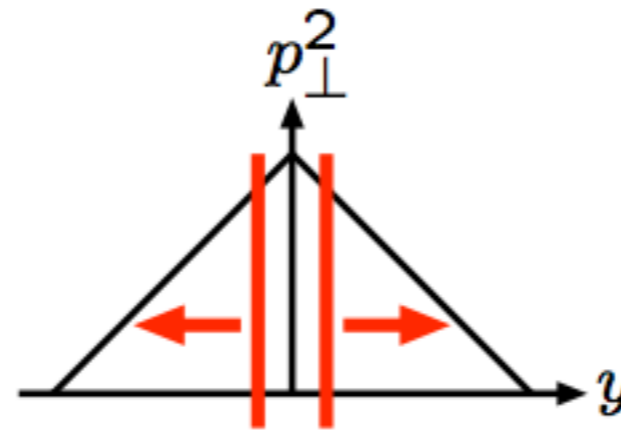
PYTHIA: $Q^2 = m^2$ HERWIG/++: $Q^2 \sim E^2\theta^2$ ARIADNE/Pythia8: $Q^2 = p_{\perp}^2$



Large mass first
[“hardness” ordered]

Covers phase space
ME merging simple
 $g \rightarrow qq$ simple
not Lorentz invariant
no stop/restart

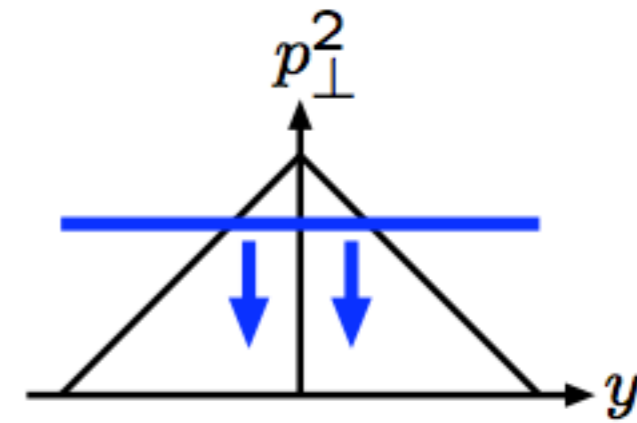
ISR: $m^2 \rightarrow -m^2$



Large angle first
[not “hardness” ordered]

Gaps in coverage
ME merging messy
 $g \rightarrow qq$ simple
not Lorentz invariant
no stop/restart

ISR: $\theta \rightarrow \theta$



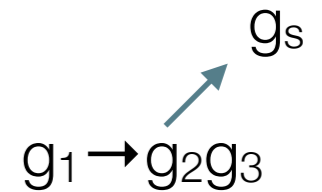
Large p_{\perp} first
[“hardness” ordered]

Covers phase space
ME merging simple
 $g \rightarrow qq$ messy
Lorentz invariant
can stop/restart

ISR: complicated

Color coherence

QED: Chudakov effect (mid-fifties)



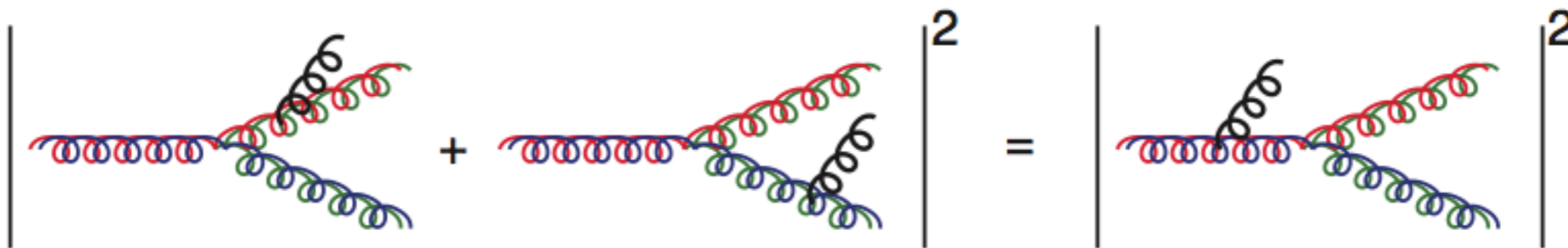
emulsion plate

reduced ionization

normal ionization

1. soft gluons see the pair of split gluons as a whole, color screening reduce their emission
2. angular ordered and p_T ordered PS reproduce the correct color coherence
3. Pythia Q^2 needs a-posteriori corrections

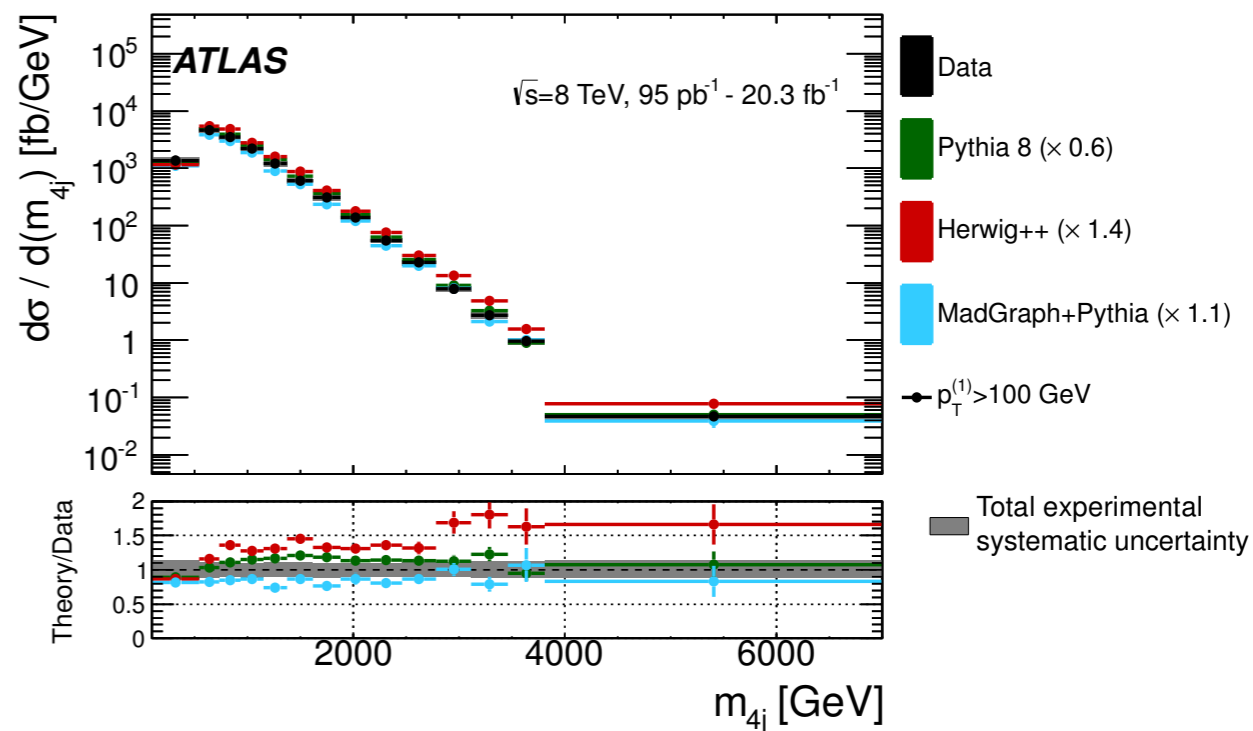
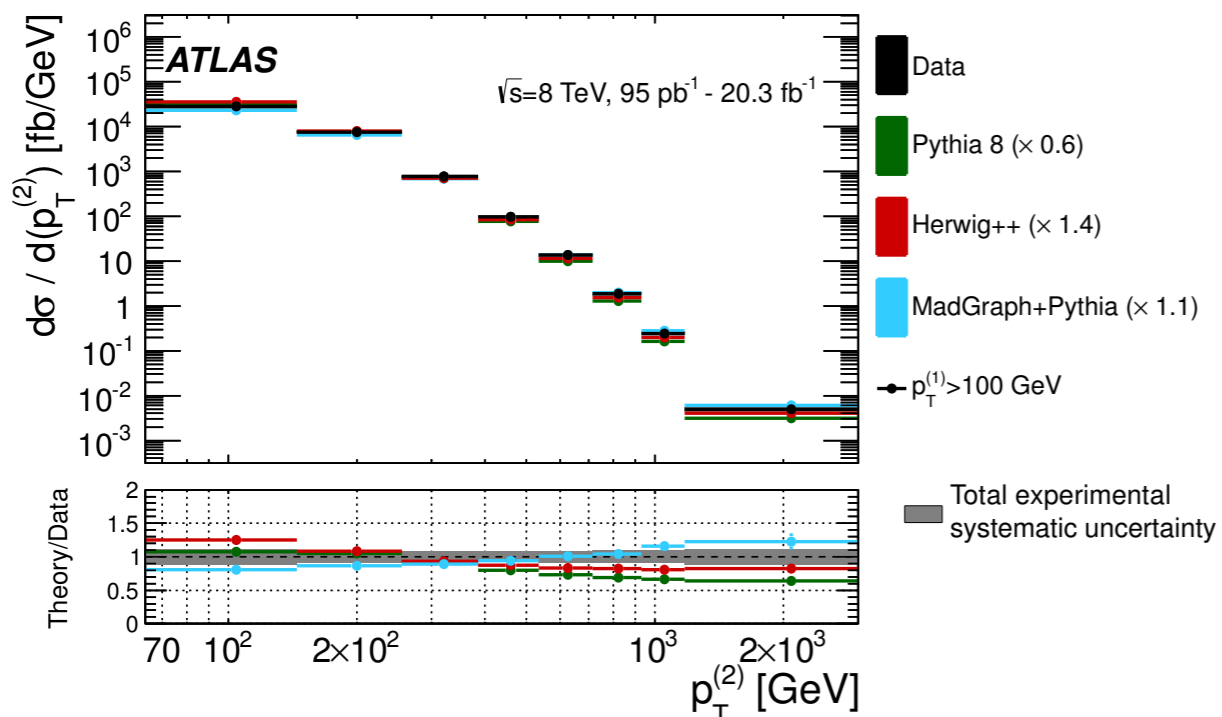
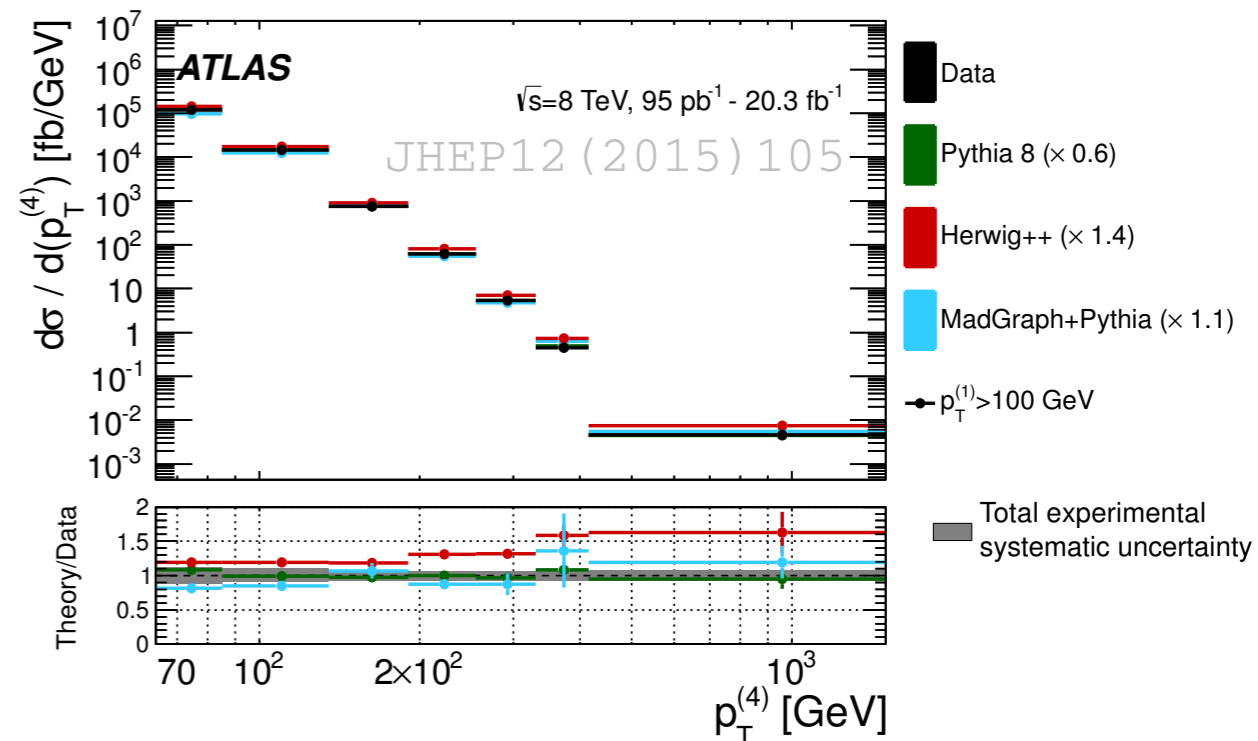
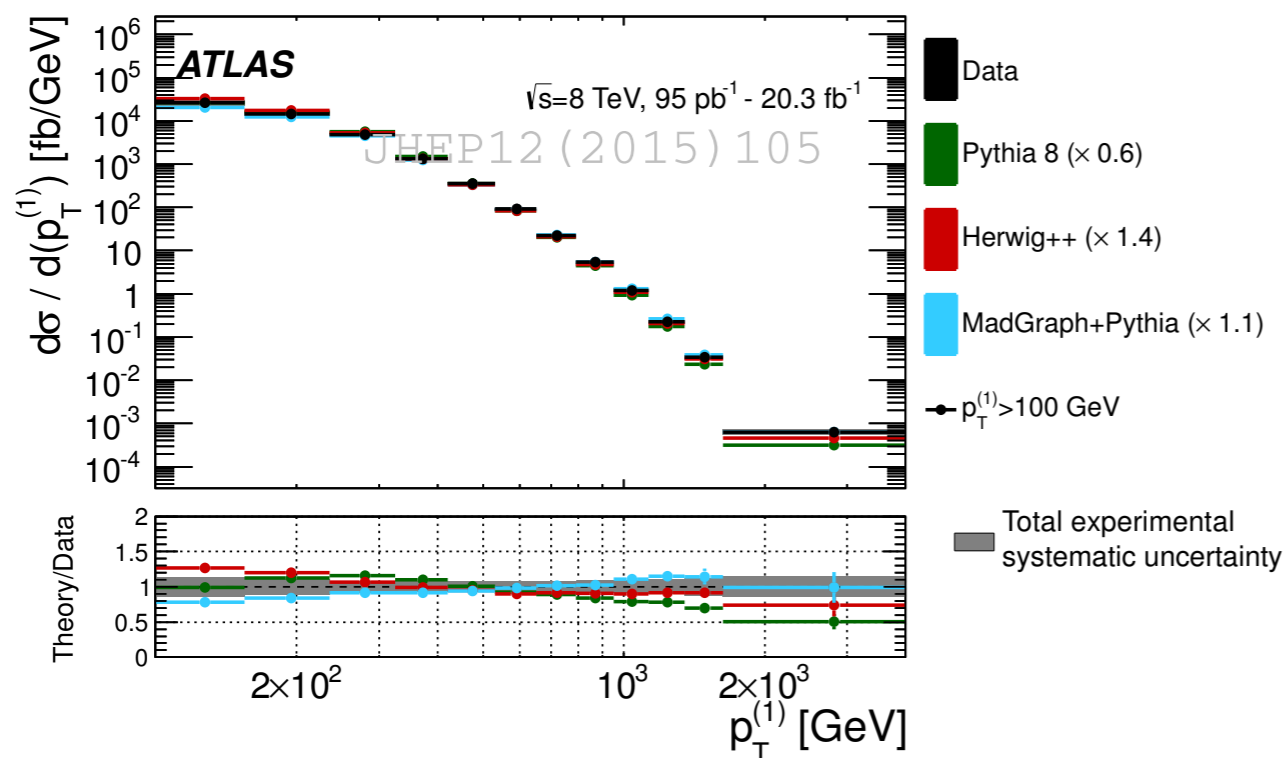
QCD: colour coherence for **soft** gluon emission



- solved by
- requiring emission angles to be decreasing
 - or
 - requiring transverse momenta to be decreasing

Compariosn to LHC data

4jets cross section: $p_T^{(1)} > p_T^{(2)} > p_T^{(3)} > p_T^{(4)}$



Process simulation

Many specialized processes already available in Pythia8/Herwig++

but, processes usually only implemented at the lowest non-trivial order ...

Need external programs that ...

1. include higher order loop corrections or, alternatively, do kinematic dependent rescaling
3. allow matching of higher order ME generators [otherwise need to trust parton shower description ...]
5. provide correct spin correlations often absent in PS ...[e.g. top produced unpolarized, while $t \rightarrow bW \rightarrow b\nu$ decay correct]
7. simulate newly available physics scenarios ...[appear quickly; need for many specialised generators]

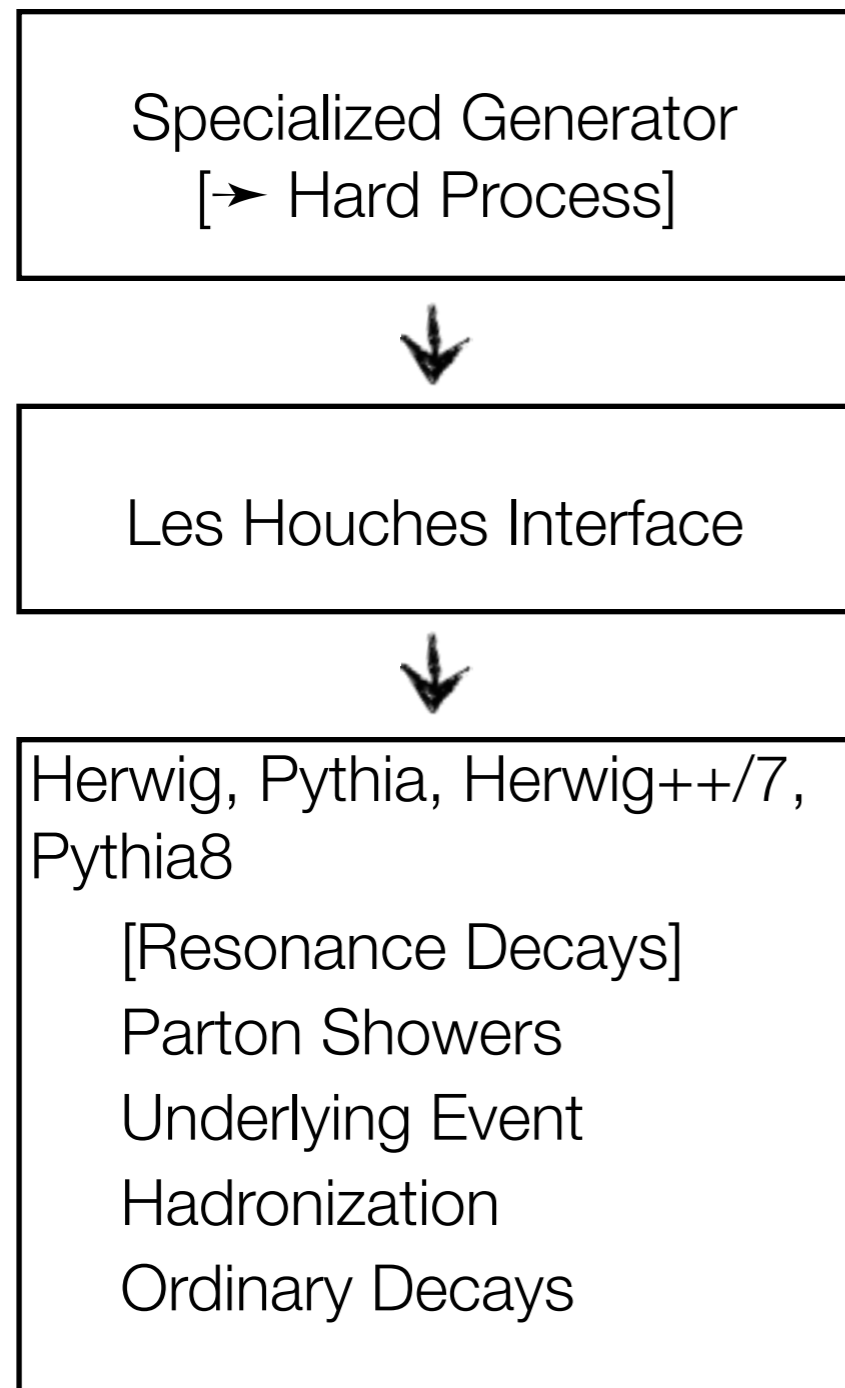
Les Houches Accord ...

Specifies how parton-level information about the hard process and sequential decays can be encoded and passed on to a general-purpose generator.

Les Houches: regular annual meeting between theoreticians and experimentalists on MC generator developments.



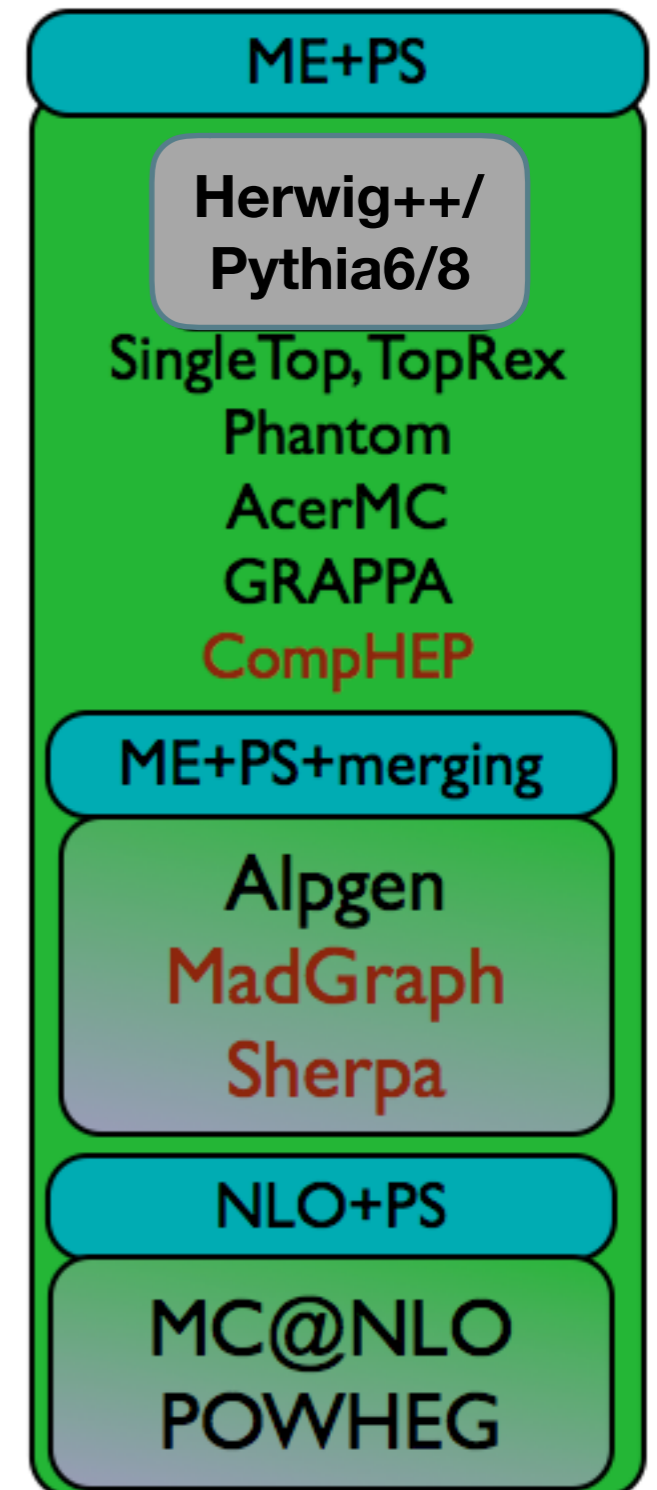
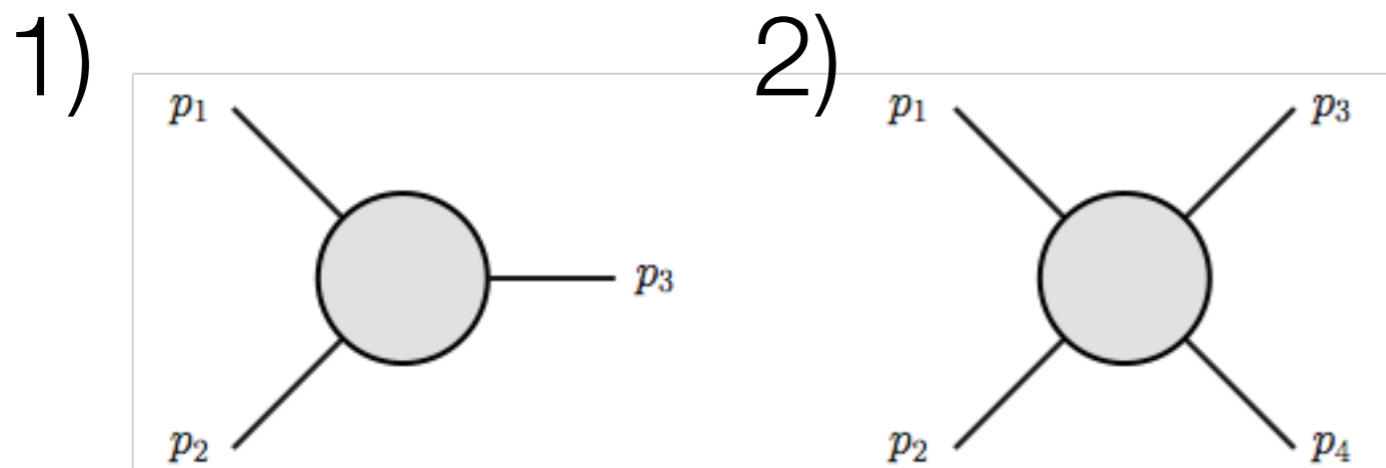
Specialised Generators [some examples]



AcerMC	:	ttbb, .single top
ALPGEN	:	$W/Z + \leq 6j$, $nW + mZ + kH + \leq 3j, \dots$
AMEGIC++	:	generic LO
CompHEP	:	generic LO
GRACE	:	generic LO
[+Bases/Spring]	:	[+ some NLO loops]
GR@PPA	:	bbbb
MadCUP	:	$W/Z+ \leq 3j$, ttbb
HELAS & MadGraph	:	generic LO
MCFM	:	NLO $W/Z+ \leq 2j$, $WZ, WH, H+ \leq 1j$
O'Mega & WHIZARD	:	generic LO
VECBOS	:	$W/Z+ \leq 4j$
HRES	:	Higgs boson production @NNLO
DYNNLO	:	W/Z production @NNLO

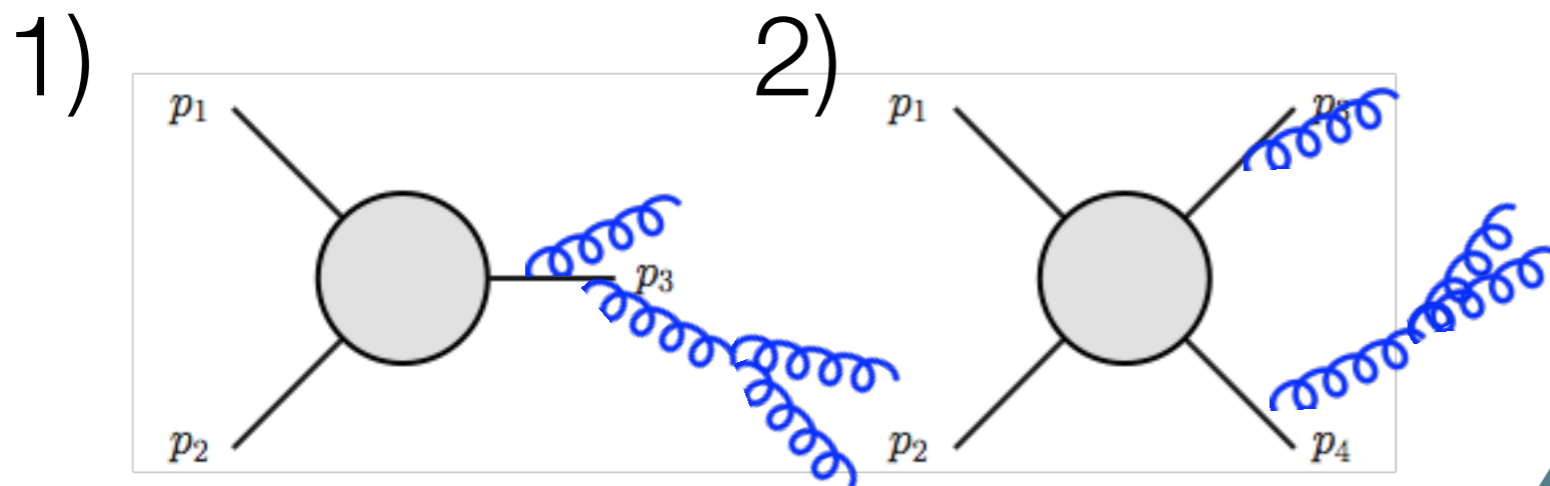
Type I : Leading order matrix element & leading log parton shower

LO ME for hard processes
[$2 \rightarrow 1$ or $2 \rightarrow 2$]

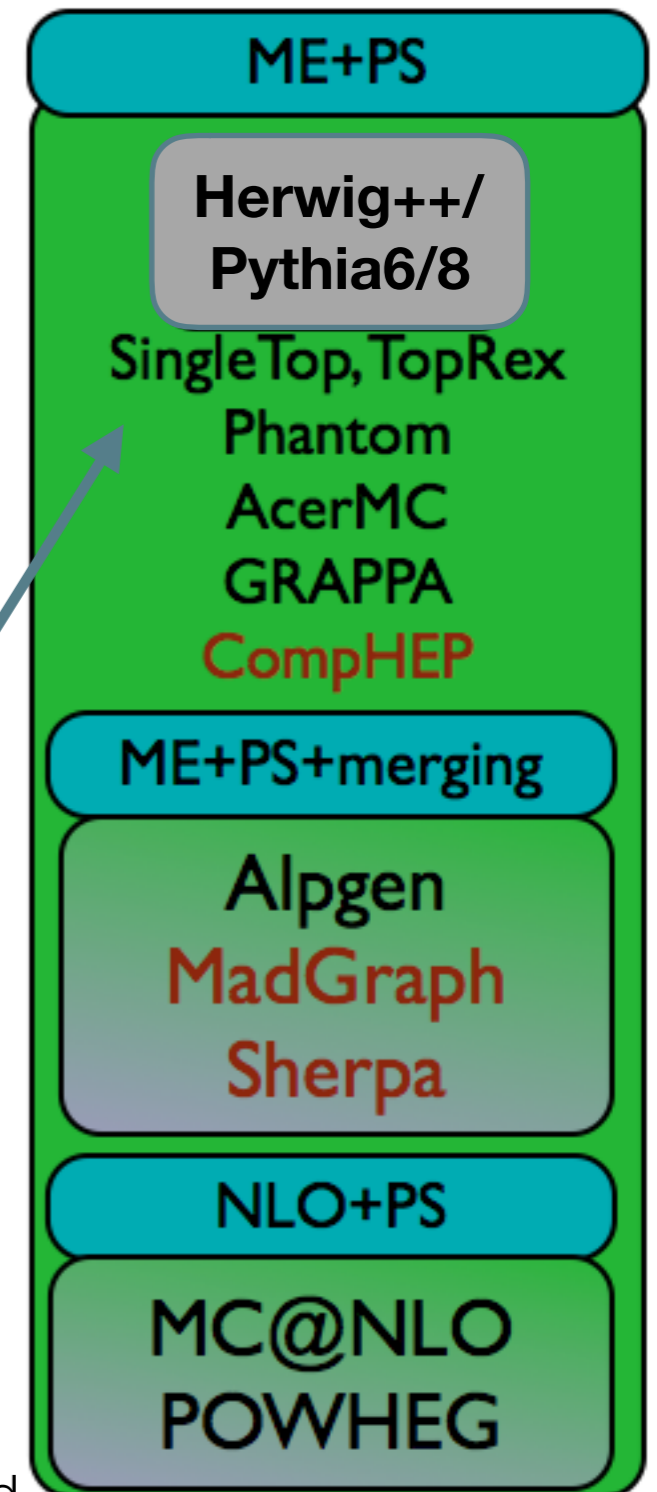


Type I : Leading order matrix element & leading log parton shower

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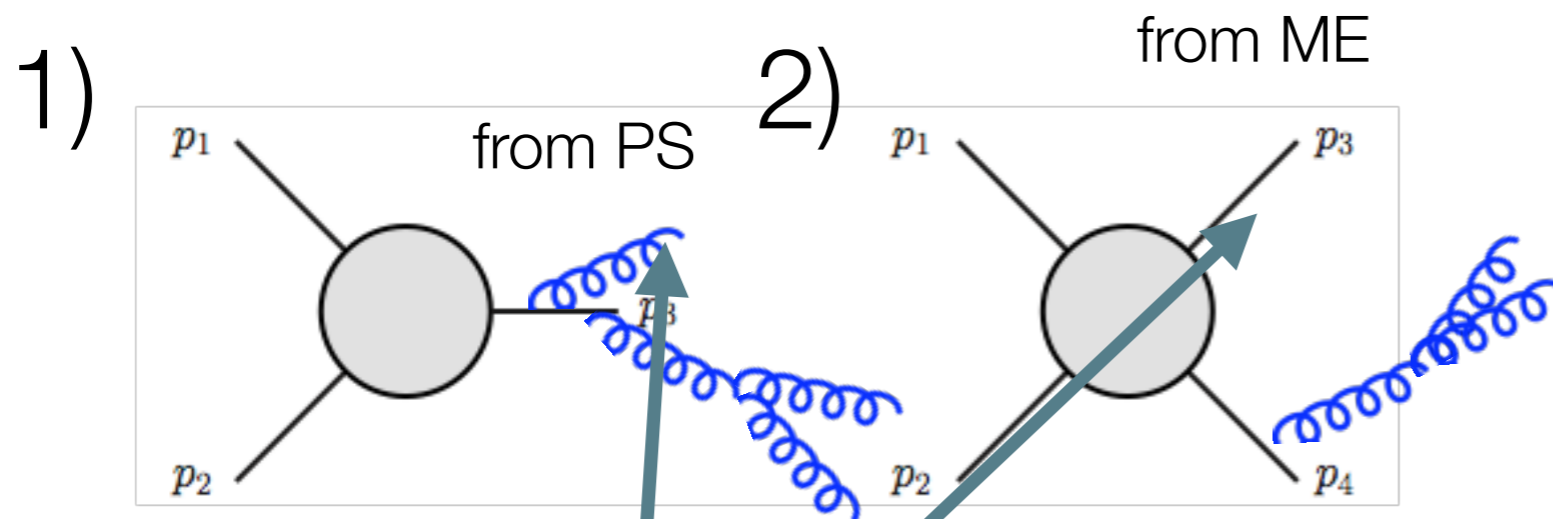


- Parton Shower: attaches gluons at each leg.
- only in soft-collinear approximation
- typically underestimate large angle/hard emission
- 1) or 2) at ME (different generations, different accuracy: cannot be combined)

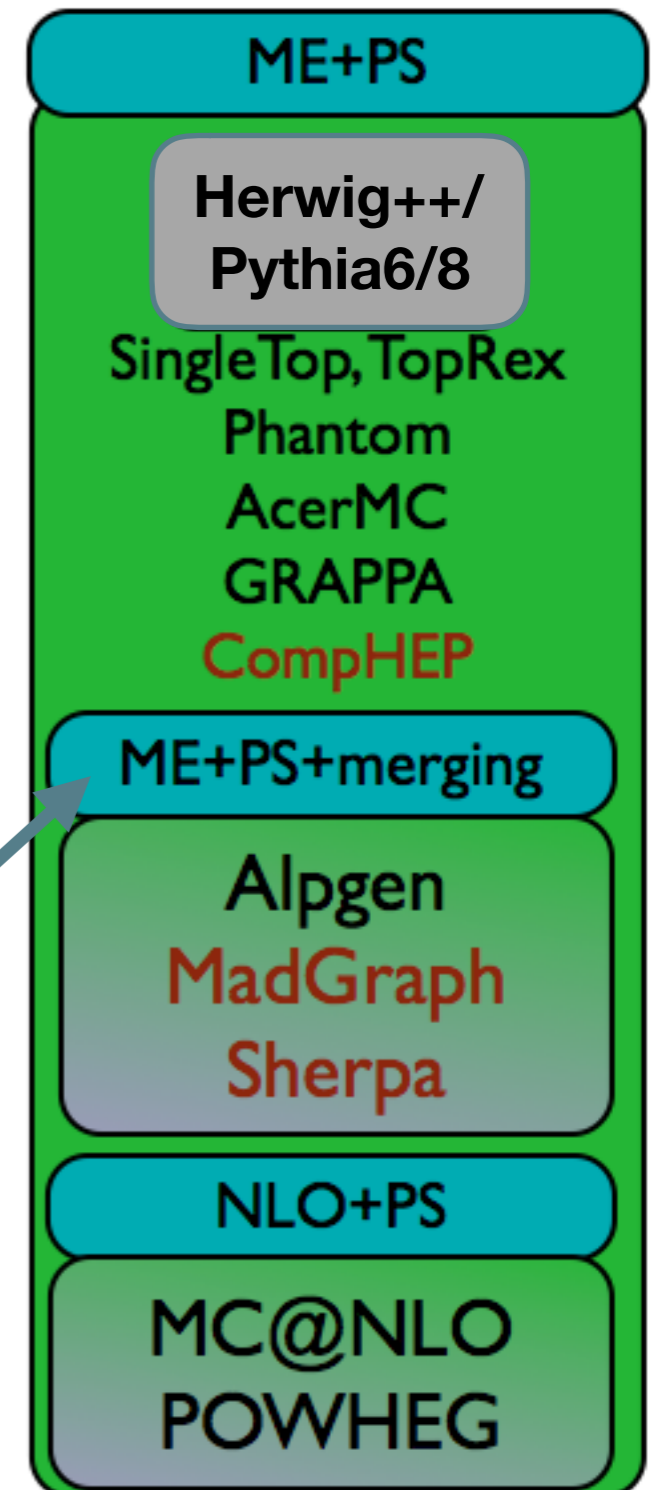


Type 2 : Leading order matrix element & leading log parton shower + merging

LO ME for hard processes
[$2 \rightarrow 1$ or $2 \rightarrow 2$]



- Type 1 can be improved using 1) + 2)
- use ME calculation for hard, large angle jets
- but needs to remove double-counting: merging
 - CKKW: Catani, Krauss, Kuhn, Weber (Sherpa)
 - MLM
- very good description of high jet multiplicity kinematics



Merging @LO

MLM matching (simplified)

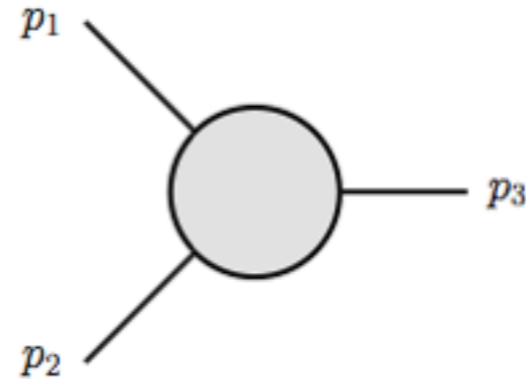
- 1) define matching cuts:
for example $p_{T^J} > 20 \text{ GeV}$, $\Delta R=0.4$

Merging @LO

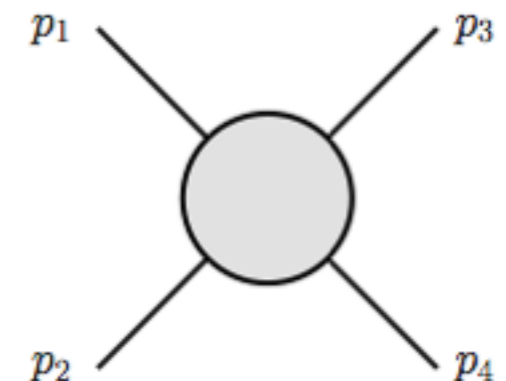
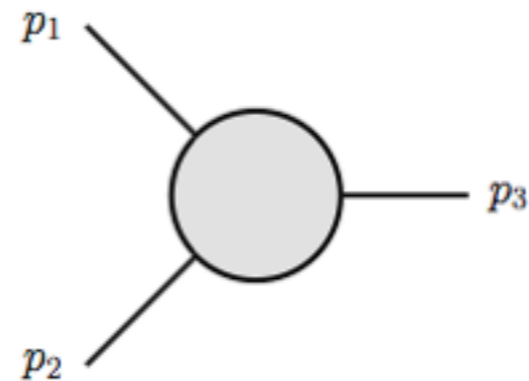
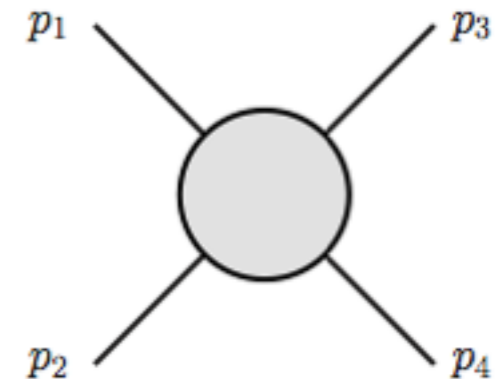
MLM matching (simplified)

- 1) define matching cuts:
for example $p_{T^J} > 20 \text{ GeV}$, $\Delta R=0.4$
- 2) generate ME with 1, 2, ...n jets

1 parton



2 partons

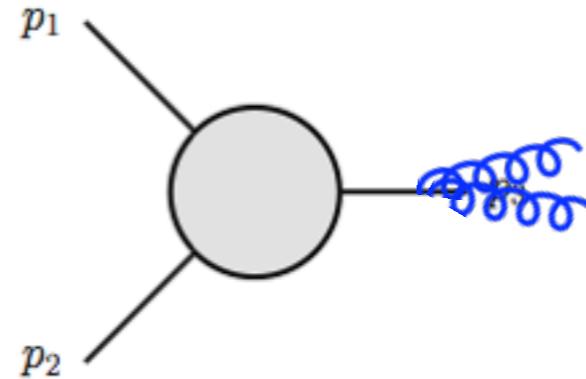


Merging @LO

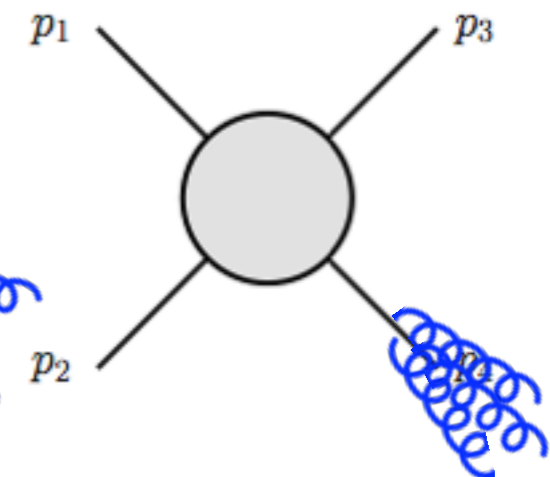
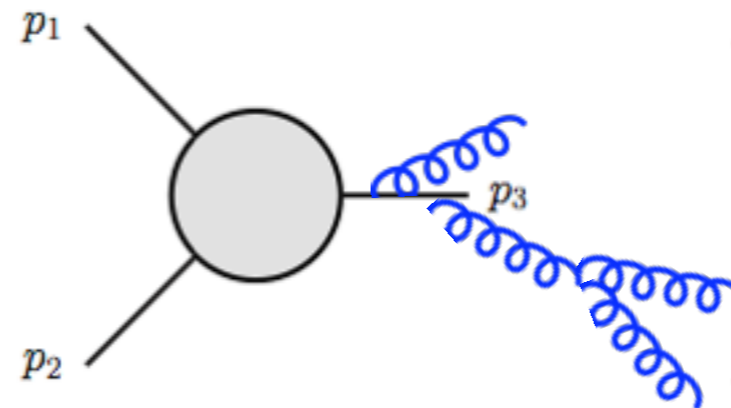
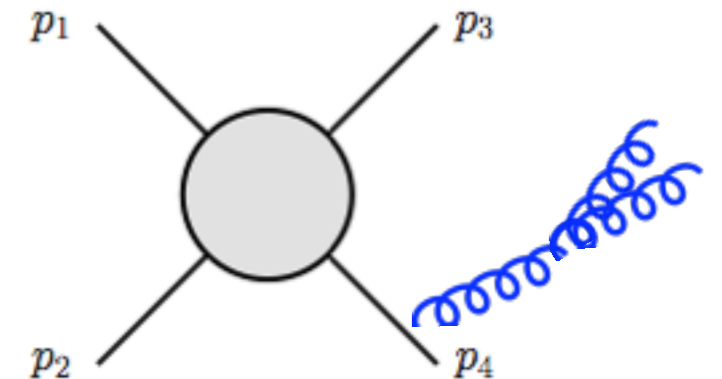
MLM matching (simplified)

- 1) define matching cuts:
for example $p_{T^J} > 20 \text{ GeV}$, $\Delta R=0.4$
- 2) generate ME with 1, 2, ...n jets
- 3) shower all events

1 parton



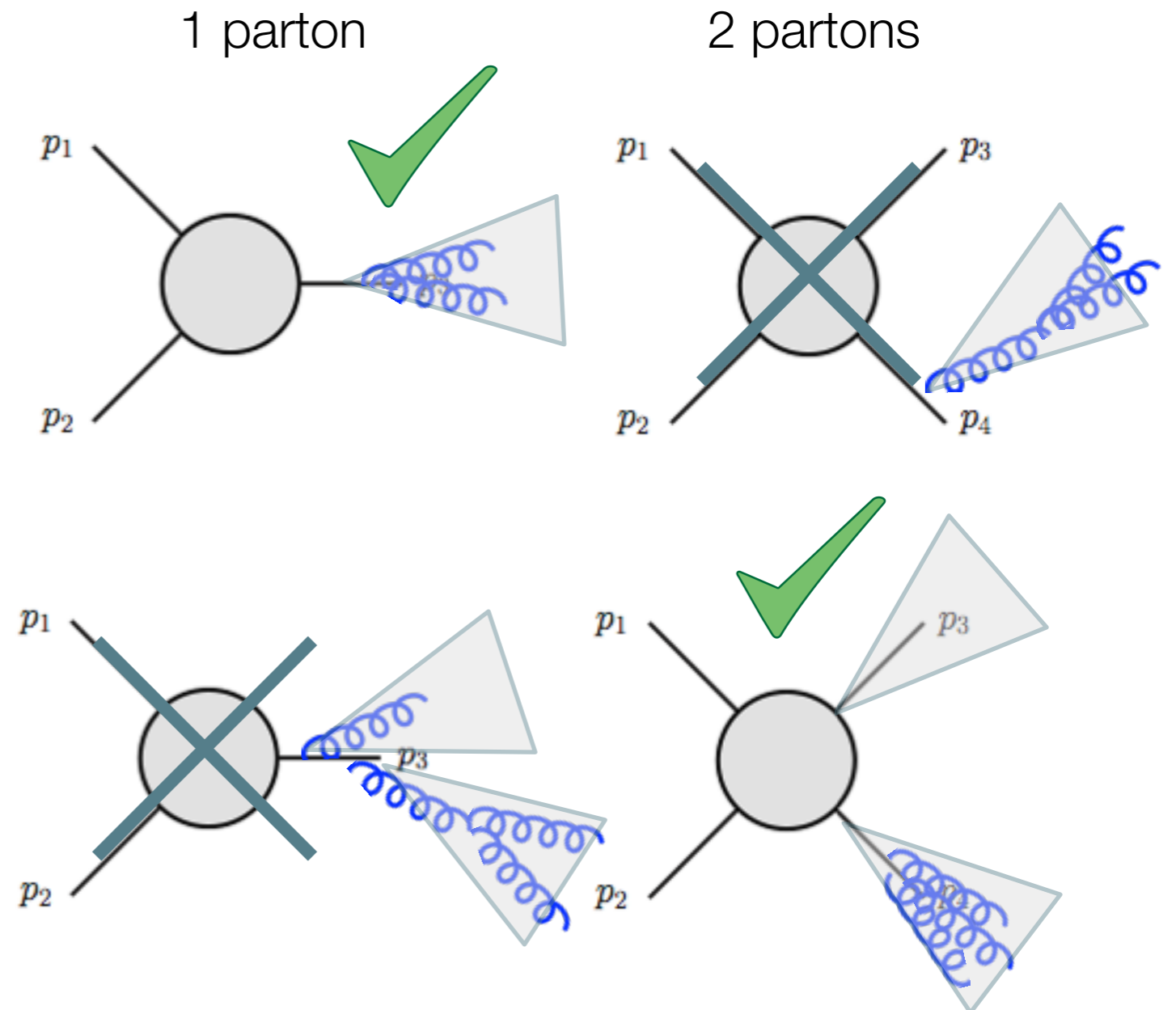
2 partons



Merging @LO

MLM matching (simplified)

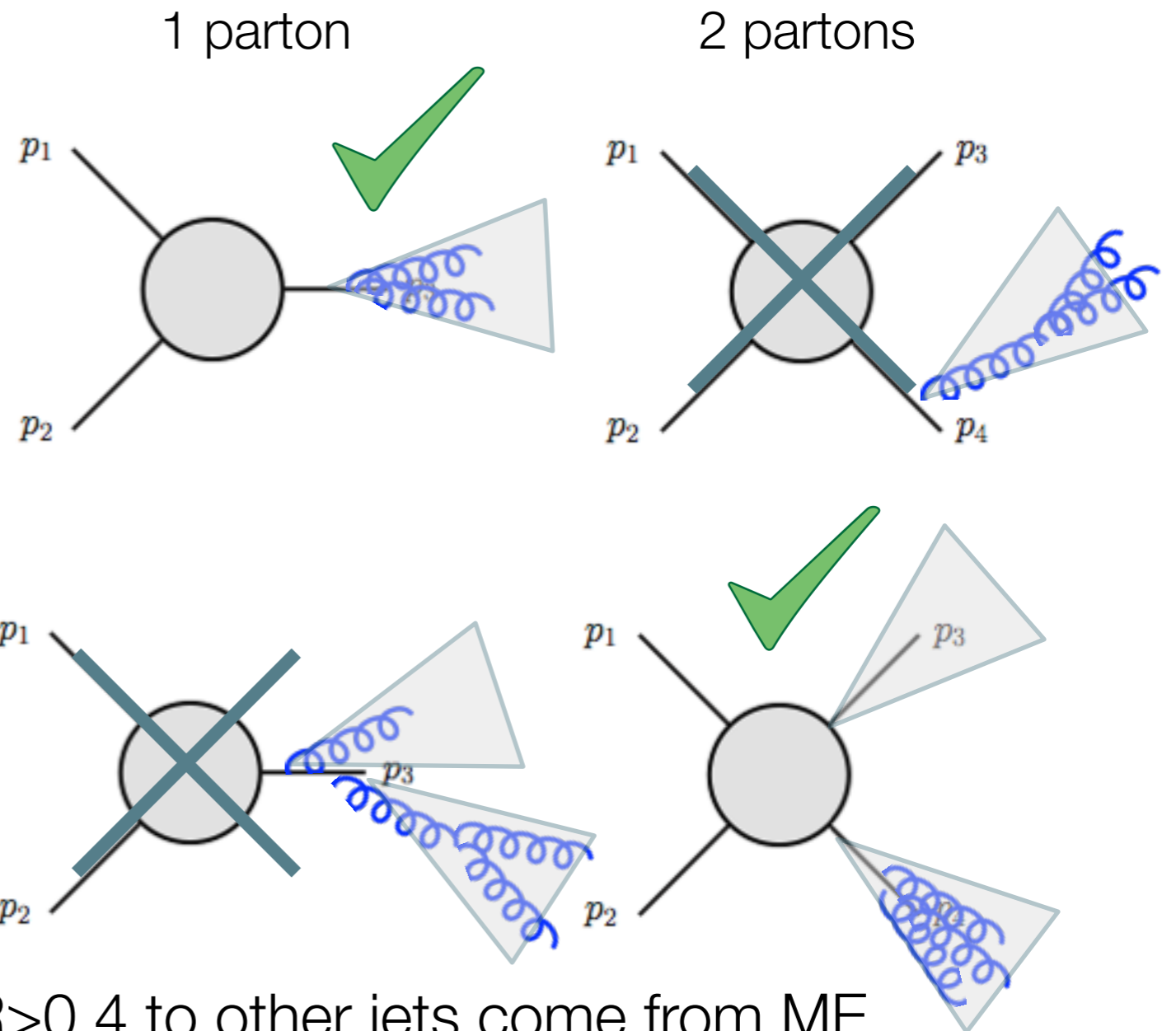
- 1) define matching cuts:
for example $p_{T^J} > 20 \text{ GeV}$, $\Delta R=0.4$
- 2) generate ME with 1, 2, ...n jets
- 3) shower all events
- 4) select only events where jets above the p_T threshold match with final partons



Merging @LO

MLM matching (simplified)

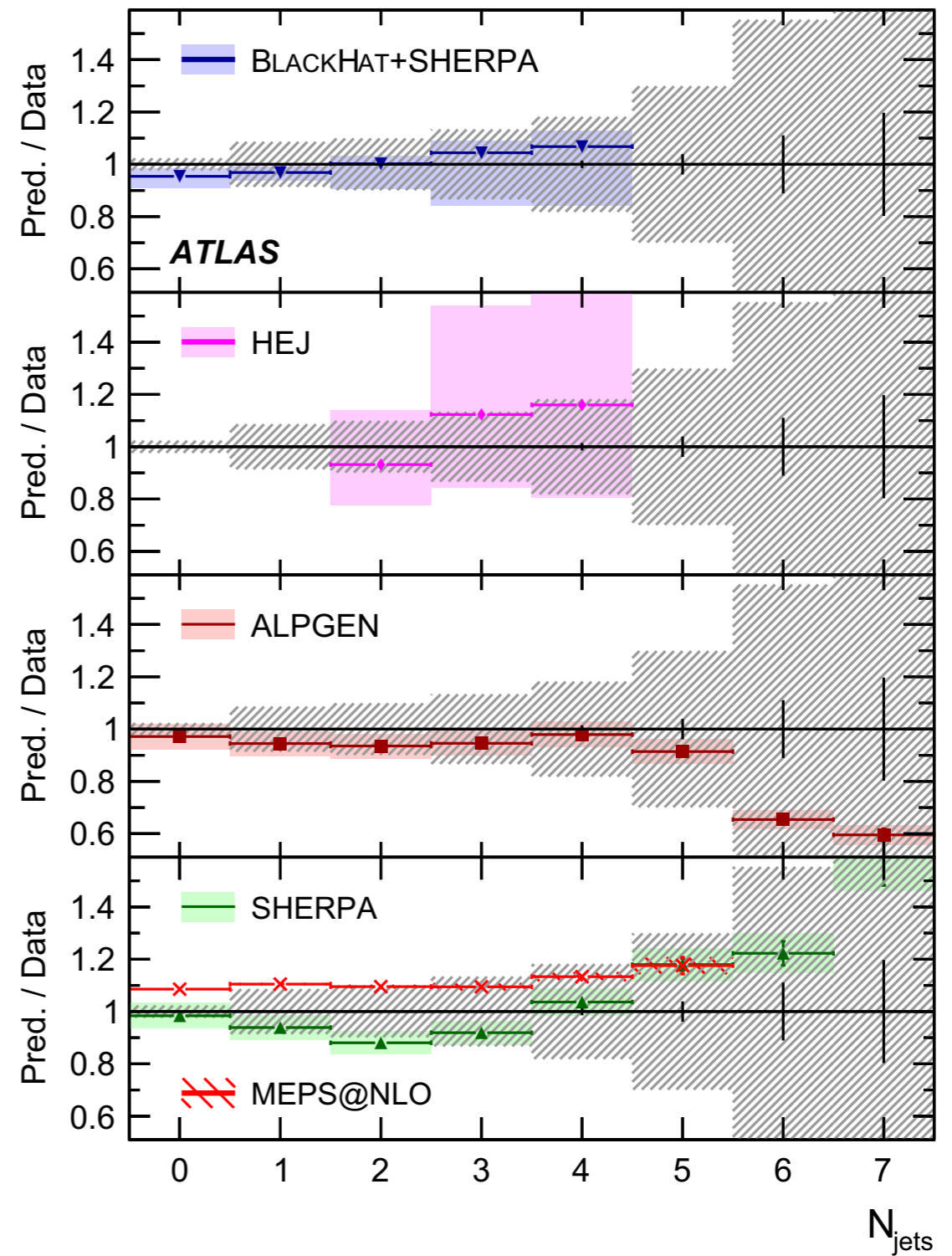
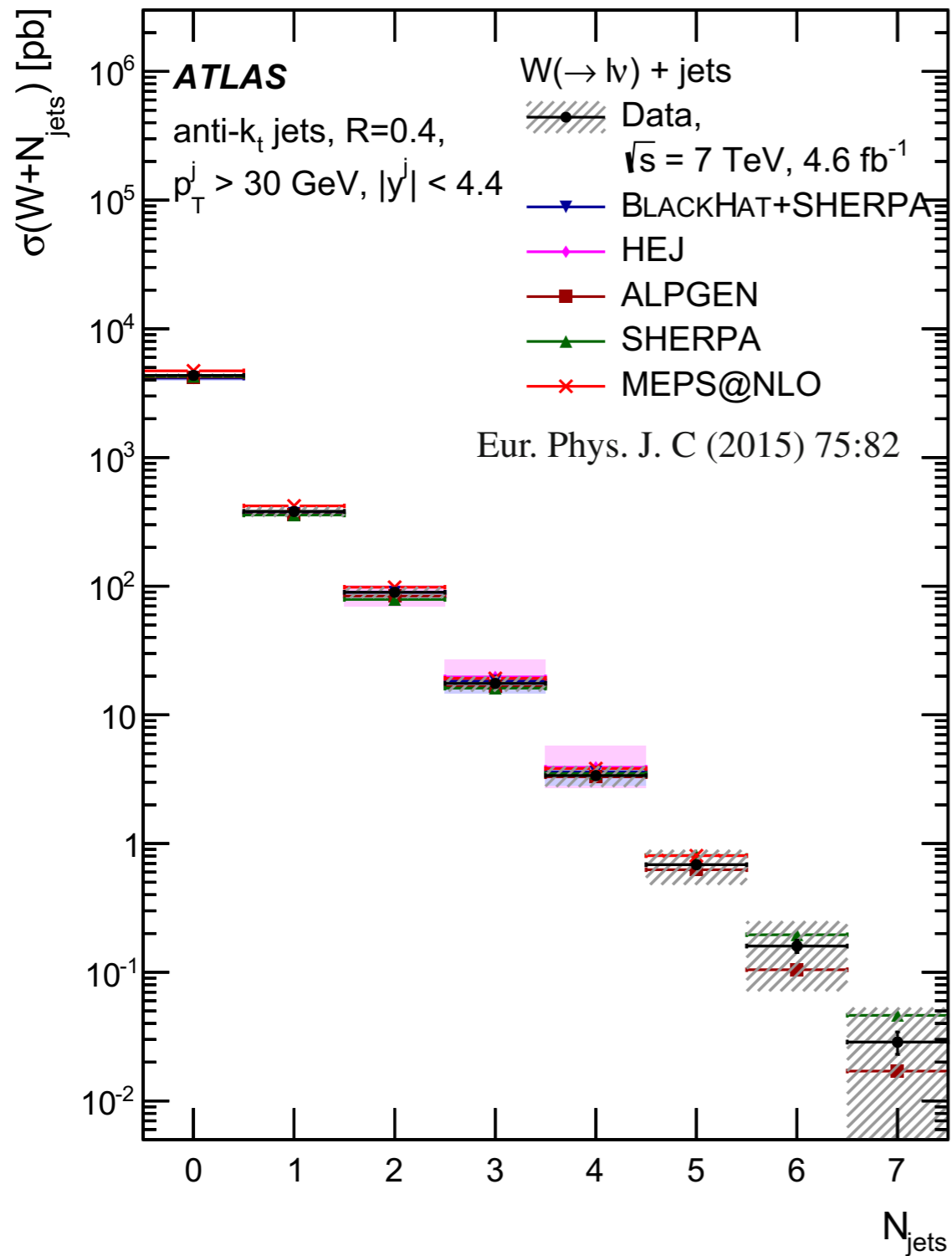
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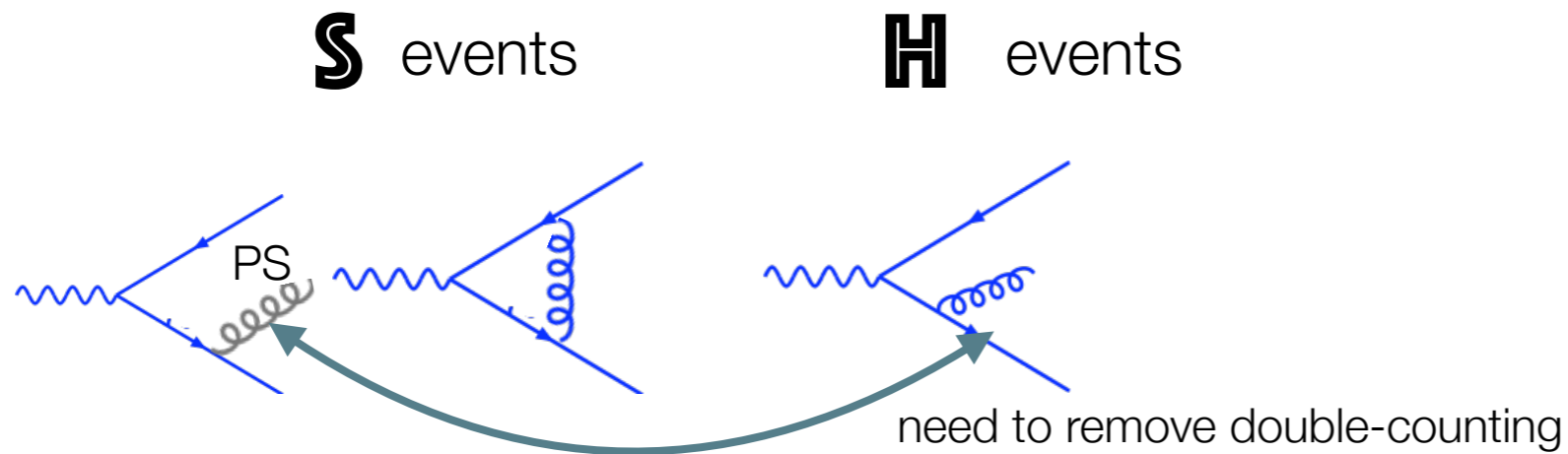
Consequences:

- all jets with $p_T > 20 \text{ GeV}$ and $\Delta R > 0.4$ to other jets come from ME
- collinear and soft jets come from PS
- Use ME and PS where they perform better.

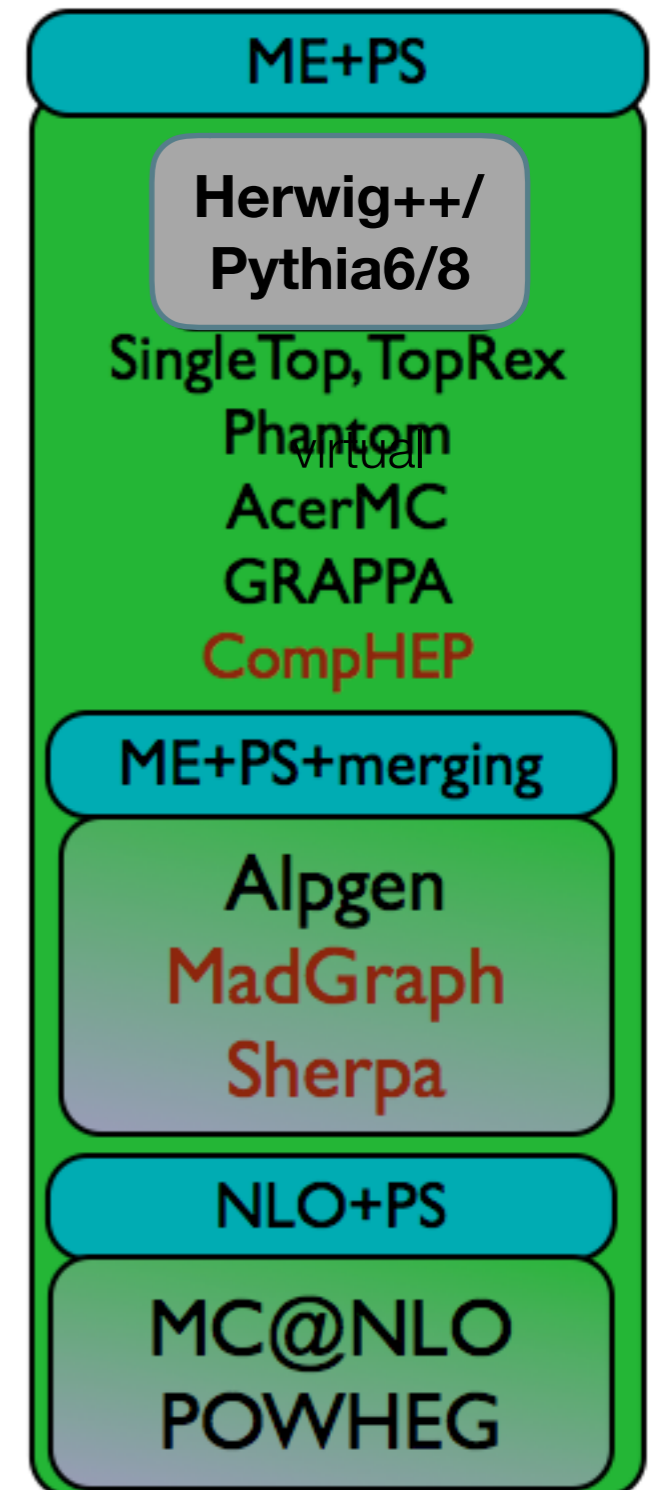
W+jets distributions



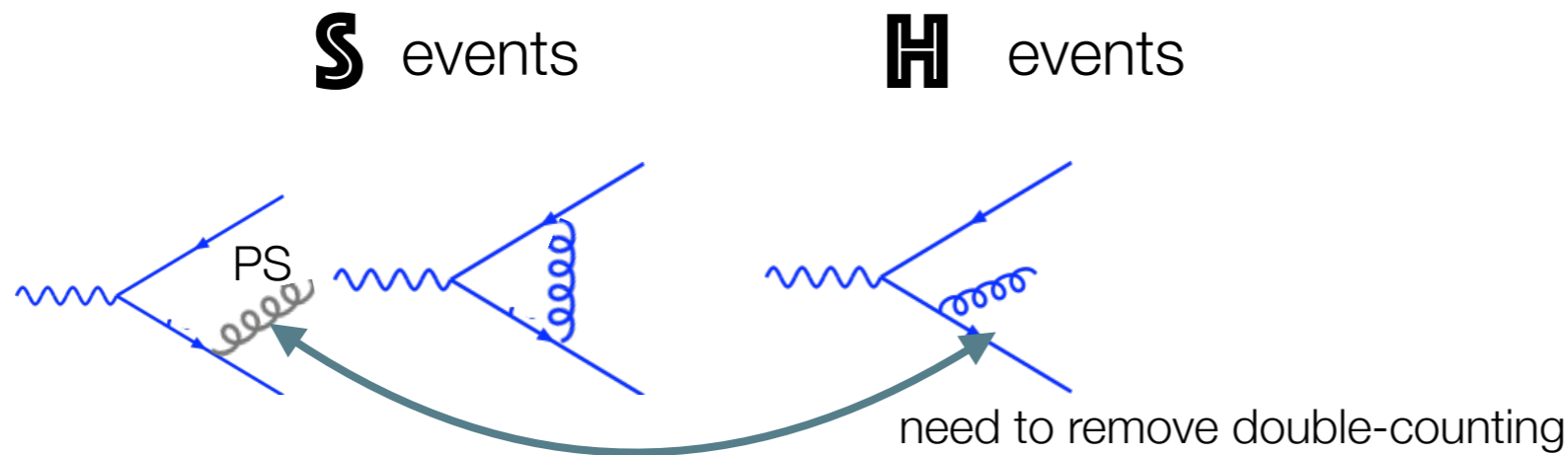
Type III : Next-to-leading order ME & leading-log parton shower



- hard processes simulated at NLO accuracy including real & virtual corrections ...
- improved description of cross sections & kinematic distributions



Type III : Next-to-leading order ME & leading-log parton shower



- hard processes simulated at NLO accuracy including real & virtual corrections ...
- improved description of cross sections & kinematic distributions

Two matching methods:

Truncated showers:

1. Powheg

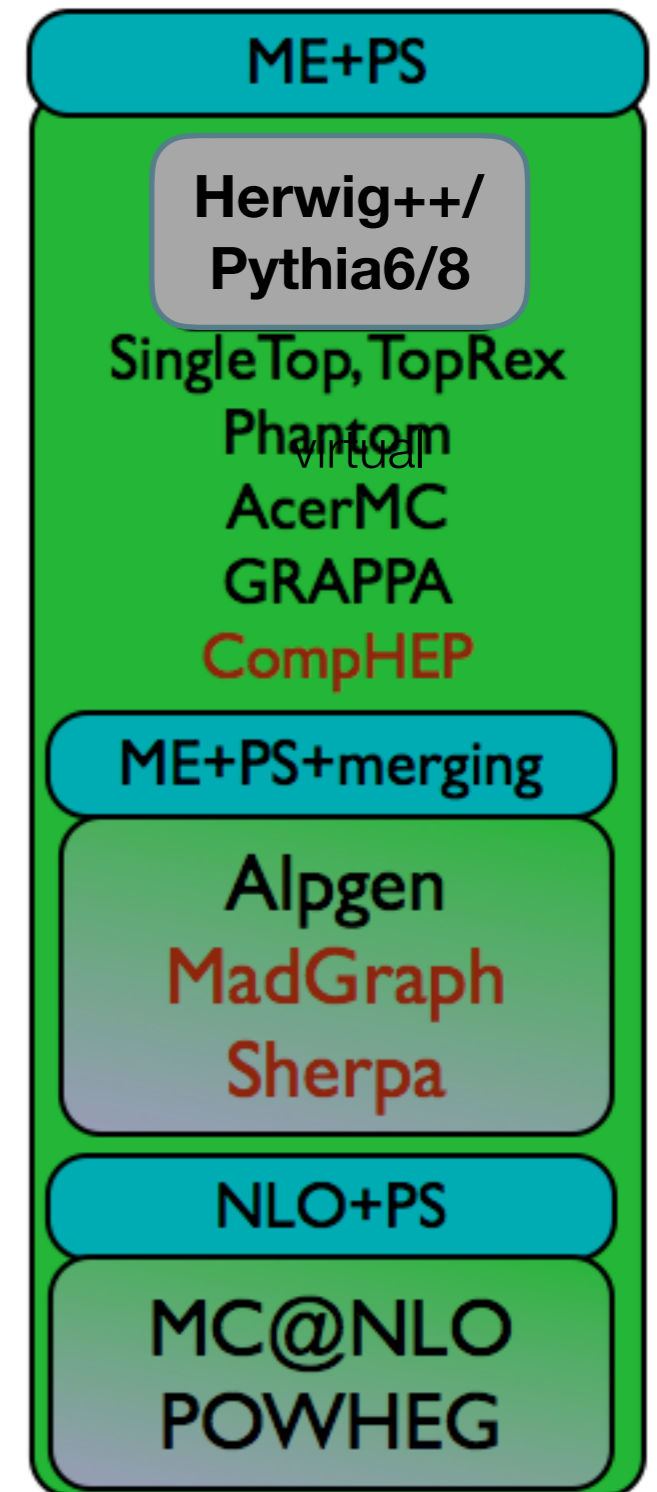
- 1) first emission produced by the ME;
- 2) don't allow the PS to produce partons harder than the first emission;
- 3) not exact at NLO (contains unbalanced higher order terms)

2. MC@NLO:

$$|ME|^2 = |ME + PS - PS(\text{up to } \alpha_s^2)|^2$$

+ Result is exact at NLO...

- produce some negative weights, need retuning for each PS

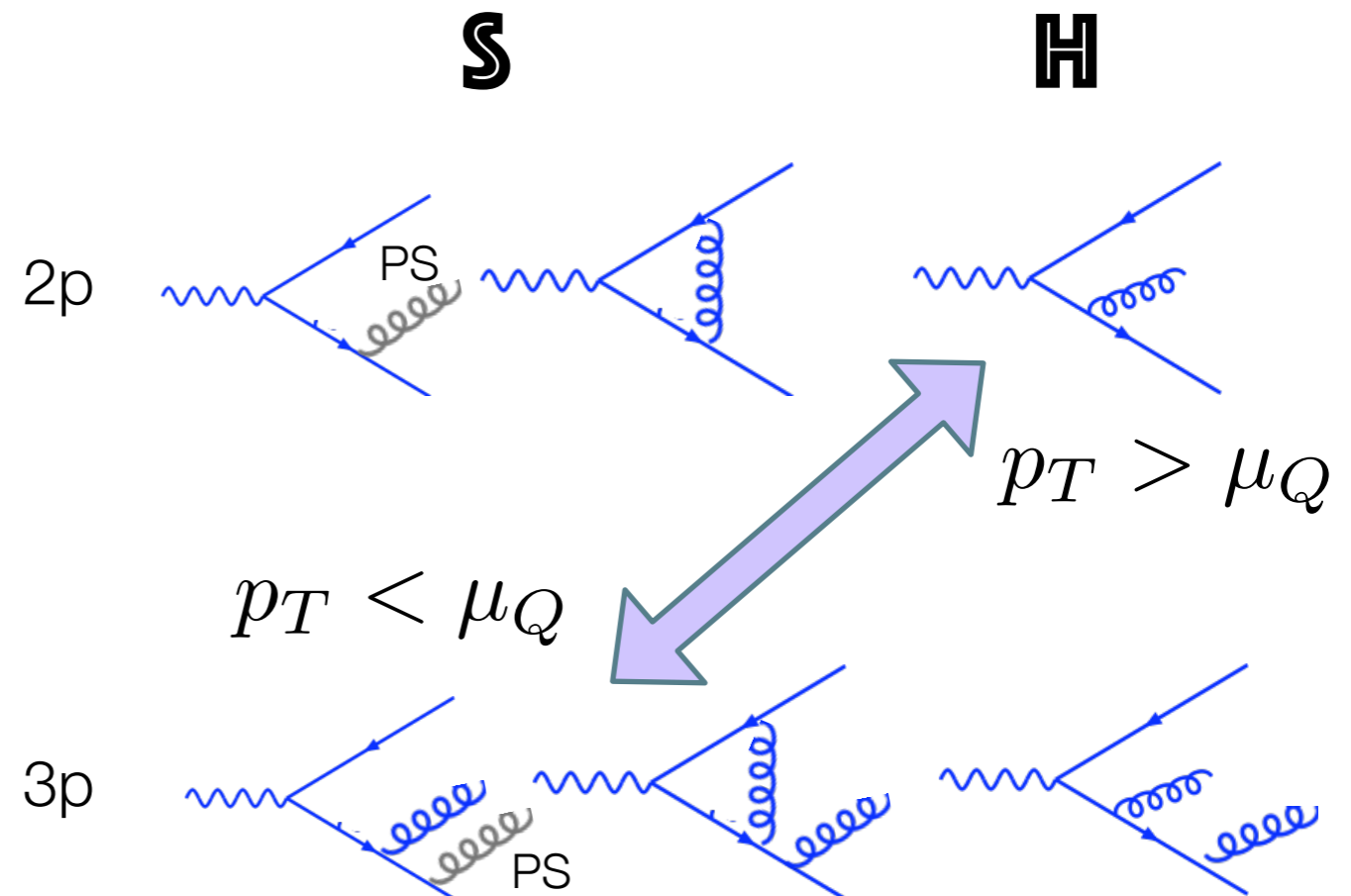


Merging @NLO (quite new, used now at 13 TeV)

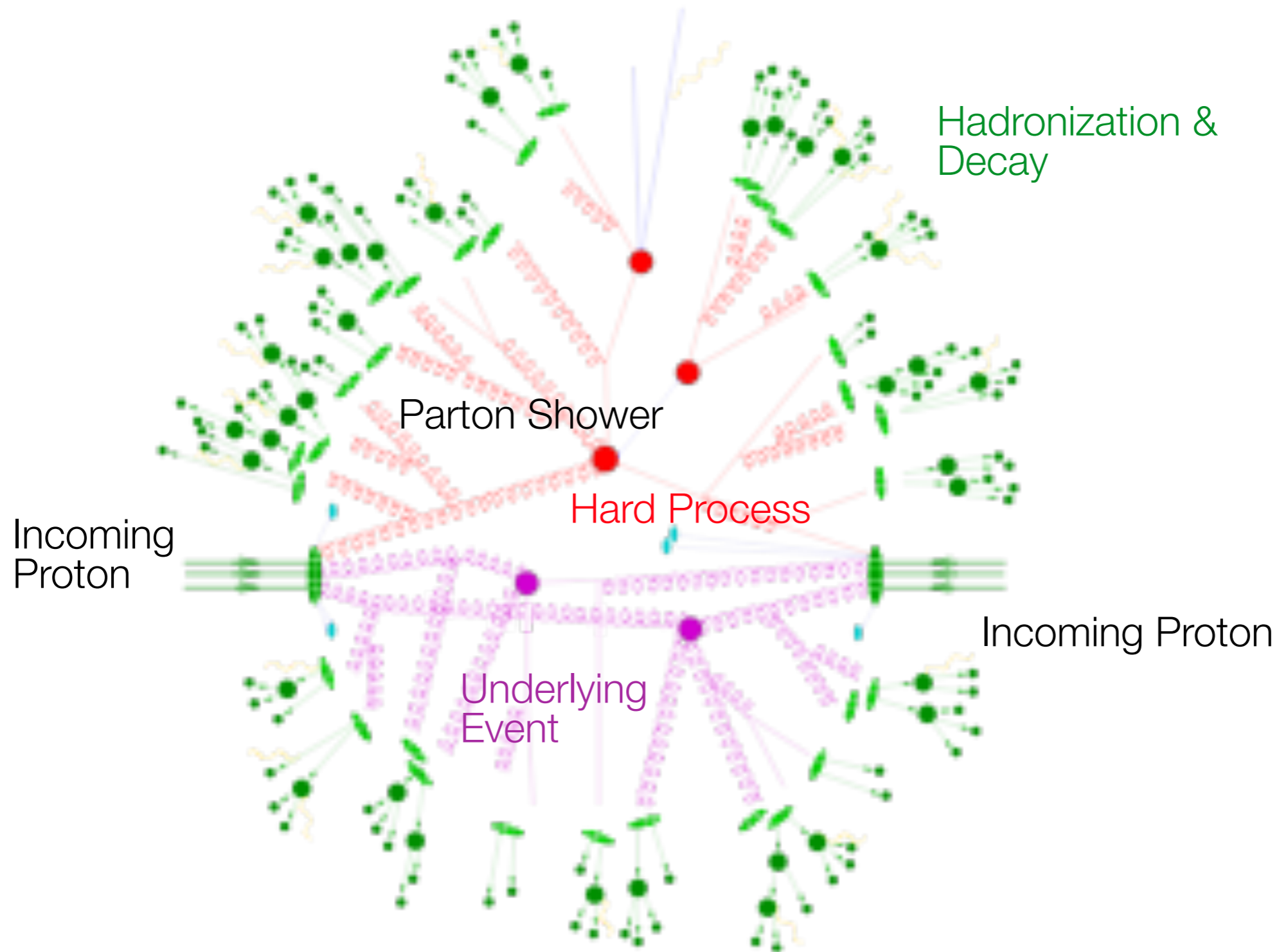
JHEP12(2012)061

FxFx (Frederix-Frixione) merging

- 1) define a matching scale μ_Q ;
- 2) don't allow **S** events with $p_T > \mu_Q$ (those will be provided by **H** events of $n-1$ partons NLO real emission); the restriction is imposed both at ME and on the shower starting scale $\mu < \mu_Q$
- 3) treat the obtained events as LO ones and apply an LO-style merging (this allow to produce smoother distributions)



Let's recap



From partons to color neutral hadrons

Fragmentation:

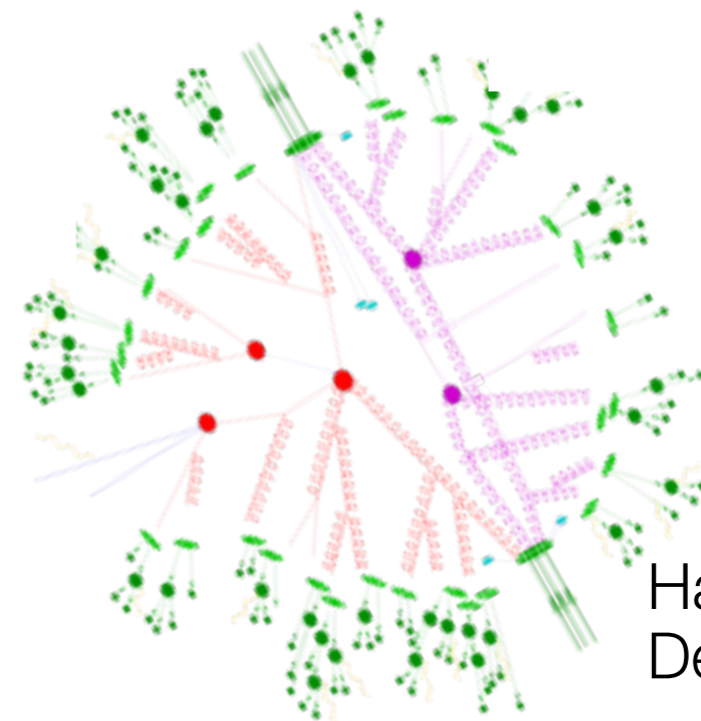
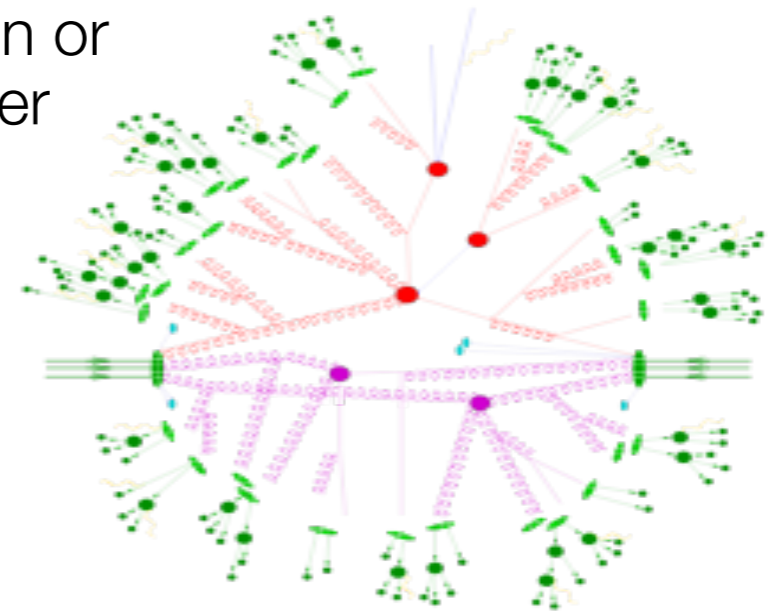
Parton splitting into other partons
[QCD: re-summation of leading-logs]
[“Parton shower”]

Hadronization:

Parton shower forms hadrons
[non-perturbative, only models]

Decay of unstable hadrons
[perturbative QCD, electroweak theory]

Fragmentation or
Parton Shower



Hadronization &
Decays

Non-perturbative transition from partons to hadrons ...

[Modelling relies on **phenomenological models** available]

Models based on MC simulations
very successful:

Generation of **complete final states** ...

[Needed by experimentalists in detector simulation]

Caveat: **tunable ad-hoc parameters**

Most popular MC models:

Pythia/8 : **Lund string model**

Herwig/++ : **Cluster model**

Independent fragmentation of each parton

Simplest approach:

[Field, Feynman, Nucl. Phys. B136 (1978) 1]

Start with original quark

Generate quark-antiquark pairs from vacuum

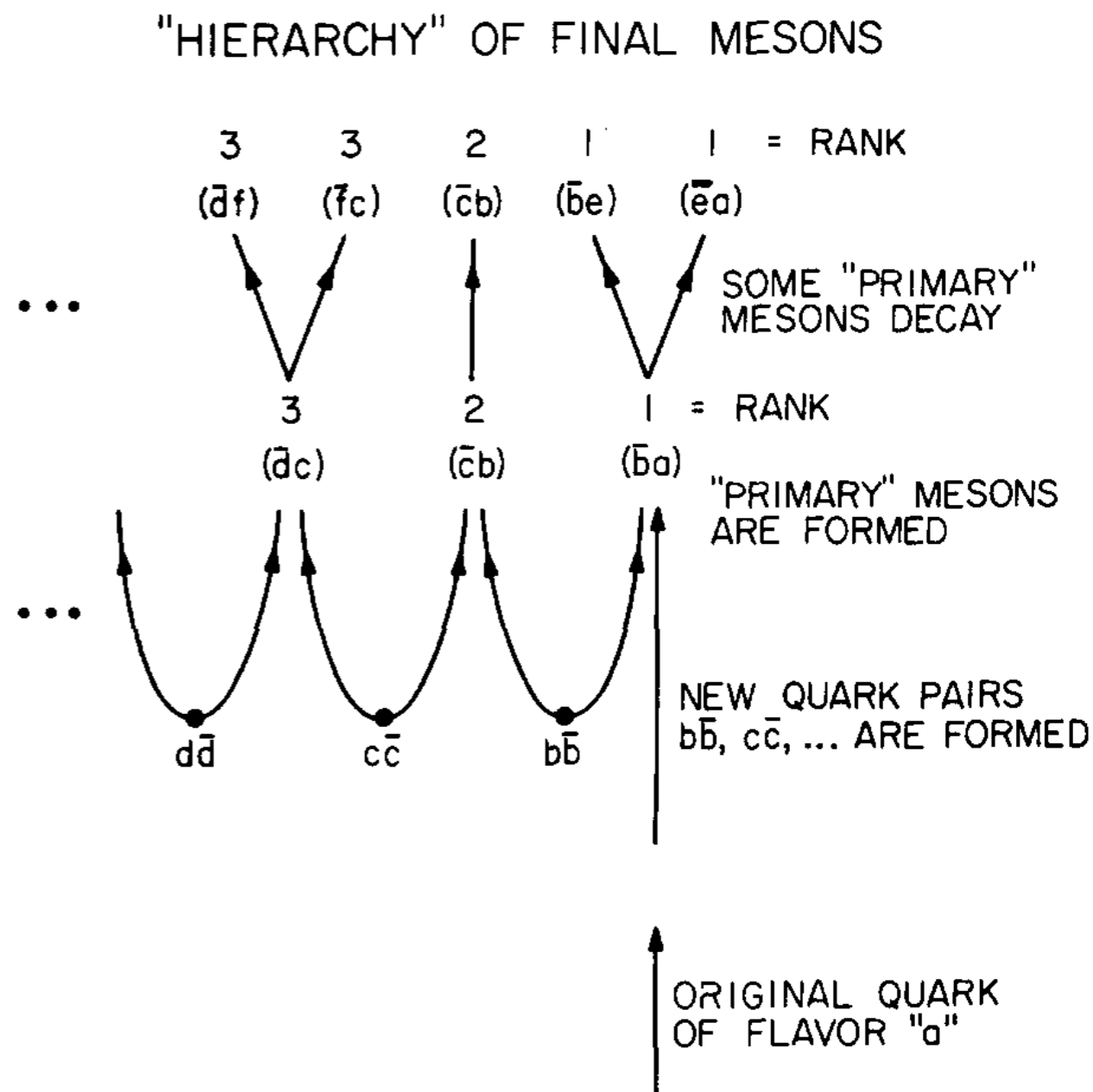
→ form "primary meson" with energy fraction z

Continue with leftover quark with energy fraction $1-z$

Stop at low energies (cut-off)

Include flavour non-perturbative fragmentation functions $D(z)$

$D(z)$: probability to find a meson/hadron with energy fraction z in jet ...

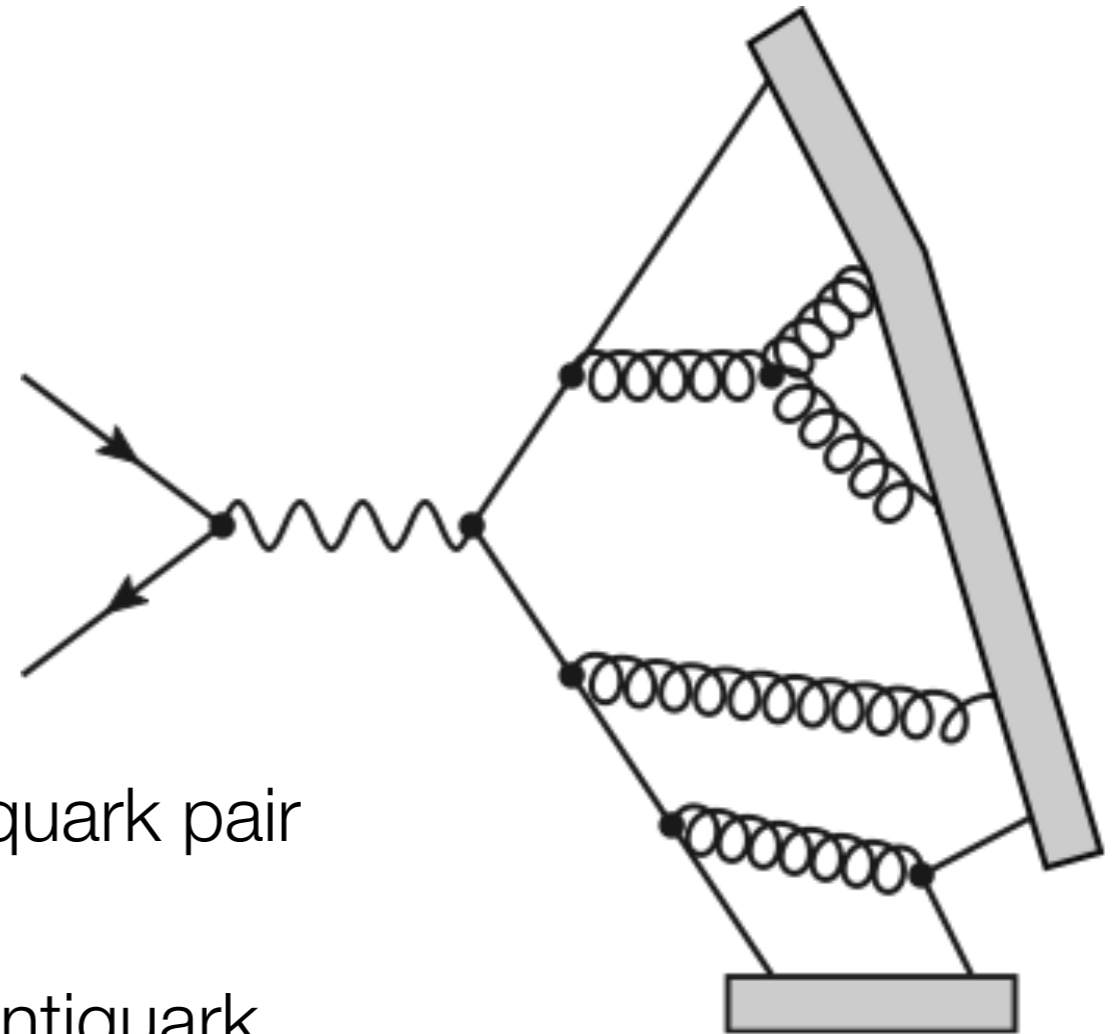


Lund String Model

[Andersson et al., Phys. Rep. 97 (1983) 31]

QCD potential:

$$V(r) = \underbrace{-\frac{4}{3} \frac{\alpha_s (1/r^2)}{r}}_{\text{neglected}} + kr$$



String formation between initial quark-antiquark pair

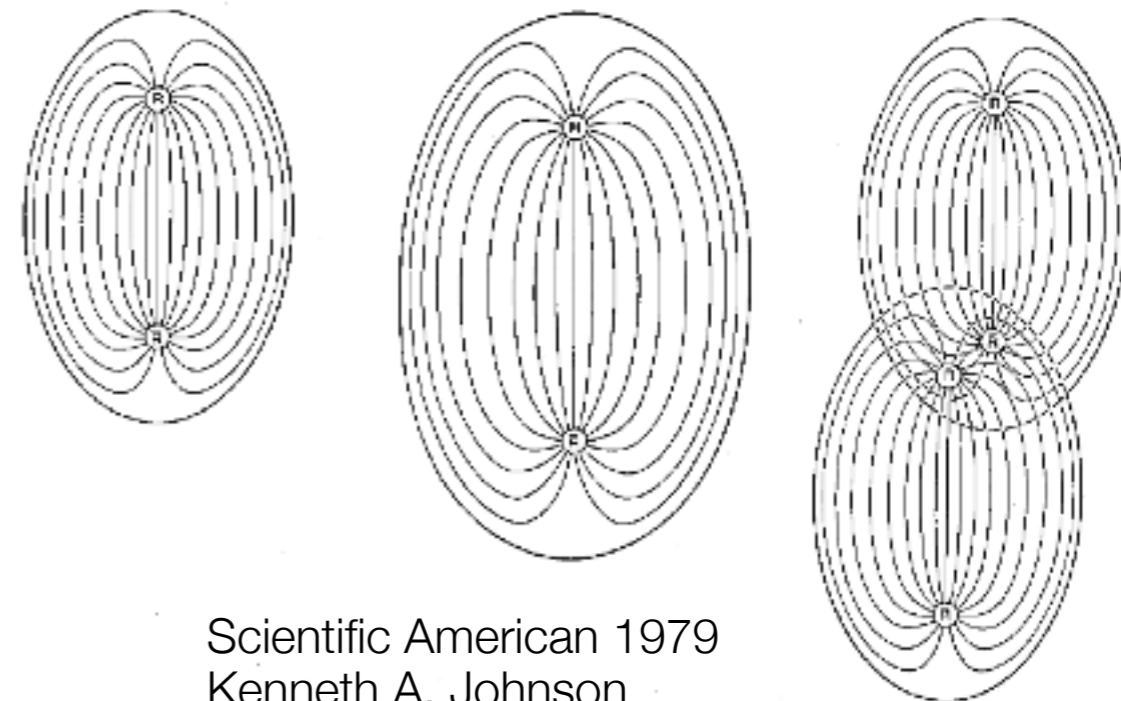
- String breaks up if potential energy large enough to produce a new quark-antiquark pair
- Gluons = 'kinks' in string
- At low energy: hadron formation
- Very widely used ... [default in Pythia 6/8]

Lund String Model

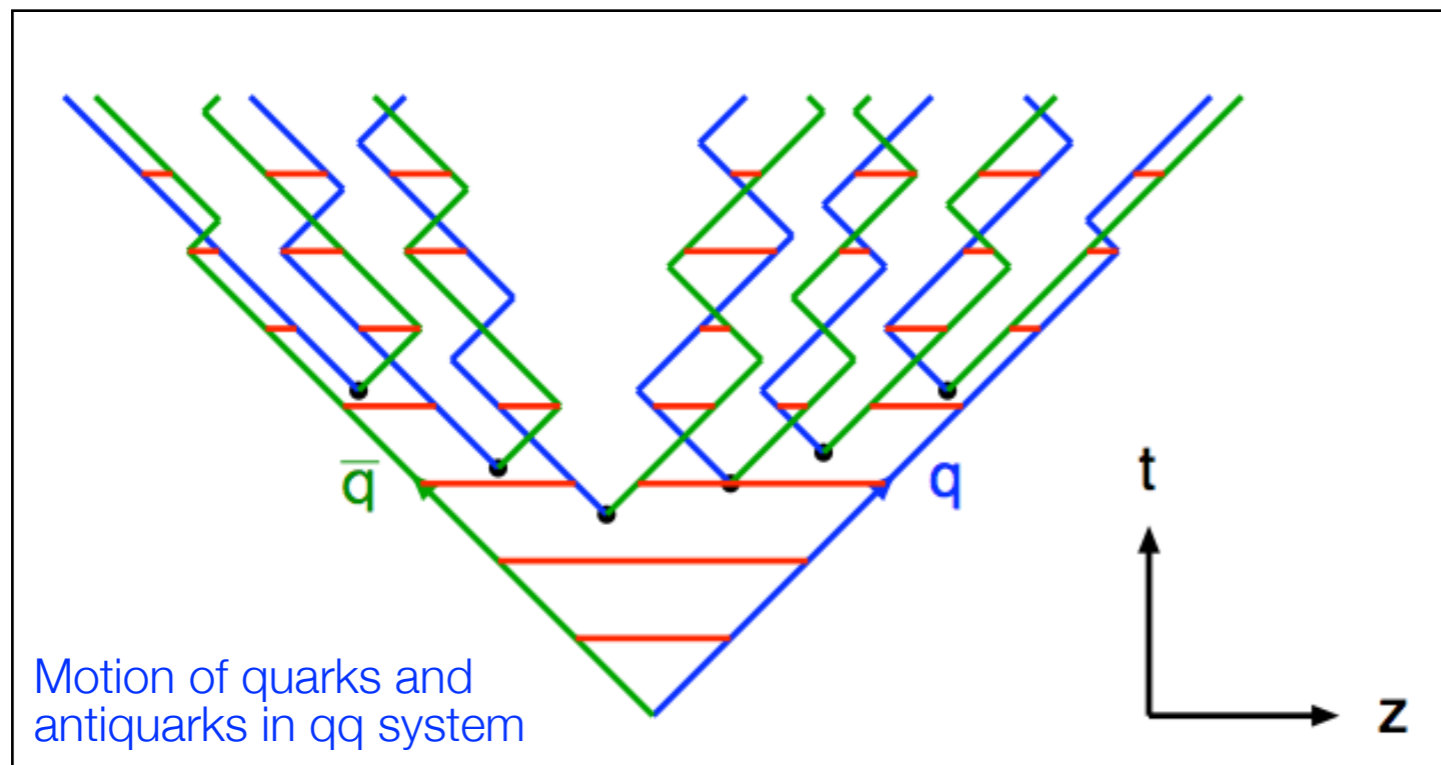
Repeated string breaks for large system with pure $V(r) = \kappa \cdot r$, i.e. neglect Coulomb part

$$\left| \frac{dE}{dz} \right| = \left| \frac{dp_z}{dz} \right| = \left| \frac{dE}{dt} \right| = \left| \frac{dp_z}{dt} \right| = \kappa$$

Energy-momentum quantities can be read from space-time quantities ...



Scientific American 1979
Kenneth A. Johnson



Simple but powerful picture of hadron production

[with extensions to massive quarks, baryons, ...]

$$\mathcal{P} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) \propto \exp\left(-\frac{\pi p_{\perp q}^2}{\kappa}\right) \exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

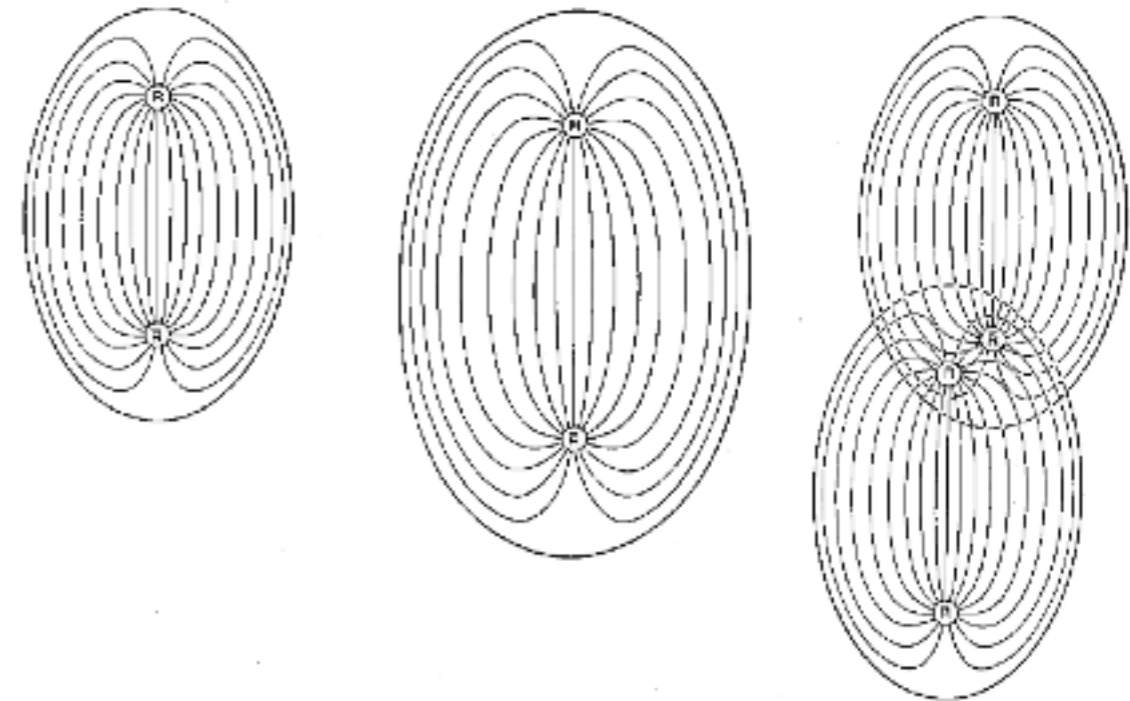
Yields: Common Gaussian p_{\perp} spectrum
Heavy quark suppression

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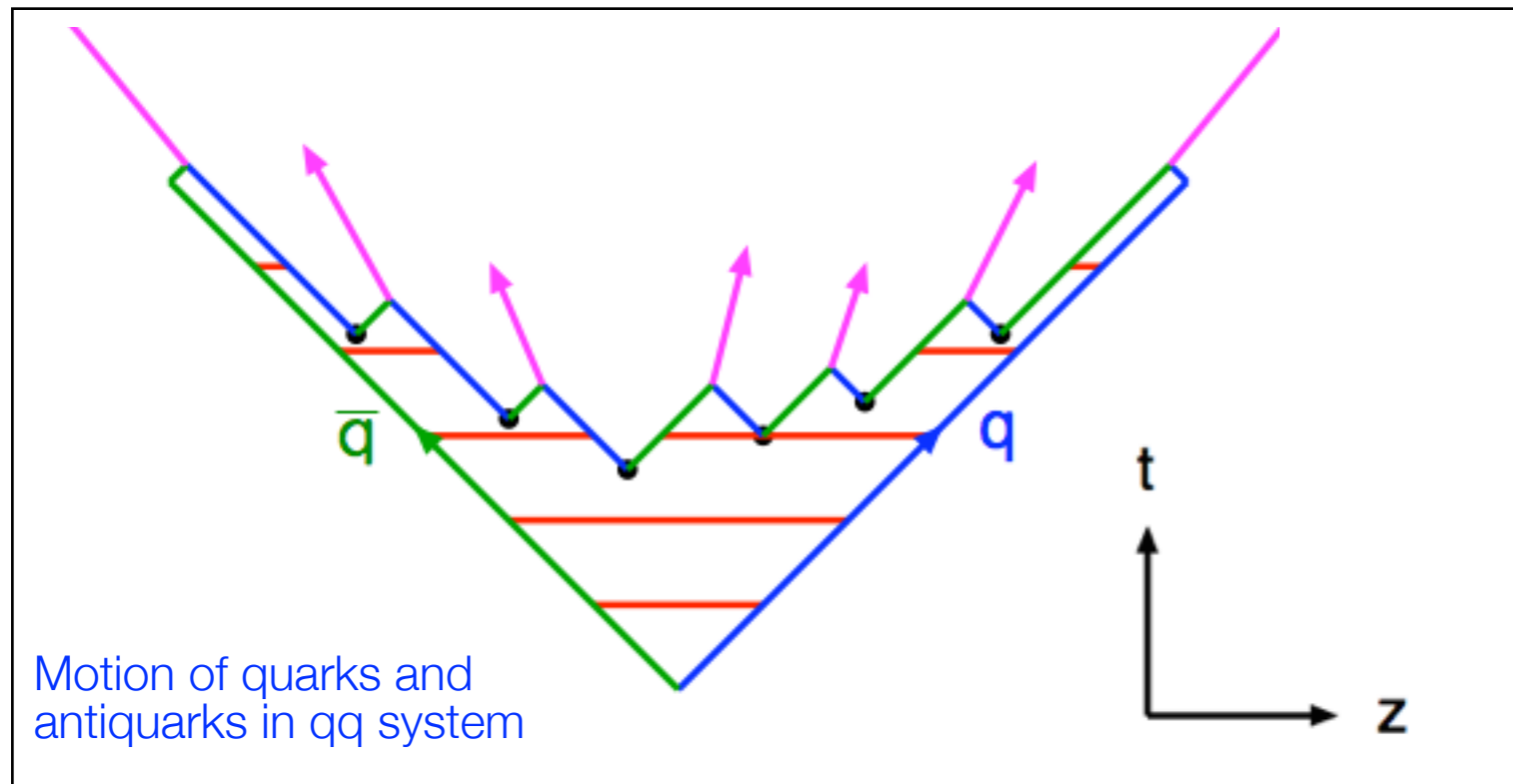
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Heavy quark suppression



Cluster Model

[Webber, Nucl. Phys. B238 (1984) 492]

Color flow confined during hadronisation process

→ Formation of color-neutral parton clusters

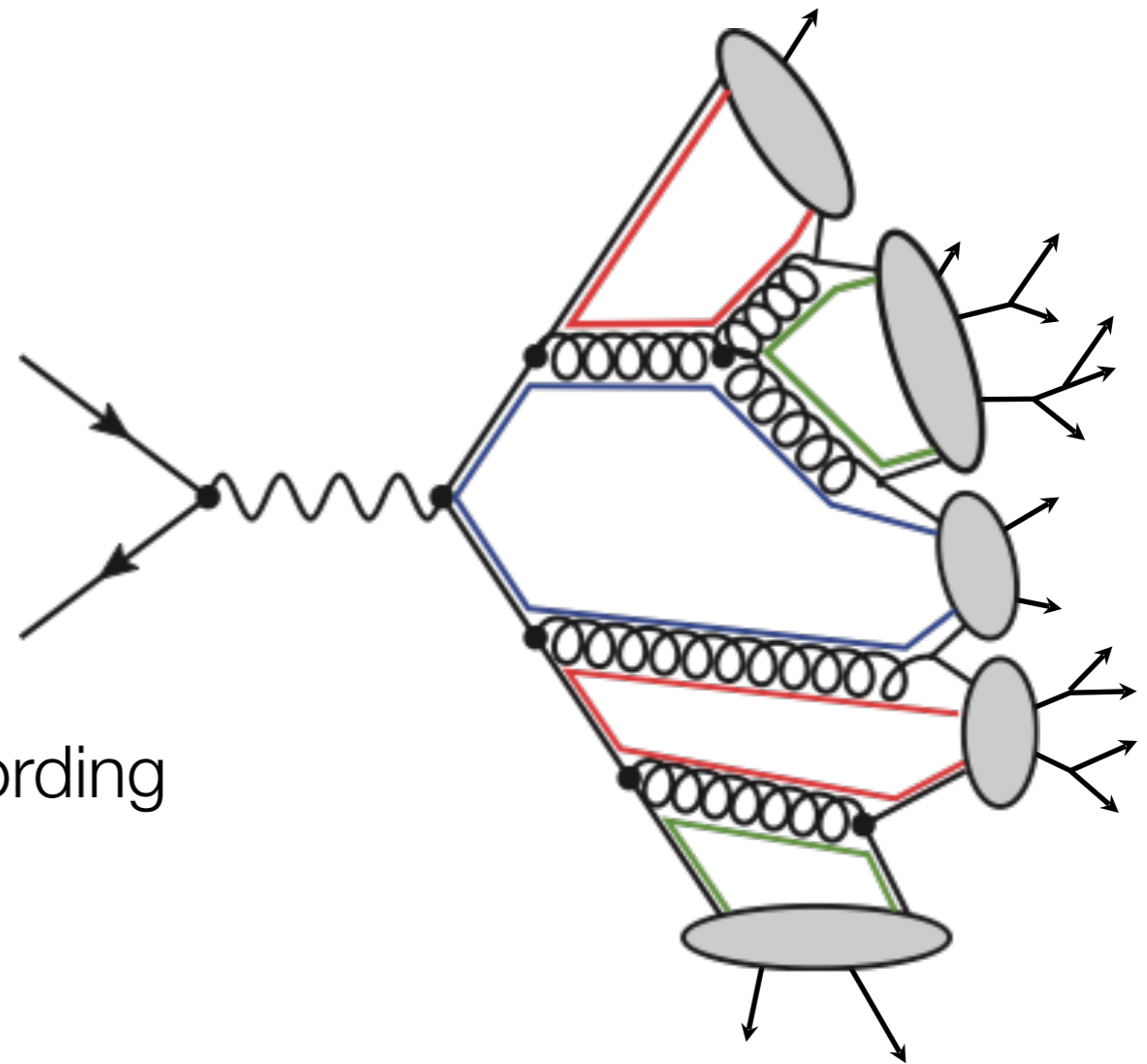
Gluons (color-anticolor) split to quark-antiquark pairs

Clusters decay into 2 hadrons according to phase-space, i.e. isotropically

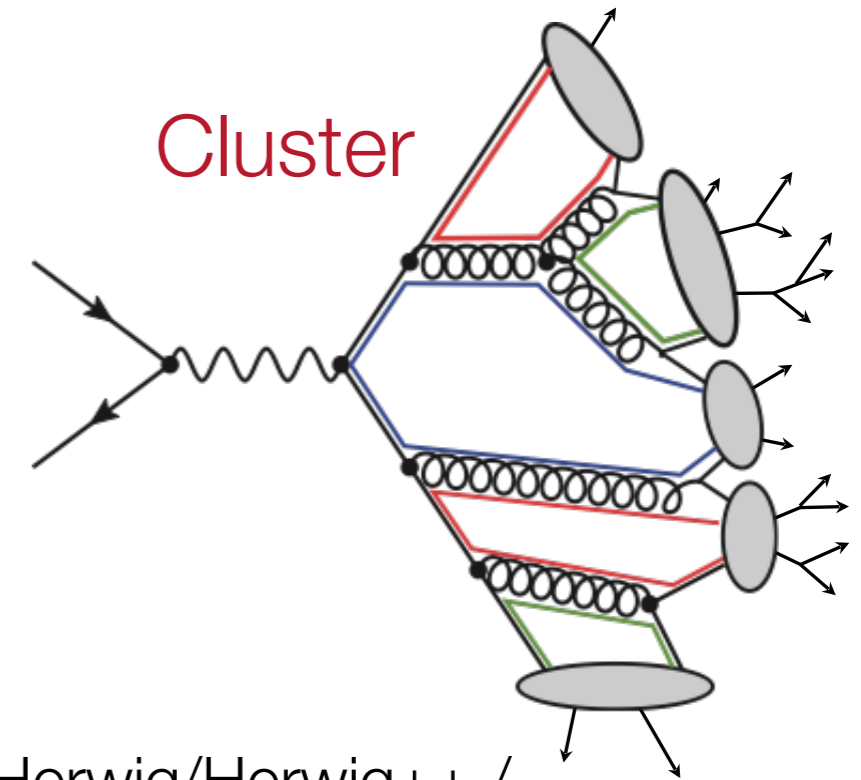
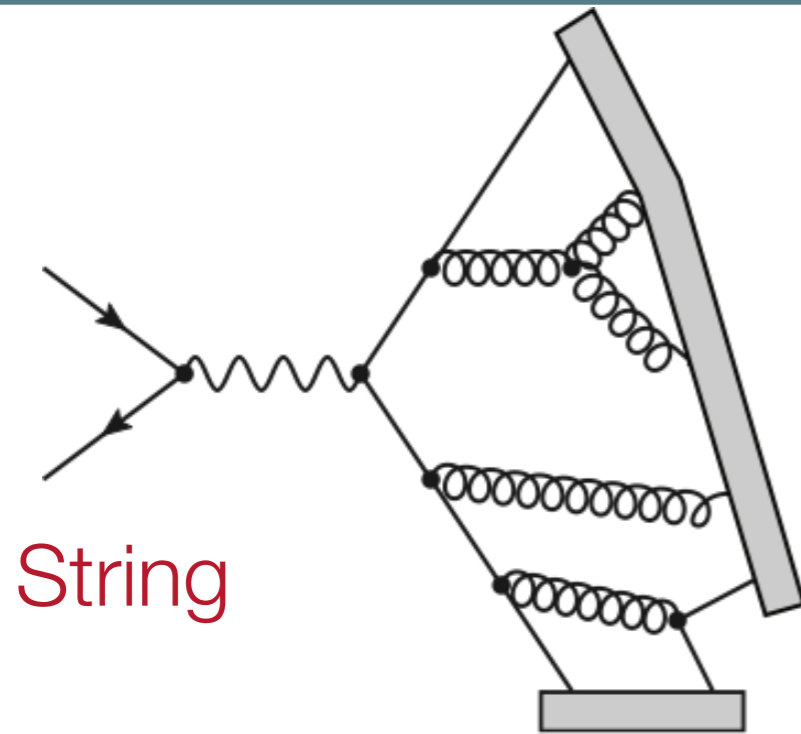
→ no free tuning parameters
parton clusters

Very widely used ...

[default in Herwig/Herwig++]



Hadronisation models summary



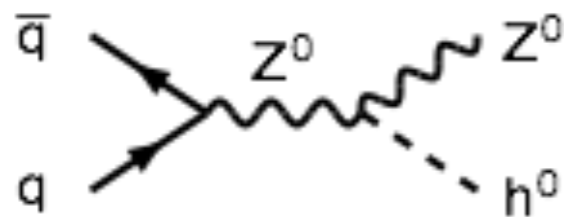
Model	Pythia6/8 (string)	Herwig/Herwig++ / Sherpa(cluster)
Energy-mom. picture	powerful predictive	simple unpredictive
Parameters	few	many
Flavour composition	messy unpredictive	simple in-between
Parameters	many	few

Structure of basic generator process [by order of consideration]

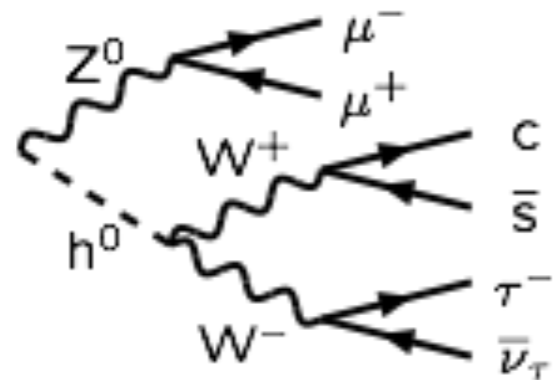
From the 'simple' to the 'complex' or from 'calculable' at large scales to 'modelled; at small

Matrix elements (ME)

1. Hard subprocess:
 $|M|^2$, Breit Wigners, PDFs

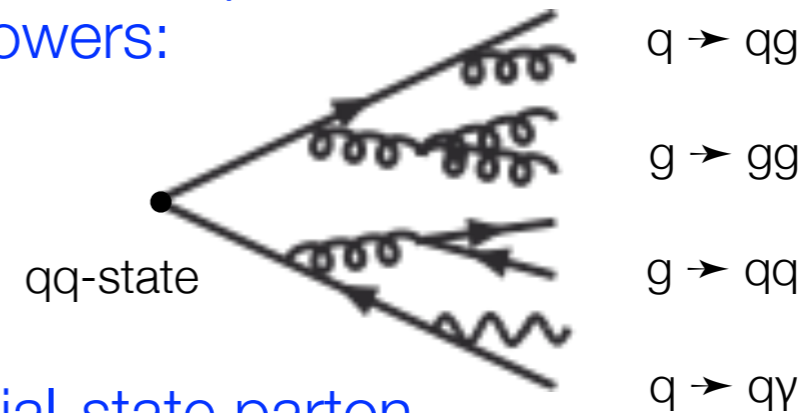


2. Resonance decays:
 Includes particle correlations

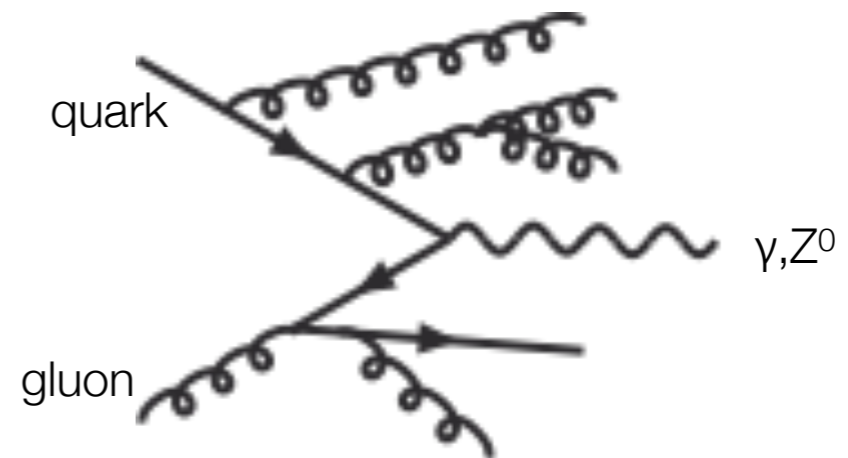


Parton Shower (PS)

3. Final-state parton showers:



4. Initial-state parton showers:

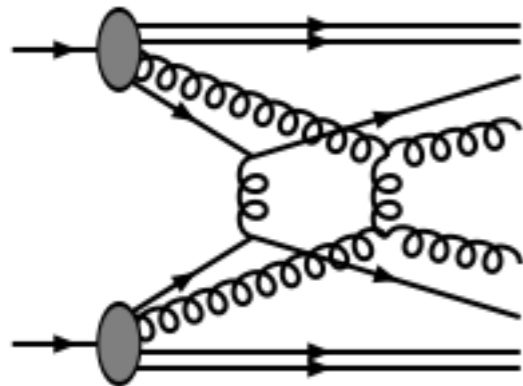


Conclusions: Structure of basic generator process

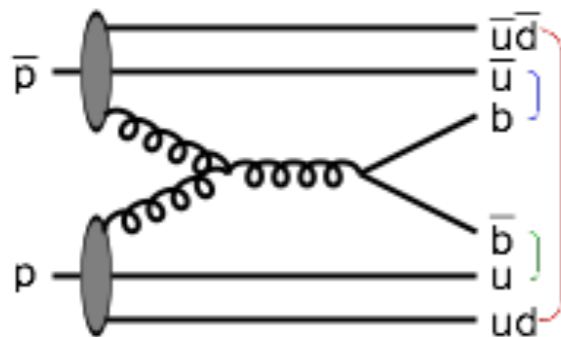
From the 'simple' to the 'complex' or
from 'calculable' at large scales to 'modelled'; at small

Underlying Event (UE)

5. Multi-parton interaction:

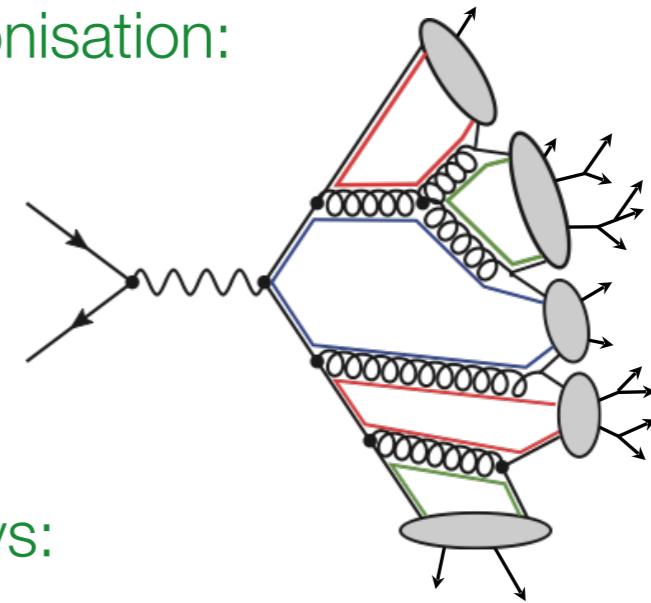


6. Beam remnants:

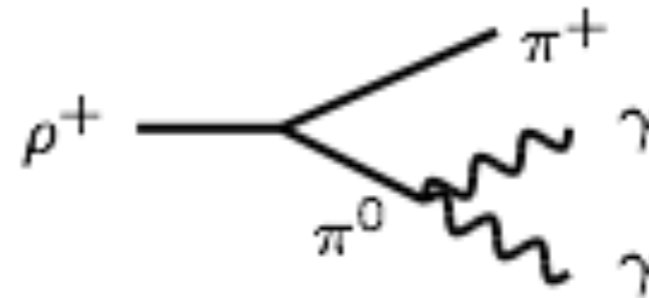


Stable Particle State

7. Hadronisation:



8. Decays:



The DGLAP evolution equation is said to **resum large collinear logarithms**. So where are these logarithms, and where is the resummation?

Let's perform the integration of the DGLAP equation and expand the result:

$$\begin{aligned}
 f(x, t) &= f_0(x) + \int_{t_0}^t \frac{dt'}{t'} \frac{\alpha_s(t')}{2\pi} \int_x^1 \frac{dz}{z} P(z) g\left(\frac{x}{z}, t'\right) \\
 &= f_0(x) + \int_{t_0}^t \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \left\{ f_0\left(\frac{x}{z}\right) + \right. \\
 &\quad \left. + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') \left[f_0\left(\frac{x}{zz'}\right) + \dots \right] \right\} \\
 &= f_0(x) + \frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right) \int_x^1 \frac{dz}{z} P(z) f_0\left(\frac{x}{z}\right) + \\
 &\quad + \frac{1}{2!} \left[\frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right) \right]^2 \int_x^1 \frac{dz}{z} P(z) \int_{x/z}^1 \frac{dz'}{z'} P(z') f_0\left(\frac{x}{zz'}\right) + \dots
 \end{aligned}$$

As suggested by the last step, it is indeed a resummation of all terms proportional to $\left[\frac{\alpha_s}{2\pi} \ln\left(\frac{t}{t_0}\right) \right]^n$.

