Accelerators

Lecture II

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Summary Lecture I

- ***History***

- ***Acceleration Concepts***

- ***Synchrotrons***

Storage Rings + Trajectories

Synchrotron Inventory

Bending Magnets

Collider Concept

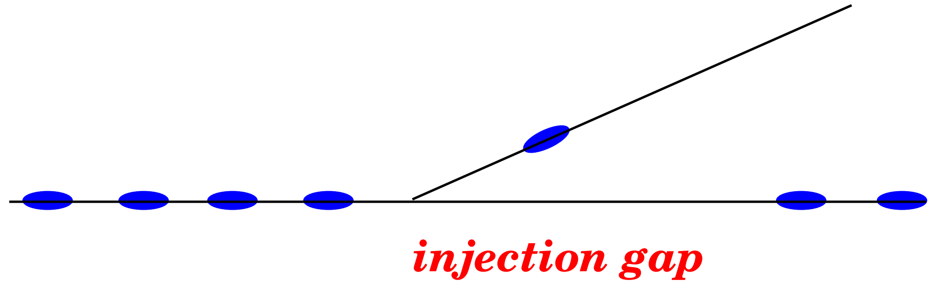
Trajectory Stability

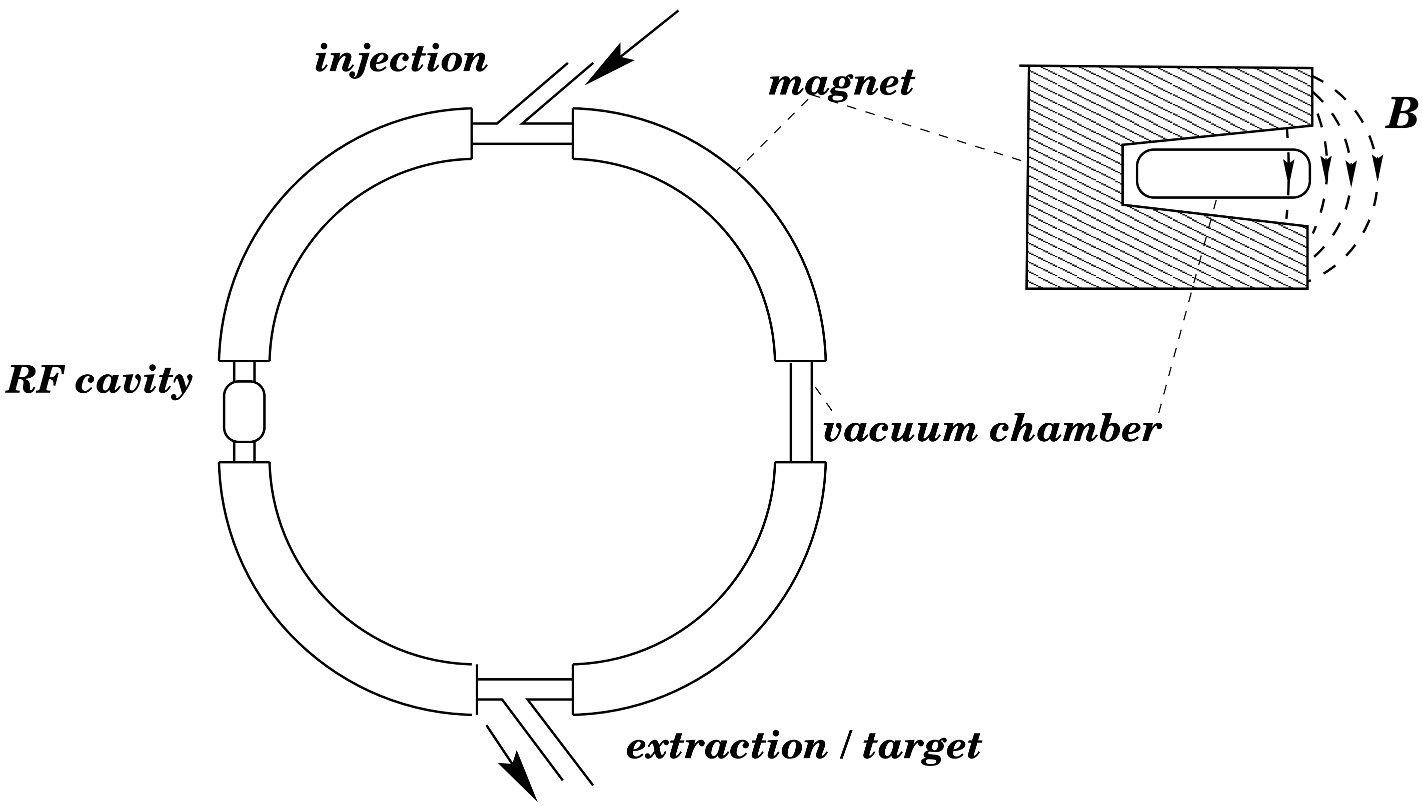
- Focusing
- Optic functions
- Longitudinal focusing
- Dispersion Orbit

Summary

Synchrotron Inventory

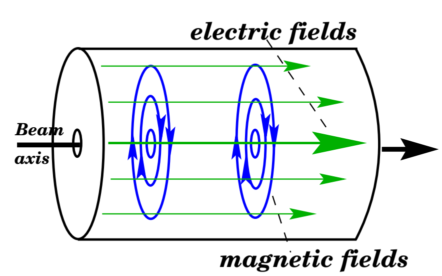
Injection:





Ejection

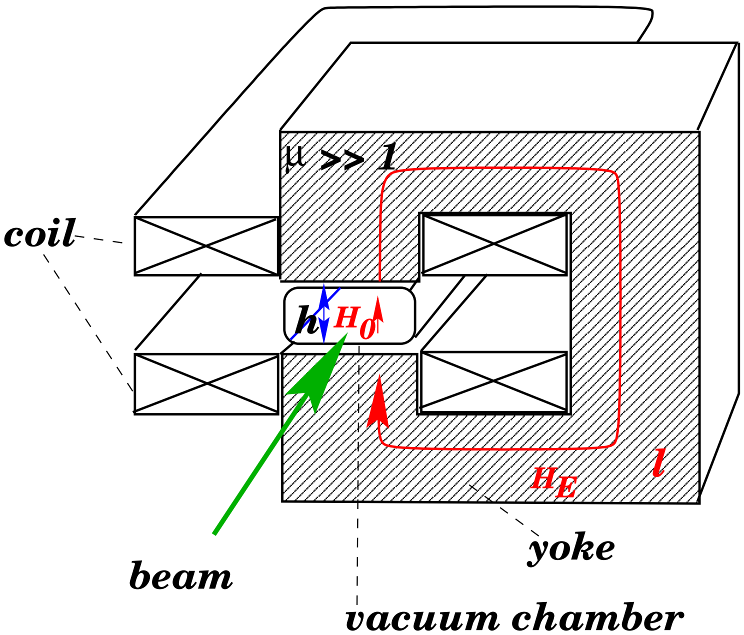
RF Cavity



Bending Magnets

Bending Magnet

- $\oint H = I \cdot N$ $\qquad B = \mu_0 \cdot \mu \cdot H$



$\mu < 1$: *Dia*

$\mu > 1$: *Para*

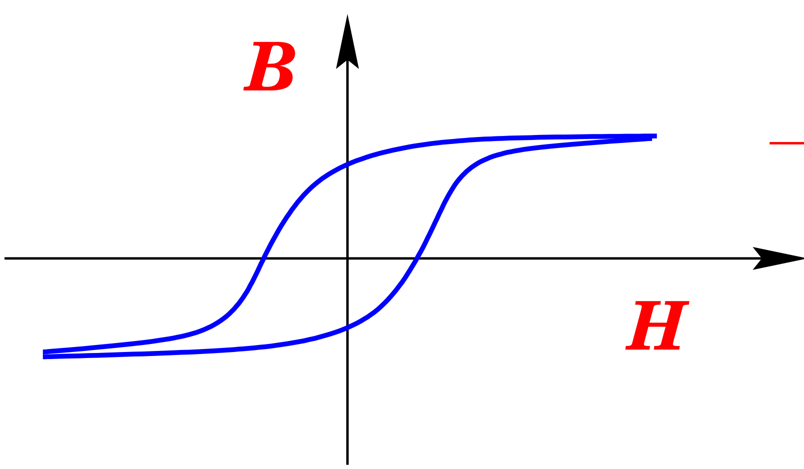
$\mu >> 1$: *Ferro*

- ***Maxwell Equations:***

$$B_{0\perp} = B_{E\perp}$$

$$\oint H = h \cdot H_0 + l \cdot H_E$$

$$H_0 = \mu \cdot H_E$$



$$B_0 = \mu_0 \frac{N\,I}{h}$$

$$\frac{1}{\rho}\,[m^{-1}] = \frac{e \cdot B}{p} = 0.3 \cdot \frac{B\,[T]}{p\,[GeV]}$$

Features (+ / -)

- **Advantages:**
  - efficient use of current
    → small gap height
  - field quality is determined by pole face
- **Limits:**
  - saturation at 2 Tesla

    (earth: $0.3 * 10^{-4}$ Tesla)

→ B > 2 Tesla requires superconducting magnets

(LHC: B = 8.4 Tesla)

→ field quality at low current?

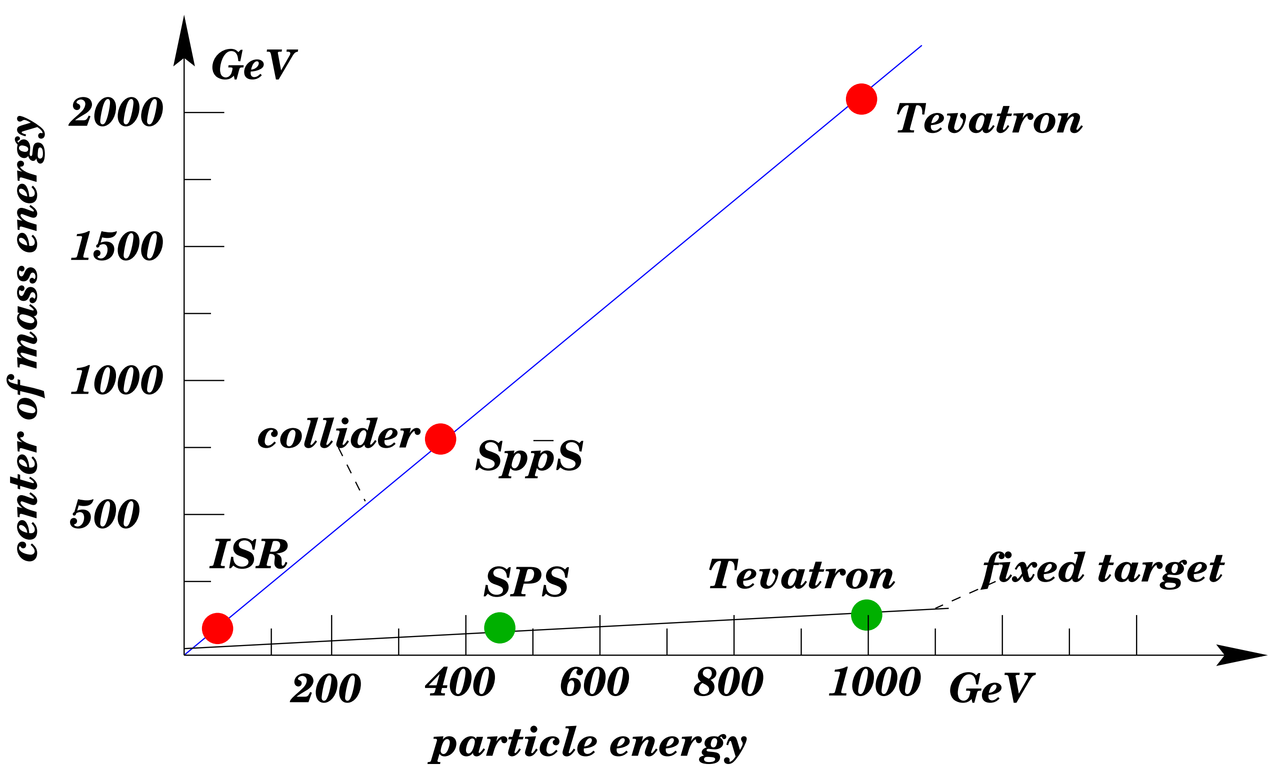
Collider Rings

→ **1960:** fixed target physics (bubble chamber)

- **But:**

$$E_{cm} = 2 \cdot m_0 \cdot c^2 \left(\sqrt{1 + \frac{E}{2 \cdot m_0 \cdot c^2}} - 1 \right)$$

- **Collider:** $E_{CM} = 2 \cdot E_p$



1960↗ : e^+ / e^- collider

1970↗ : p^+ / p^- collider

Features (+ / -)

Advantages:

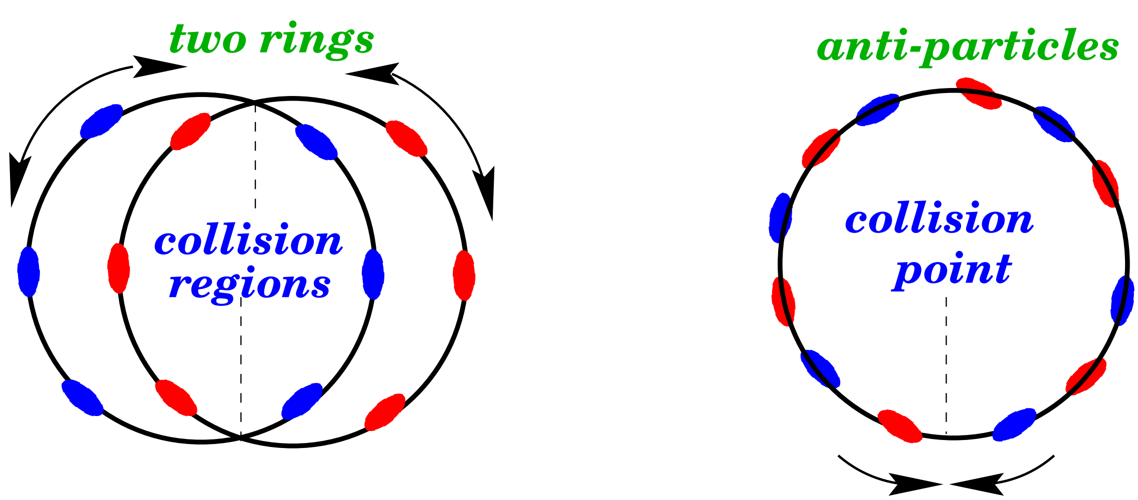
- $E_{CM} = 2 \cdot E_p$

Disadvantages:

- not all particles collide in one crossing

  → long storage times

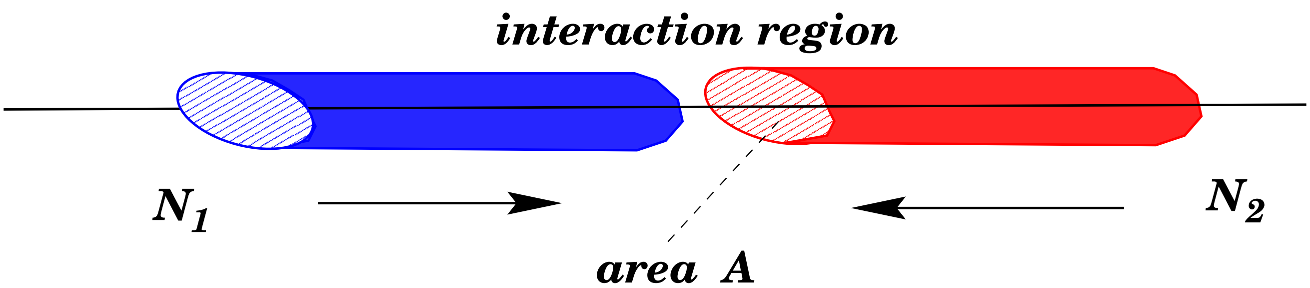
- requires 2 beams:



- beam-beam interaction

Luminosity

- $N_{ev} / sec = \sigma \cdot L \qquad [\, L \,] = cm^{-2} \cdot s^{-1}$



$$L = \frac{n_b \cdot N_1 \cdot N_2 \cdot f_{rev}}{A}$$

- **high bunch current**
  beam-beam; collective effects

- **many bunches**
  total current (RF); collective effects

- **small beam size**
  coupling; dispersion; hardware

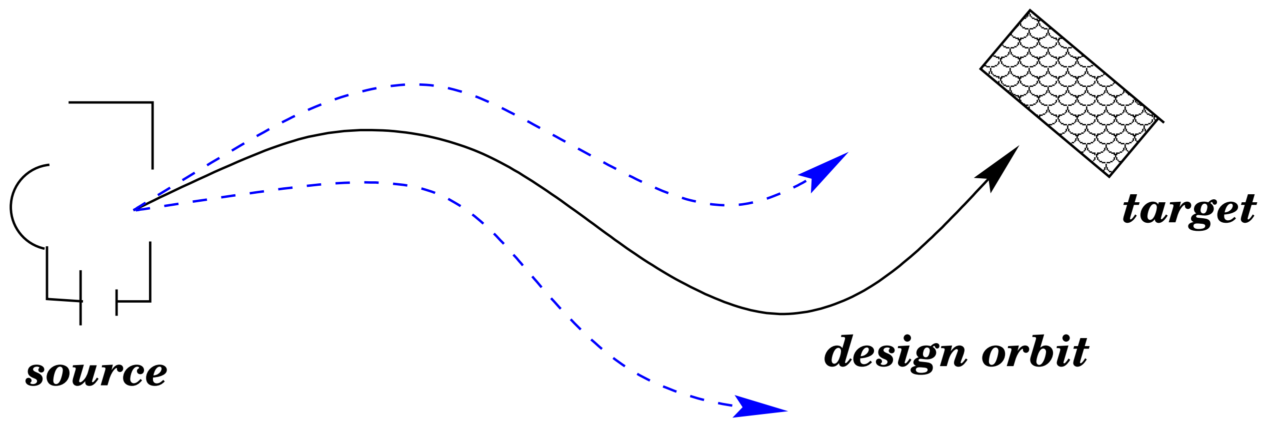
Lepton versus Hadron Collider

- **Leptons:**
  - elementary particles
    → well defined energy
  - light particles (γ >> 1)
    → synchrotron radiation
    (size, damping, magnet type)
- **Hadrons:**
  - multi particle collisions
    → energy spread
    (discovery range vs. background)
  - heavy particles (γ < 10000)
    → no synchrotron radiation
    (no damping, superconducting magnets)
- **Example:** Z_0

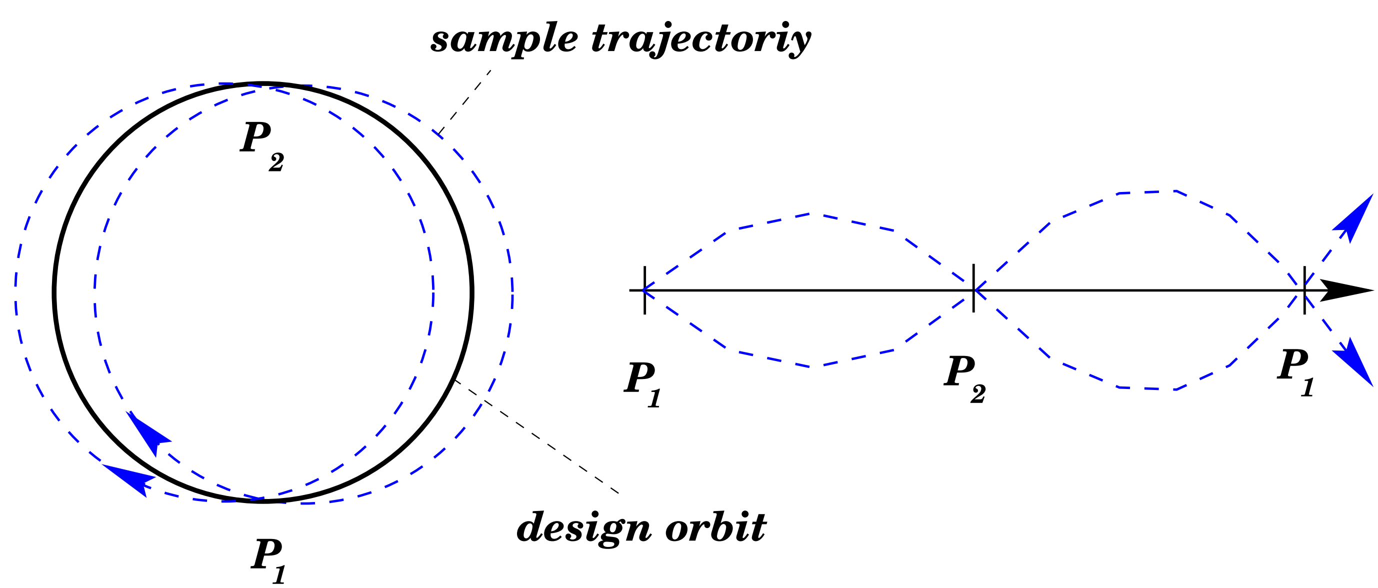
1985 Sp$\bar{p}$S	p^+p^-
1990 LEP	e^+e^-

Trajectory Stability

Beam Divergence:



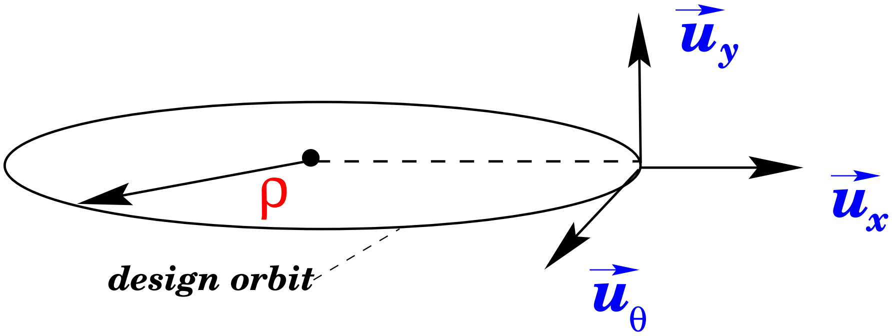
Geometrical Focusing:



Equation of Motion

Rotating Coordinate System:

$$x(t) = a \cdot \sin(\omega \cdot t + \phi_0)$$



$$\omega = \omega_{rev}$$

$$\omega_{rev} = 2 \cdot \pi \cdot \frac{v}{L}$$

$$\omega_{rev} = \frac{v}{\rho}$$

$$\frac{d^2x}{dt^2} = -v^2 \cdot \frac{1}{\rho^2} \cdot x$$

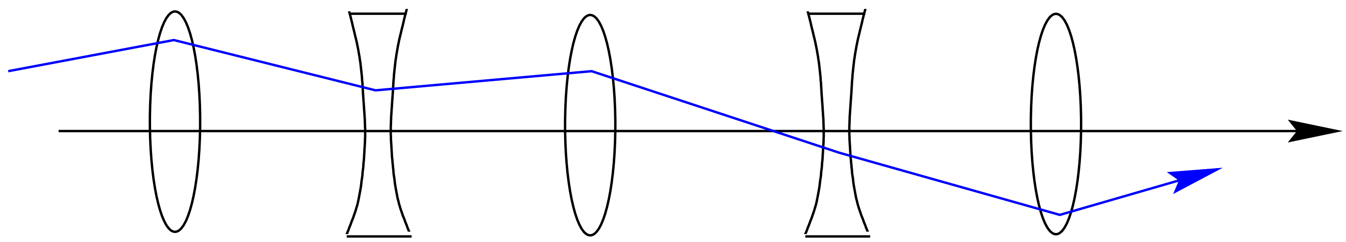
$$\frac{d}{dt} = \frac{ds}{dt} \cdot \frac{d}{ds} \qquad \frac{ds}{dt} \rightarrow v$$

$$\rightarrow \quad \frac{dx}{ds} = \frac{p_x}{p_0}$$

$$\rightarrow \quad \frac{d^2x}{ds^2} = -\frac{1}{\rho^2} \cdot x$$

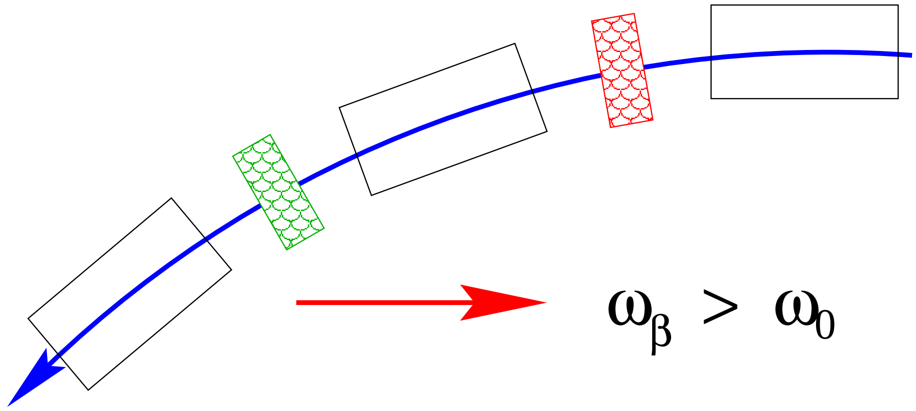
However: *no focusing in vertical plane!*

Alternate Gradient Focusing



Idea: ***cut the arc sections in***

focusing ***and*** ***defocusing*** ***elements***



$\omega_\beta > \omega_0$

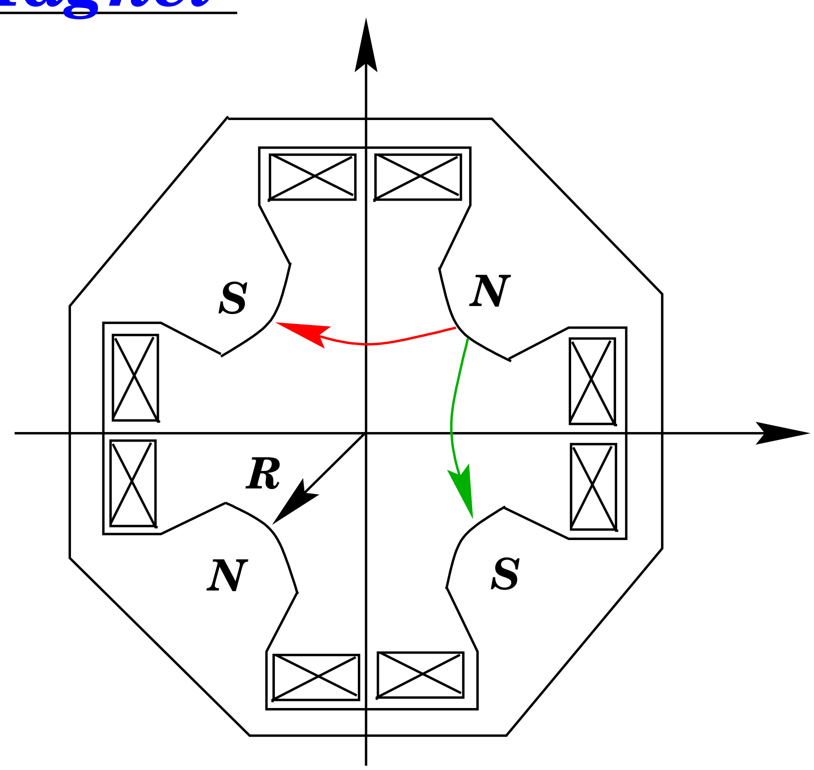
Quadrupole Magnet

$B_x = -g \cdot y$

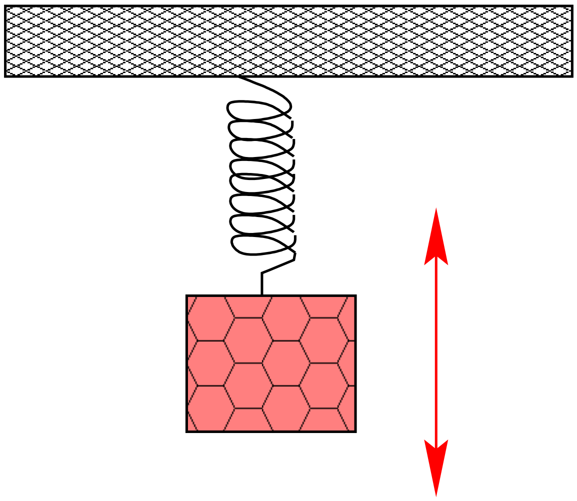
$B_y = -g \cdot x$

$F_x = -q \cdot v \cdot B_y$

$F_y = q \cdot v \cdot B_x$



oscillator (spring):



$$\boldsymbol{F = -g \cdot y}$$

$$\Omega^2 \propto g \qquad A \propto \frac{1}{g}$$

for a fixed energy

strong focusing:

→ **small amplitudes**

→ **small vacuum chamber**

→ **efficient magnets**

→ **high oscillation frequency**

Optic Functions

Hills Equation:

$$\frac{d^2x}{d\,s^2} + K(s)\cdot x = 0; \qquad K(s) = \begin{cases} 0 & \text{drift} \\ 1/\rho^2 & \text{dipole} \\ \text{general: } \dfrac{F}{p\cdot v} & \end{cases}$$

$$K(s) = K(s + L)$$

$$K(s) = const. \longrightarrow x = A\cdot \sin(\sqrt{K}\cdot s + \phi_0)$$

Floquet Theorem:

$$x = \sqrt{A\cdot \beta(s)}\cdot \sin(\phi\,(s) + \phi_0\,)$$

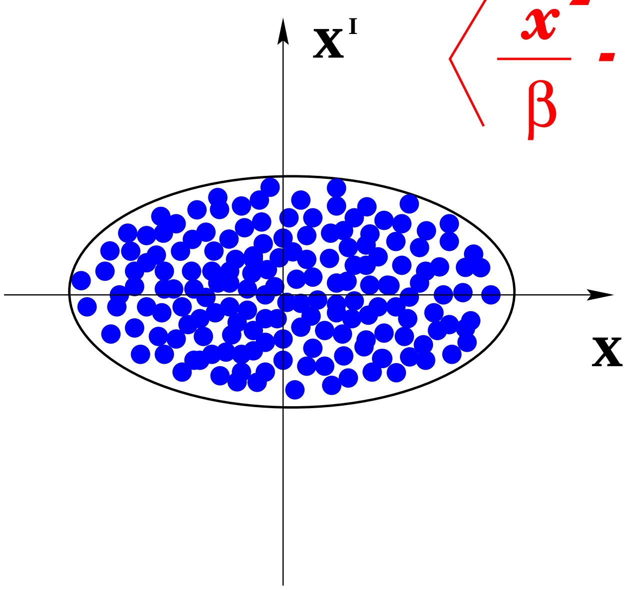
$$\beta(s) = \beta\,(s + L); \qquad \phi\,(s) = \int \frac{1}{\beta}\,ds$$

→ ***differential equation for*** β **!**

- β ***and*** ϕ ***are determined by the arrangement of the magnets in the tunnel***

- ***individual trajectories are determined by*** A ***and*** ϕ_0

- ***beam ensemble:***



$$\left\langle \frac{x^2}{\beta} - \beta' \cdot x \cdot x' + \beta \cdot x'^2 \right\rangle = \varepsilon$$

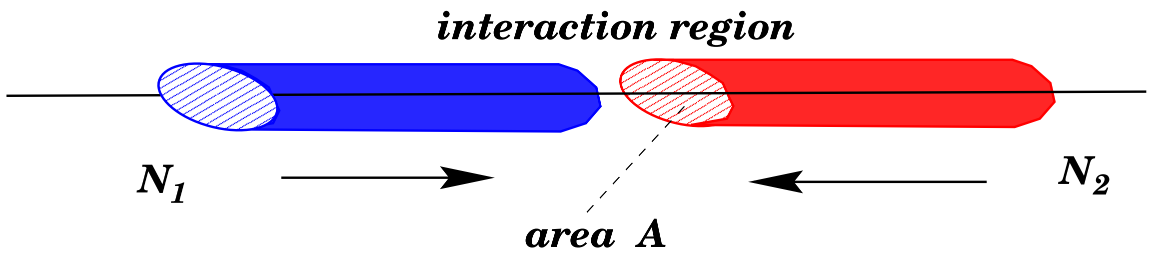
$$\varepsilon = \frac{area}{\pi}$$

→ ε ***describes the beam quality***

→ $\sigma = \sqrt{\varepsilon \cdot \beta}$ ***describes the beam size***

Beam Size

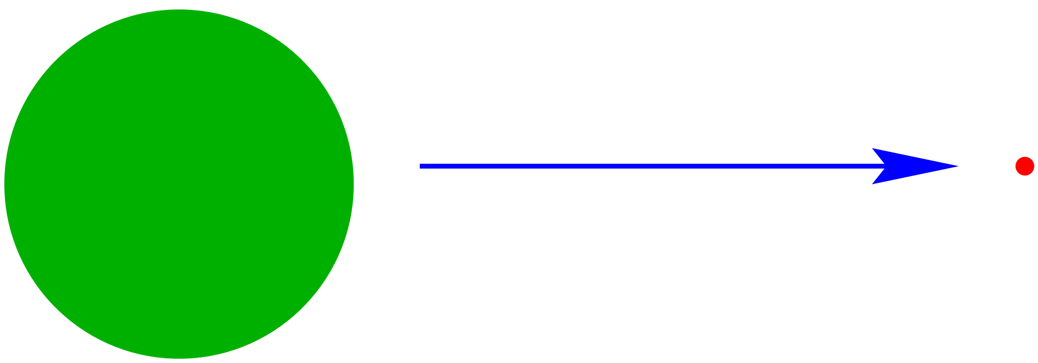
Luminosity:



$$L = \frac{n_b \cdot N_1 \cdot N_2 \cdot f_{rev}}{A}$$

$$A = \pi \cdot \beta \cdot \varepsilon$$

LHC:



$$\langle\beta\rangle_{arc} = 80 \text{ meter}$$

$$\beta_{IP} = 0.5 \text{ meter}$$

Limit:

- magnet strength
- aperture

$$x = \sqrt{A \cdot \beta} \cdot \sin(\phi)$$

$$x' = \sqrt{\frac{A}{\beta}} \cdot \sin(\phi)$$

Summary Focusing

- ***beam divergence***

- ***geometrical focusing***

  → ***horizontal stability***

- ***strong focusing***

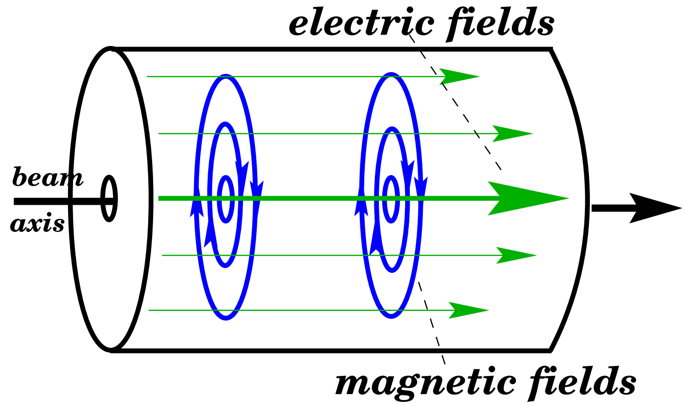
  → ***horizontal and vertical stability***

- ***optic functions:*** β, ϕ

- ***beam size:*** $\sigma = \sqrt{\beta \cdot \varepsilon}$

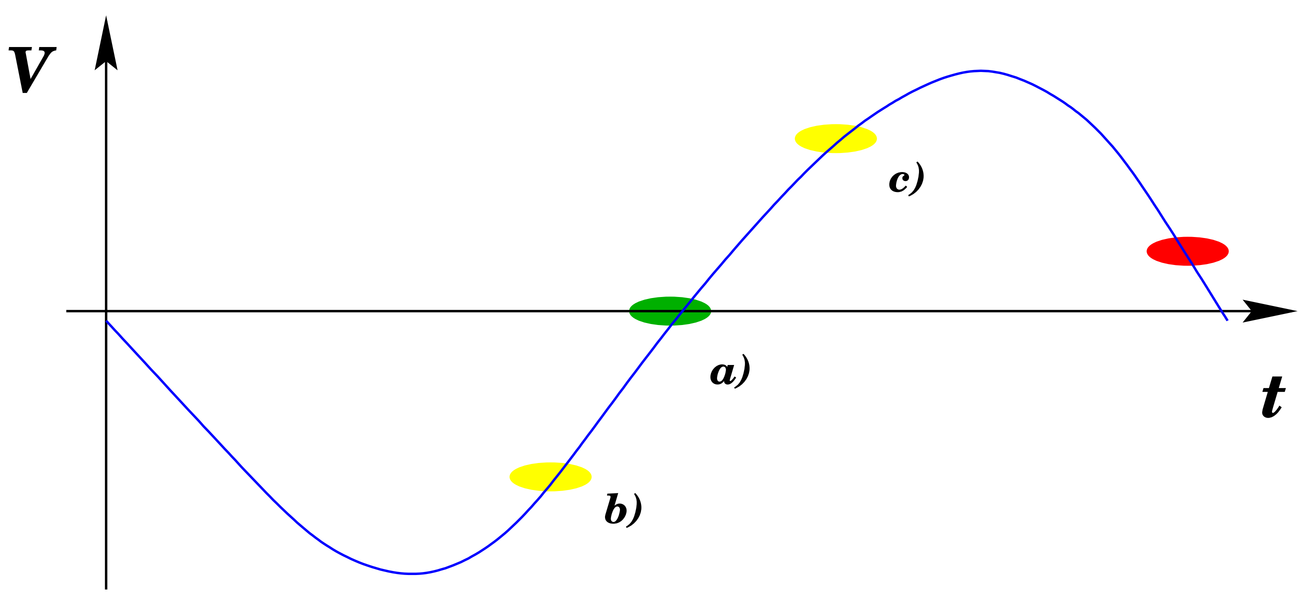
Longitudinal Stability

RF Cavity



assume: $p = p_0 + \Delta p \longrightarrow \omega = \omega_0 + \Delta\omega$

voltage in cavity:



$\longrightarrow$ ***longitudinal stability***

Dispersion Orbit

- **Dipole:** $$\frac{1}{\rho} = \frac{q \cdot B}{p}$$

→ *energy error leads to orbit error*

- **Equation of motion:**

$$x'' - K(s) \cdot x = \frac{1}{\rho} \cdot \frac{\Delta p}{p_0}$$

→ $$x(s) = x_0(s) + D(s) \cdot \frac{\Delta p}{p_0}$$

- **Beam Distribution:**

$$\langle p \rangle = p_0$$

$$p = p_0 + \Delta p \cdot \cos(\omega_s \cdot s)$$

→ *each particle has its own orbit*