Accelerators

Lecture III

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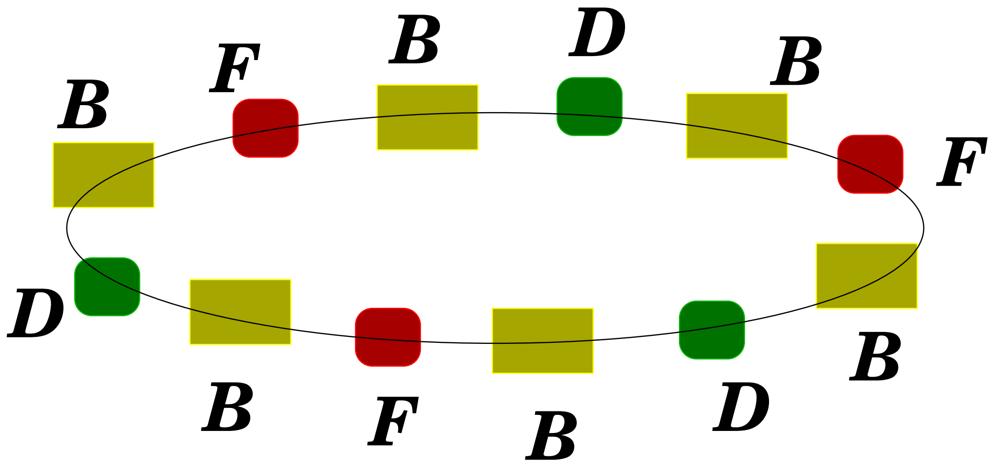
Summary Lecture II

- Collider Concept
- Weak and Strong Focusing
- Beam Optics
- Longitudinal Focusing

III) Orbit Stability + Long Term Stability

- Closed Orbit
- Linear Resonances + Orbit Stability
- Chromaticity + Sextupoles
- Long Term Stability
  - Non-linear resonances
  - Detuning with amplitude
- Summary

Closed Orbit



$$B_x = -g \cdot y$$

$$B_y = -g \cdot x$$

Orbit Offset in Quadrupole:

$$x = x_0 + \tilde{x}$$

quadrupole

$$B_x = -g \cdot \tilde{y}$$

$$B_y = -g \cdot x_0 - g \cdot \tilde{x}$$

dipole component

→ orbit error

Sources for Orbit Offsets in Quadrupoles

- **Alignment:** +/- 0.1 mm
- **Ground motion**
  - slow drift
  - civilisation
  - moon
  - seasons
  - civil engineering
- **Error in dipole strength**
  - power supplies
  - calibration
- **Energy error of particles**

Orbit error:

→

- *x-y coupling*
- *aperture*
- *ernergy error*
- *field imperfections*

Aim:

$\Delta x, \; \Delta y < 4$ mm

rms < 0.5 mm

→ beam monitors and orbit correctors

LEP:

784 quadrupoles

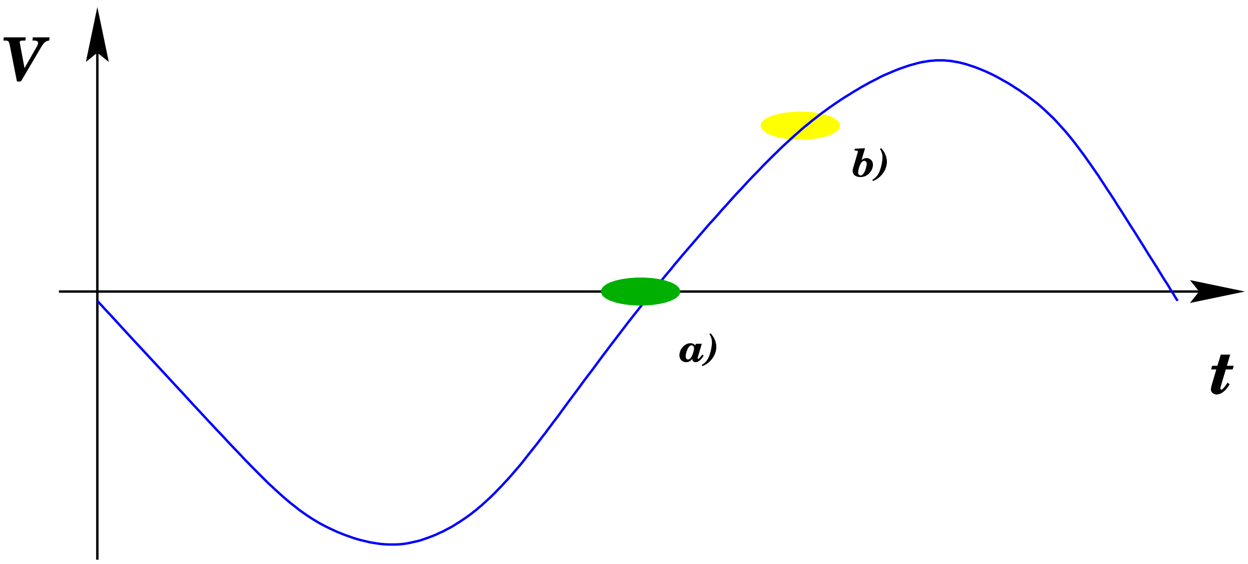
$\approx \; 2 \cdot 540$ monitors

$\approx \; 2 \cdot 300$ correctors

Synchrotron:

→ ***the orbit determines the particle energy!***

assume: L > design orbit



→ ***energy increase***

Equilibrium:

$$f_{RF} = h \cdot f_{rev}$$

$$f_{rev} = \frac{1}{2 \cdot \pi} \cdot \frac{q}{m \cdot \gamma} \cdot B$$

→ ***E depends on orbit and magnetic field!***

Particle Motion:

- find closed orbit

  $x_0(s); y_0(s)$

- transverse oscillations around closed orbit

→ complete description of particle motion

CO: periodic: $x_0(s + L) = x_0(s)$
common to all particles

ϕ, β: periodic: $\beta_0(s + L) = \beta_0(s)$
common to all particles

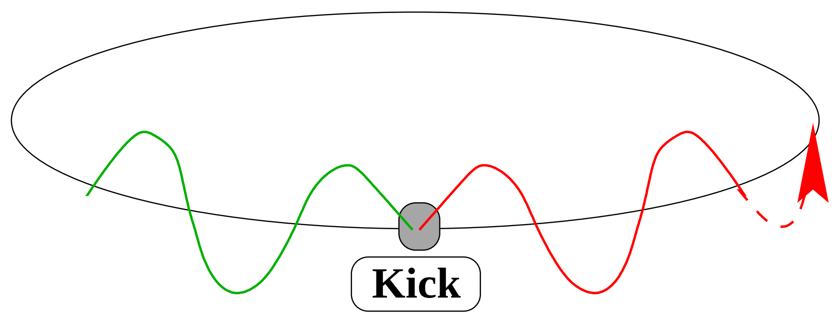
ϕ_0, A: individual particle motion

ε: beam ensemble
beam quality

Dipole Error and Orbit Stability

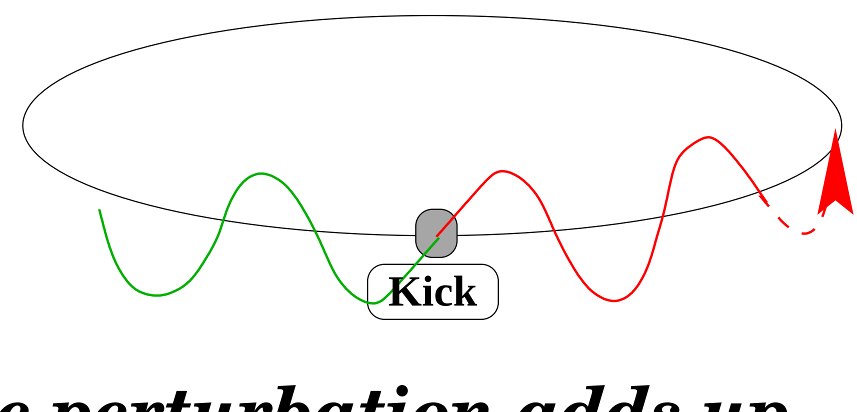
- ***Q:*** ***number of β-oscillations per turn***

$Q = N + 0.5$



→ ***the perturbation cancels after each turn***

$Q = N$



→ ***the perturbation adds up***

→ ***watch out for integer tunes!***

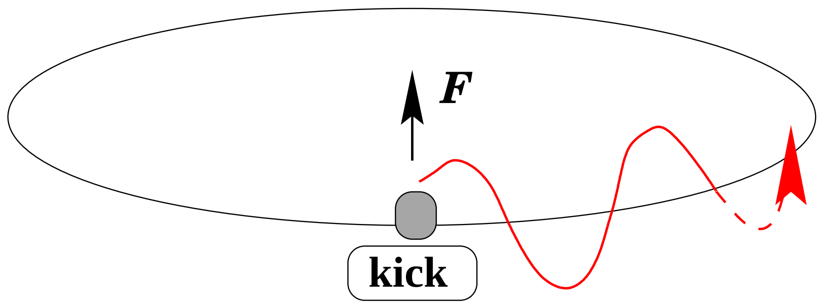
Quadrupole Error and Orbit Stability

Quadrupole Error:

→ ***orbit kick proportional to beam offset in quadrupole***

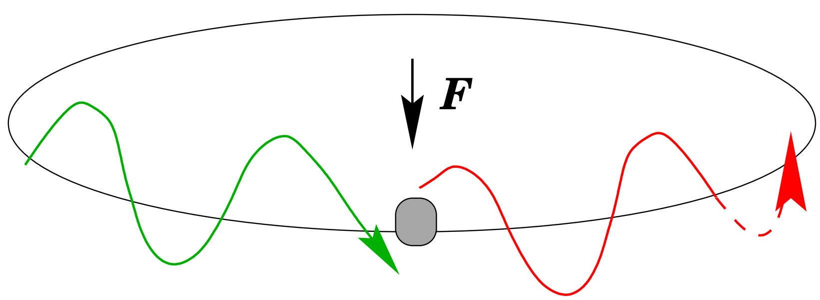
$Q = N + 0.5$

1. Turn: $x > 0$



→ ***amplitude increase***

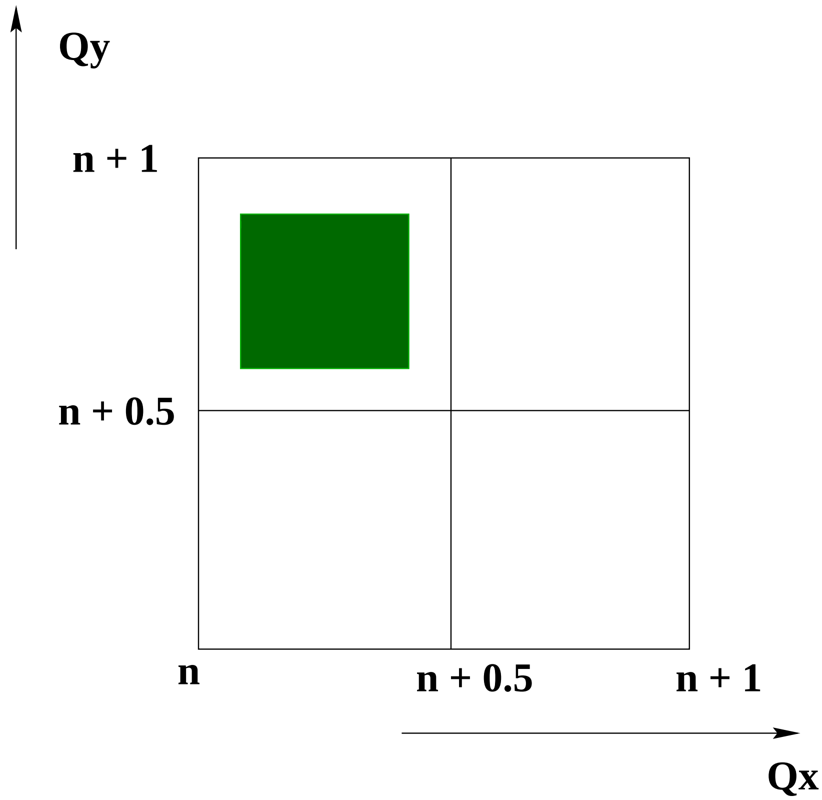
2. Turn: $x < 0$



→ ***amplitude increase***

→ ***watch out for half integer tunes!***

Tune Diagram



Problem:

$$K\,(quadrupole) = \frac{e \cdot g}{p}$$ (lecture II)

→ $$Q = Q_0 + \xi \cdot \frac{\Delta p}{p_0}$$

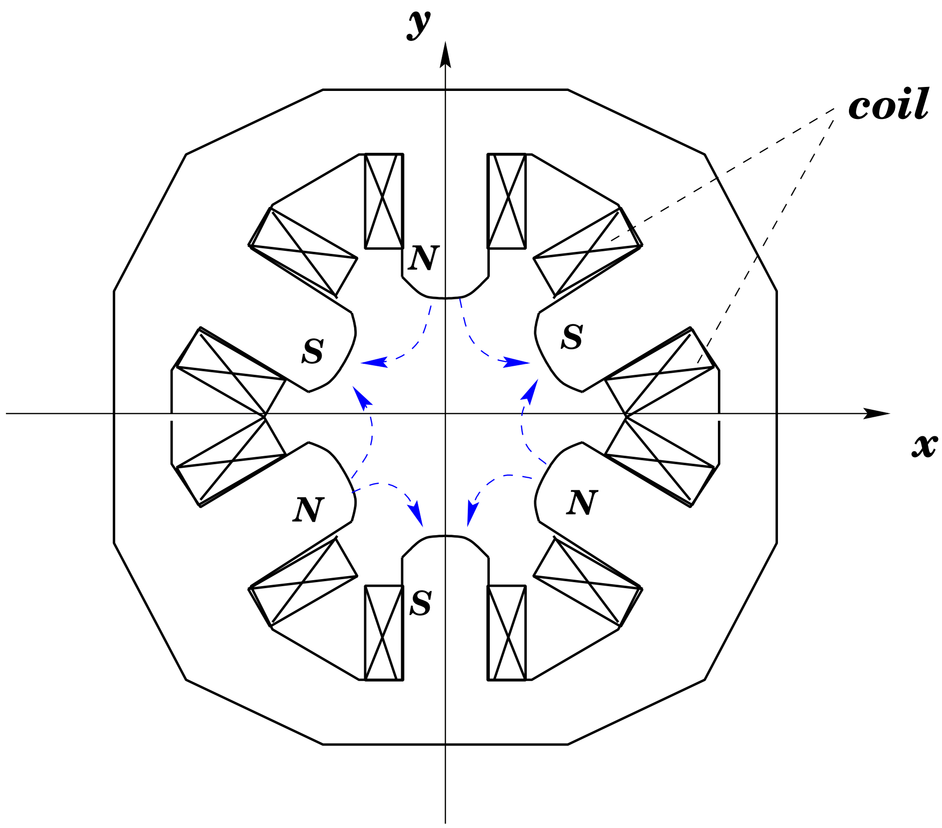
Large Machine (LEP / LHC):

$$\xi \approx 100 \div 500$$

$$\frac{\Delta p}{p_0} \approx 10^{-3}$$

→ **requires correction!**

Sextupole Magnet



$$B_x = \tilde{g} \cdot x \cdot y$$

$$B_y = \frac{1}{2} \cdot \tilde{g} \cdot (x^2 - y^2)$$

$$[\tilde{g}] = T / m^2$$

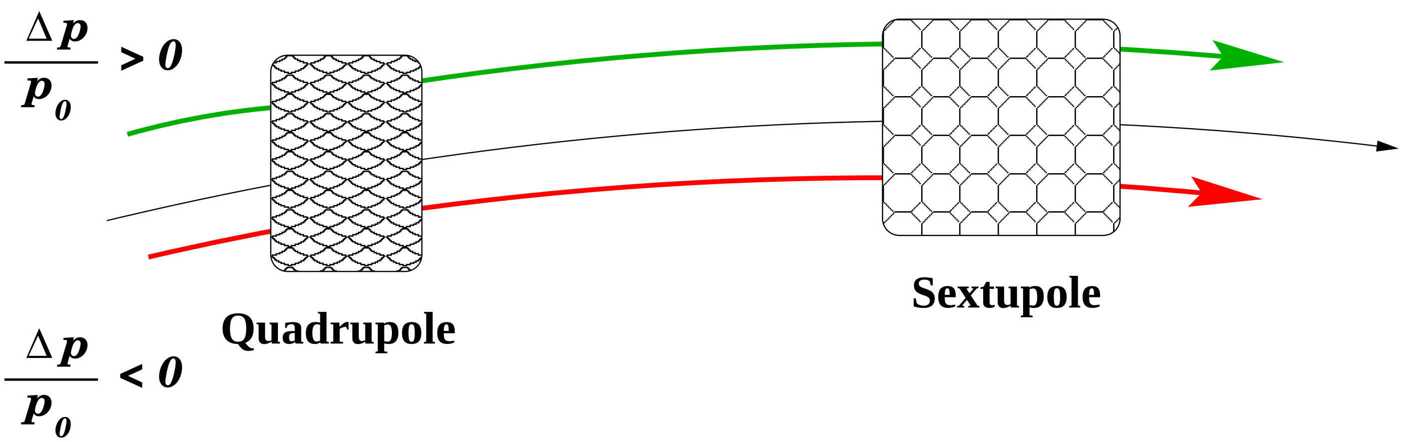
Orbit Offset: $y = y_0 + \tilde{y}$

$$B = \tilde{g} \cdot x \cdot y_0 + \tilde{g} \cdot x \cdot \tilde{y}$$

quadrupole component

$$B = \frac{1}{2} \cdot \tilde{g} \cdot (x^2 - \tilde{y}^2) - \tilde{g} \cdot y_0 \cdot \tilde{y}$$

Chromaticity Correction



$$x(s) = x_0(s) + D(s) \cdot \frac{\Delta p}{p_0}$$

→ *offset in sextupole*

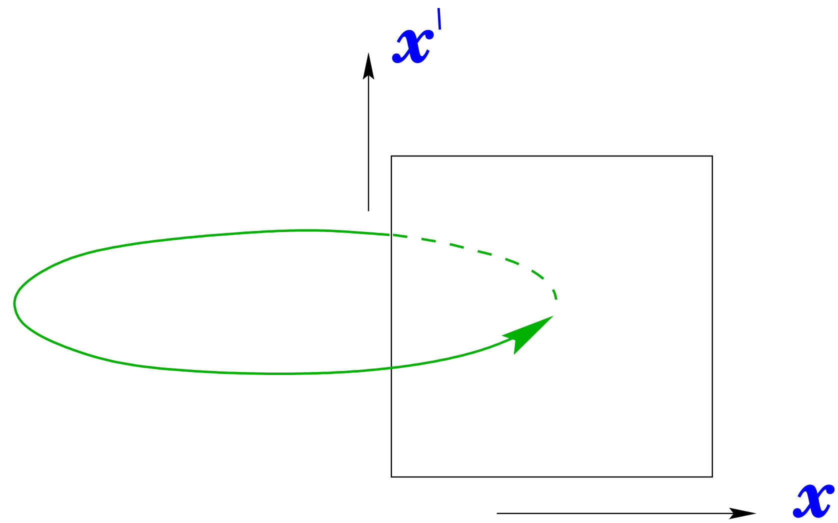
→ $$Q = Q_0 + \underbrace{\Delta Q_Q\left(\frac{\Delta p}{p_0}\right) + \Delta Q_S\left(\frac{\Delta p}{p_0}\right)}_{\approx 0}$$

- ***Problem:***

  non-linear resonances

Poincare Section

Display coordinates after each turn:



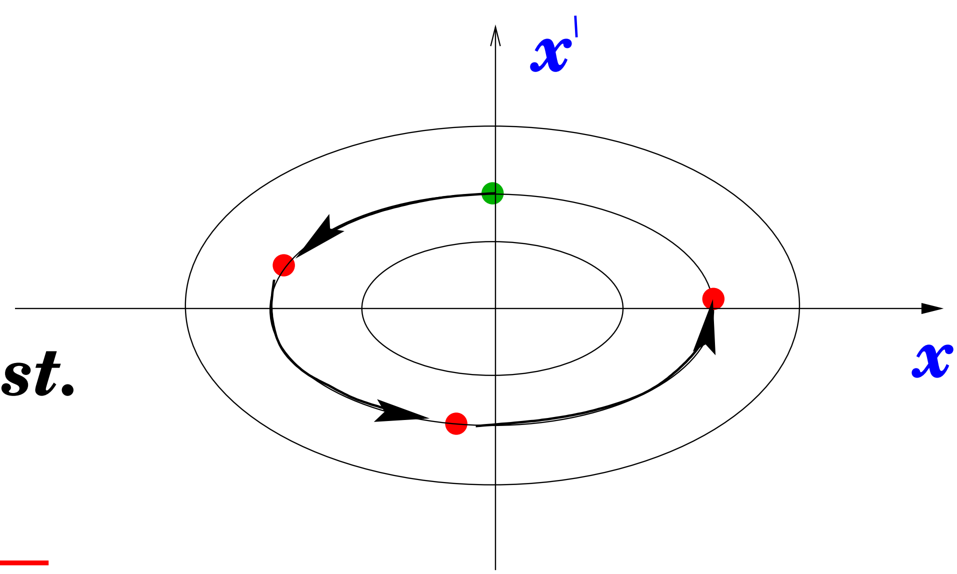
Linear β - motion:

$$x = a \cdot \sqrt{A} \cdot \sin\left(\frac{2\pi}{L} \cdot Q \cdot s + \phi_0\right) \qquad (\beta = const)$$

$$x' = b \cdot \sqrt{A} \cdot \cos\left(\frac{2\pi}{L} \cdot Q \cdot s + \phi_0\right)$$

→ ***ellipse***

$$\frac{x^2}{a^2} + \frac{x'^2}{b^2} = const. = A$$



Sextupole Perturbation

Lorentz Force:

$$x' = \frac{F}{v \cdot p} \longrightarrow x' = q \cdot \frac{B_y}{p}$$

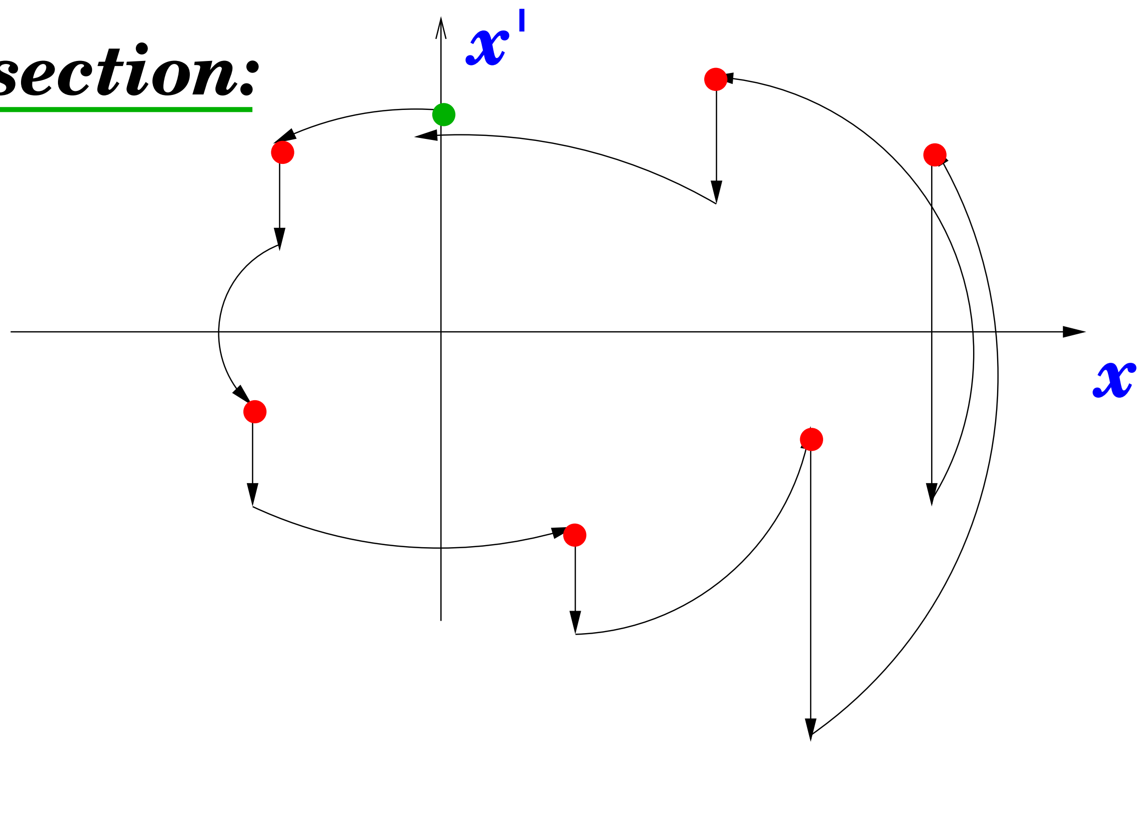
Sextupole Magnet:

$$B_y = \frac{1}{2} \cdot \tilde{g} \cdot x^2$$

$$\longrightarrow \Delta x' = \int_l \frac{F}{v \cdot p} \, ds$$

$$= \frac{1}{2} \cdot \frac{l}{v} \cdot \frac{q}{m} \cdot \tilde{g} \cdot x^2$$

Poincare section:



Amplitude Growth

$$R = x^2 + x'^2$$

$$\frac{dR}{ds} = 2 \cdot x \cdot x' + 2 \cdot x' \cdot x''$$

- **Sextupole Kick:**

$$\frac{\Delta R}{Turn} = \frac{1}{4} \cdot \frac{q}{m} \cdot \frac{l}{v} \cdot \tilde{g} \cdot \left[3\cos(\phi) + \cos(3\phi)\right]$$

$(\Delta\phi / Turn = 2\pi Q)$

- **Many Turns:**

$\Delta R = 0$ unless: $Q, 3 \cdot Q = n$

- **Perturbation Theory:**

→ *all resonances are driven!*

Detuning with Amplitude

Non-linear Perturbation:

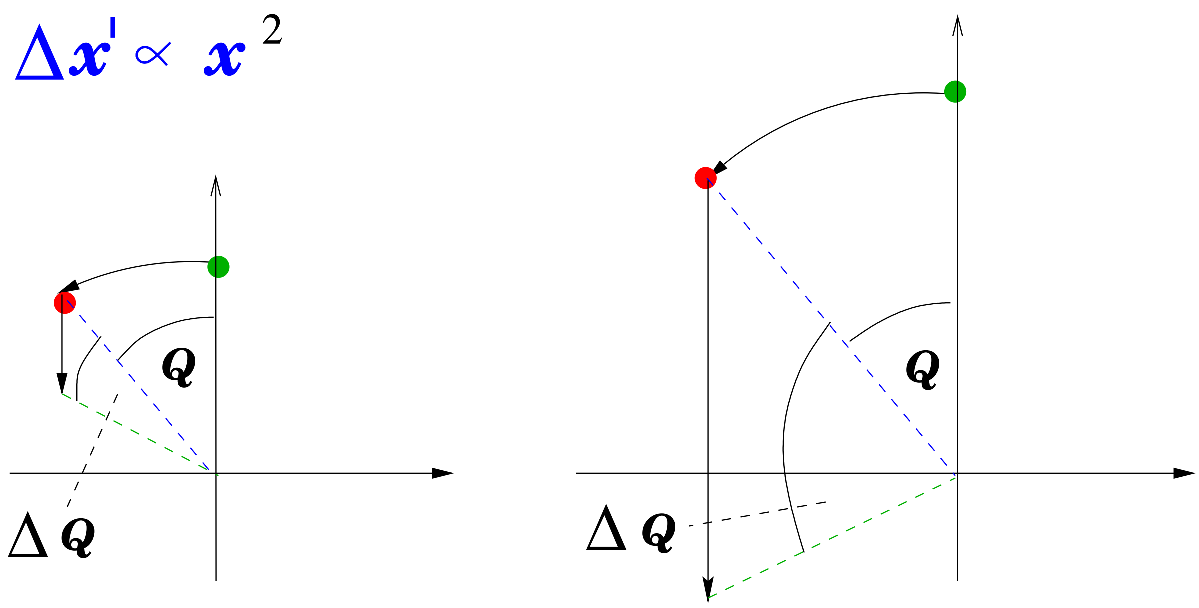
avoid resonances: $r \cdot Q = n$!

Problem:

there are resonances everywhere!

Stabilisation Mechanism:

$\Delta x' \propto x^2$



Poincare section

Long Term Stability

- ***Non-linear Perturbation:***
  - *amplitude growth*
  - *detuning with amplitude*
  - *coupling*

sextupole: $(B_y = g \cdot [x^2 - y^2])$

→ ***Complex dynamics:***

3 degrees of freedom

\+ *1 invariant of the motion*

\+ *non-linear dynamics*

→ ***no analytical solution!***

→ *analysis of long term stability relies on numerical simulations*

Sources for Non-Linear Fields

- **Sextupoles**

- **Magnet errors:**

  pole face accuracy

  geometry errors

  eddy currents

  edge effects

- **Vacuum chamber:**

  LEP I welding

- **Beam-beam interaction**

→ *careful analysis of all components*

Summary Resonances

- **Linear Optics:**

$$Q = n \cdot \pi \; ; \; n \cdot \pi + \frac{1}{2}$$

- **Chromaticity:**

  → *sextupoles*

  → *resonance driving terms*

- **Non-Linear Resonances:**

  → *amplitude growth*

  → *detuning with amplitude*

  long term stability?

→ **classical mechanics**

+

chaos theory

Accelerator Model

- **Toy Model:** → *simple*

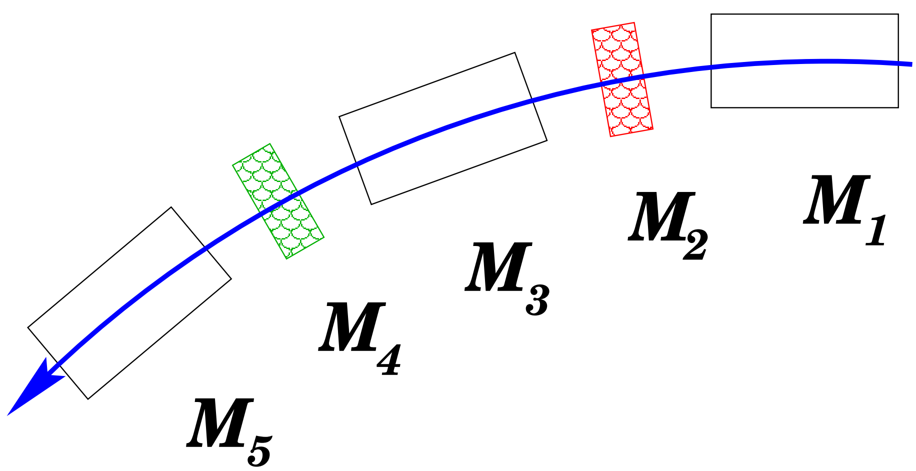
  HO + perturbation

  → **Hamilton Function**

  # > 1000 elements!

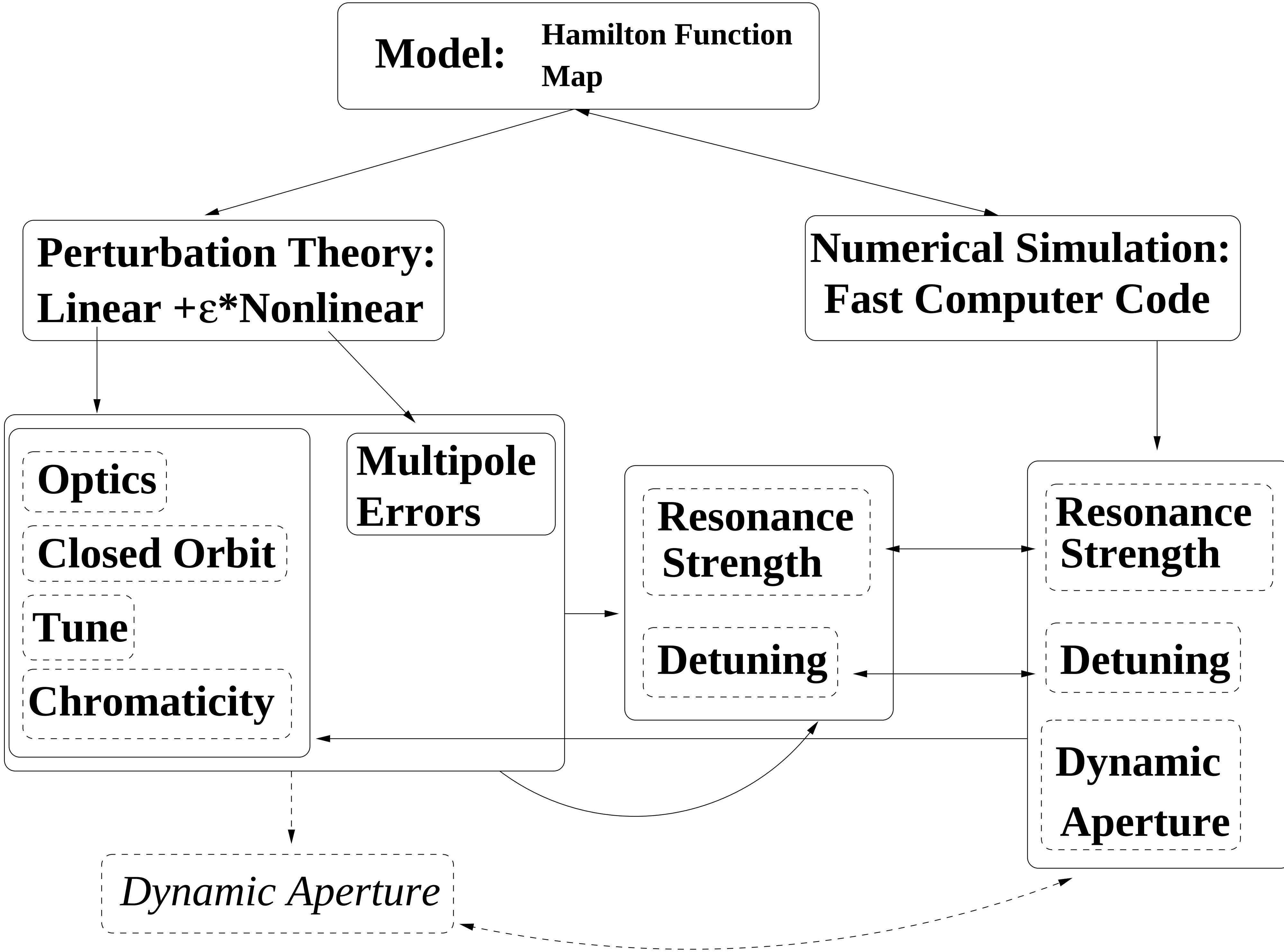
- **Element by Element Tracking**

  → **numberical analysis**



→ **One Turn Map (Taylor Series)**

→ **Hamilton Function**


Model:
Hamilton Function
Map
Perturbation Theory:
Linear +ε*Nonlinear
Numerical Simulation:
Fast Computer Code
Optics
Closed Orbit
Tune
Chromaticity
Multipole
Errors
Resonance
Strength
Detuning
Resonance
Strength
Detuning
Dynamic
Aperture
Dynamic Aperture