Backflows in AGNs

Self-regulation of accretion and of jet emission

V. A.-D. , U. Becciani	*INAF-Catania*
J. Silk	*Johns Hopkins/Univ. P.M. Curie*
S. Cielo	*MPIA, Heidelberg*
	(made also the movies!)
C. Tortora	*ETH*
A. Macciò	*MPIA, Heidelberg*
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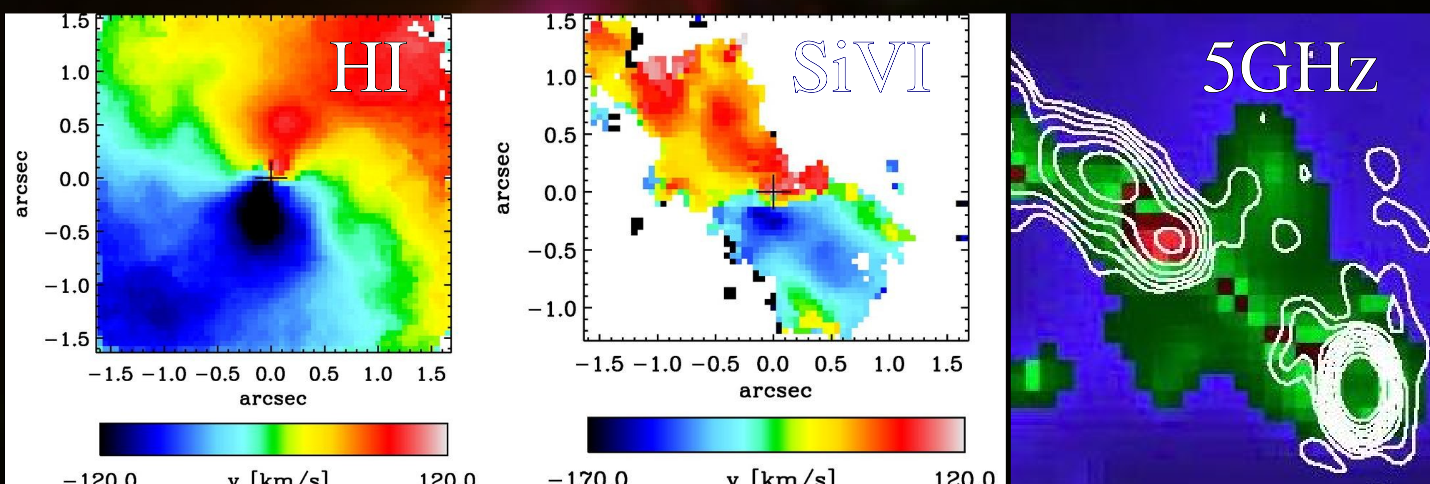
V. A.-D. & Silk, MN 389, 1750 (2008)
Tortora et al., MN 396, 61 (2009)
V. A.-D. & Silk, MN 405, 1303 (2010)
Tortora et al., MN 411, 627 (2011)
Crockett et al., MN 412, 1603 (2012)
Silk et al., arxiv1209.1175

Backflows: theory

- Mizuta et al. (ApJ 709, L83, 2010): 2D simulations of relativistic jets - *Backflows along relativistic jets*
- V. A.-D. & Silk (MN 405, 1303, 2010): 2D simuls. of jet prop. inrealistic E's models *With cooling and BH's radiation field*

Backflows: observations

- Neumeyer et al. (ApJ 671, 1329, 2007): SiVI line within ~ 100 pc of Cen A – *Redshifted gas in NE part*



- Laing and Bridle (MN 424, 1149, 2012): Counterflows detected in 2 FRI's – V_{bck} *~ -700 km/sec (5" ~ 3.7 kpc)*

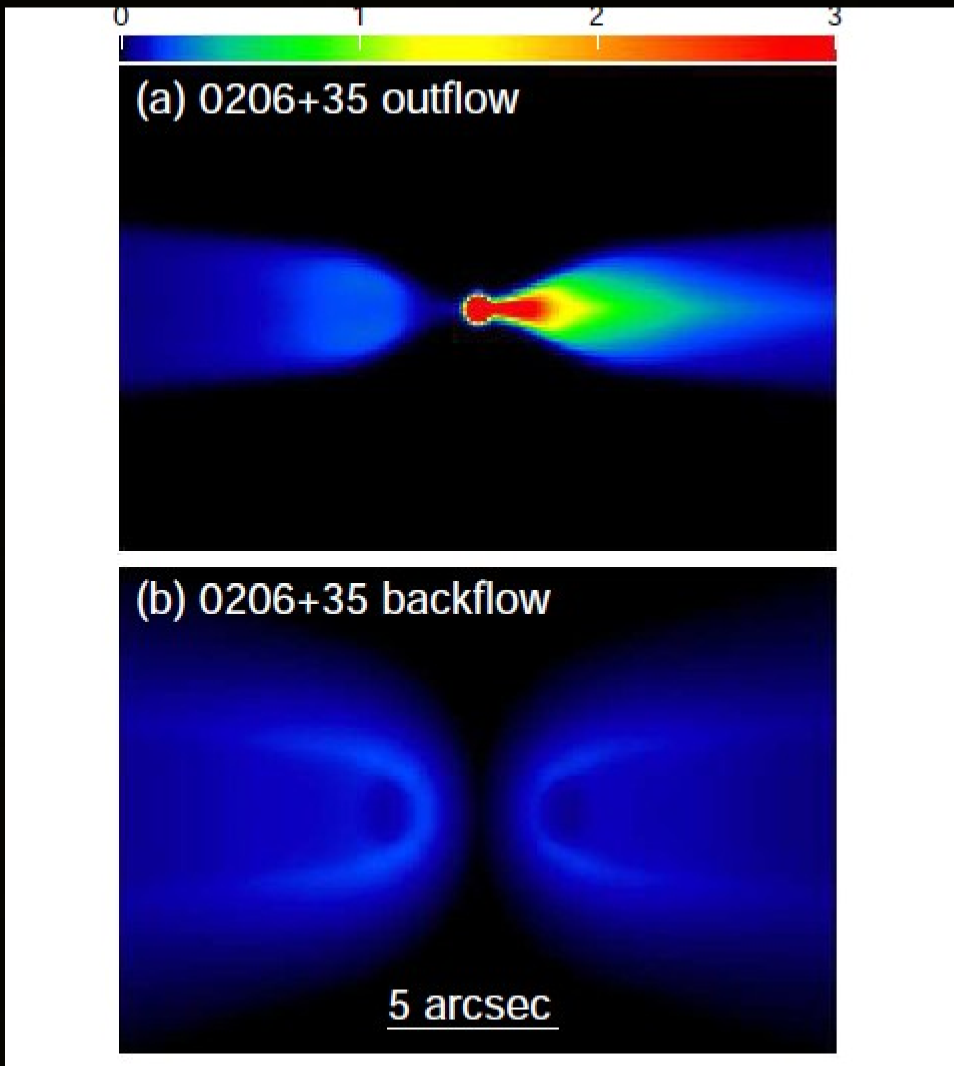
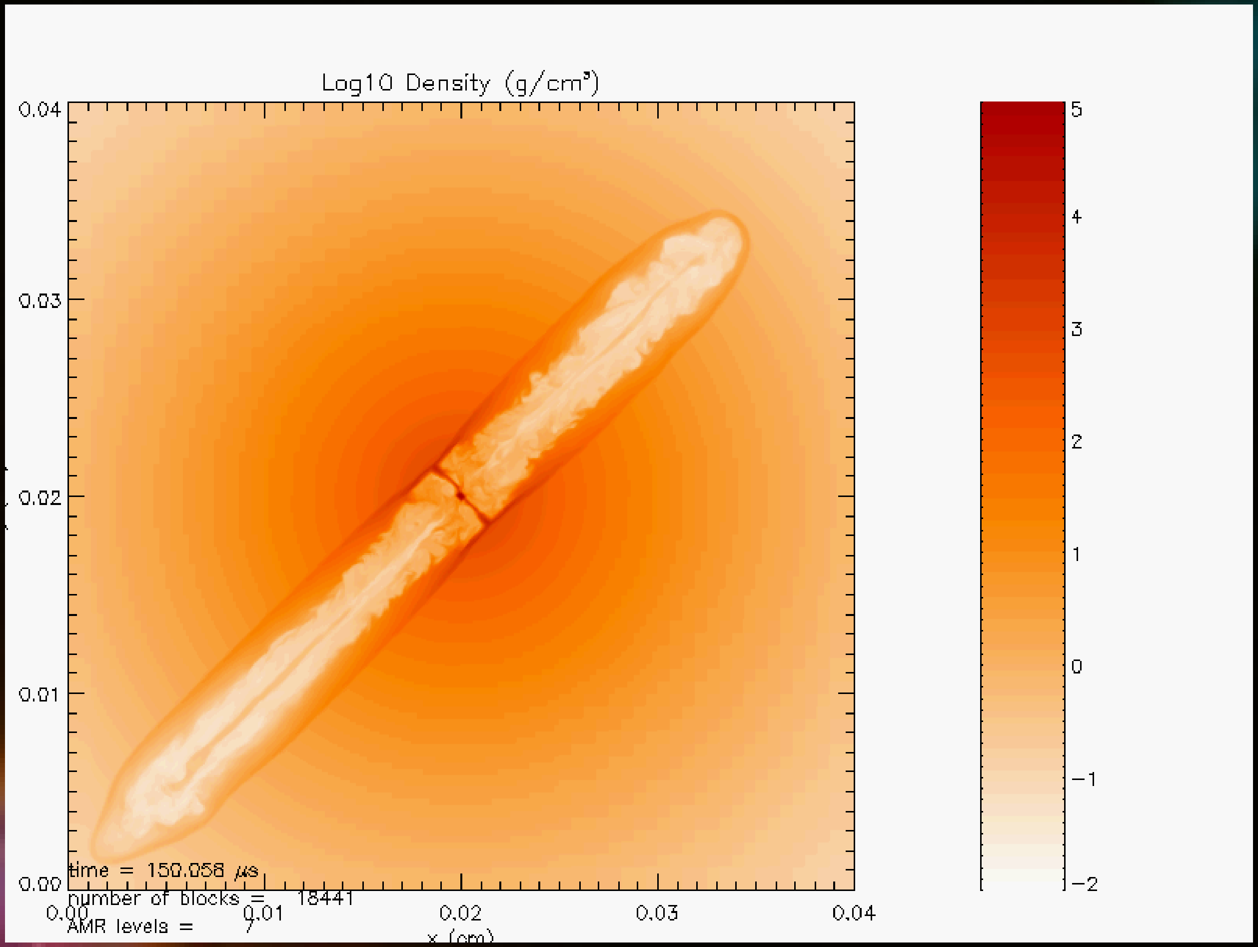


Figure 9. Predicted brightness distributions for the outflowing and back-flowing parts of the model for 0206+35. (a) outflow; (b) backflow.

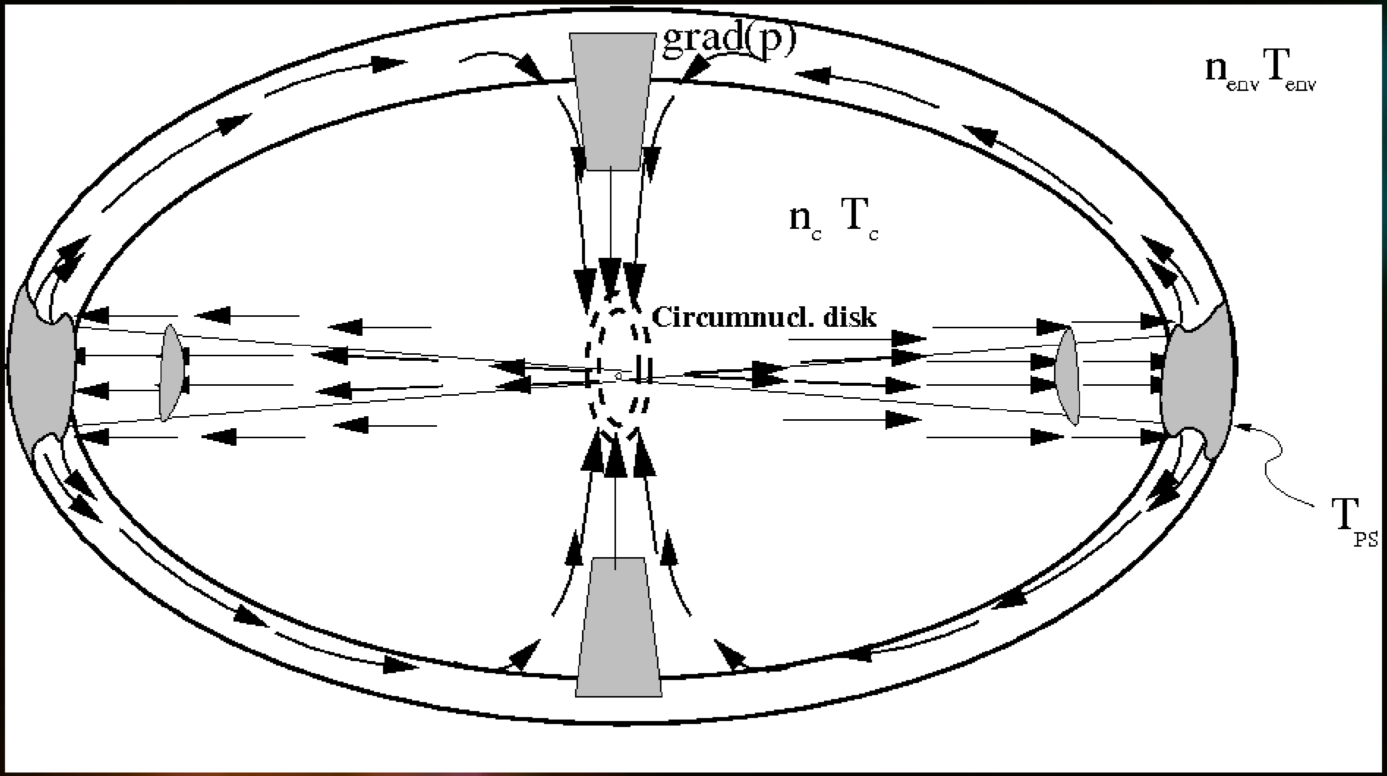
- Counter jet gas streaming along the bow shock

Dynamics of jet propagation into host galaxy



- Jet/cocoon system, $T_{coc} \simeq 10^{9}$-4×10^{11} K, $n_{coc} \simeq 10^{-4}$-10^{-1} cm^{-3}
- Bow shock+contact discontinuity – *Hot spot* at jet
- termination

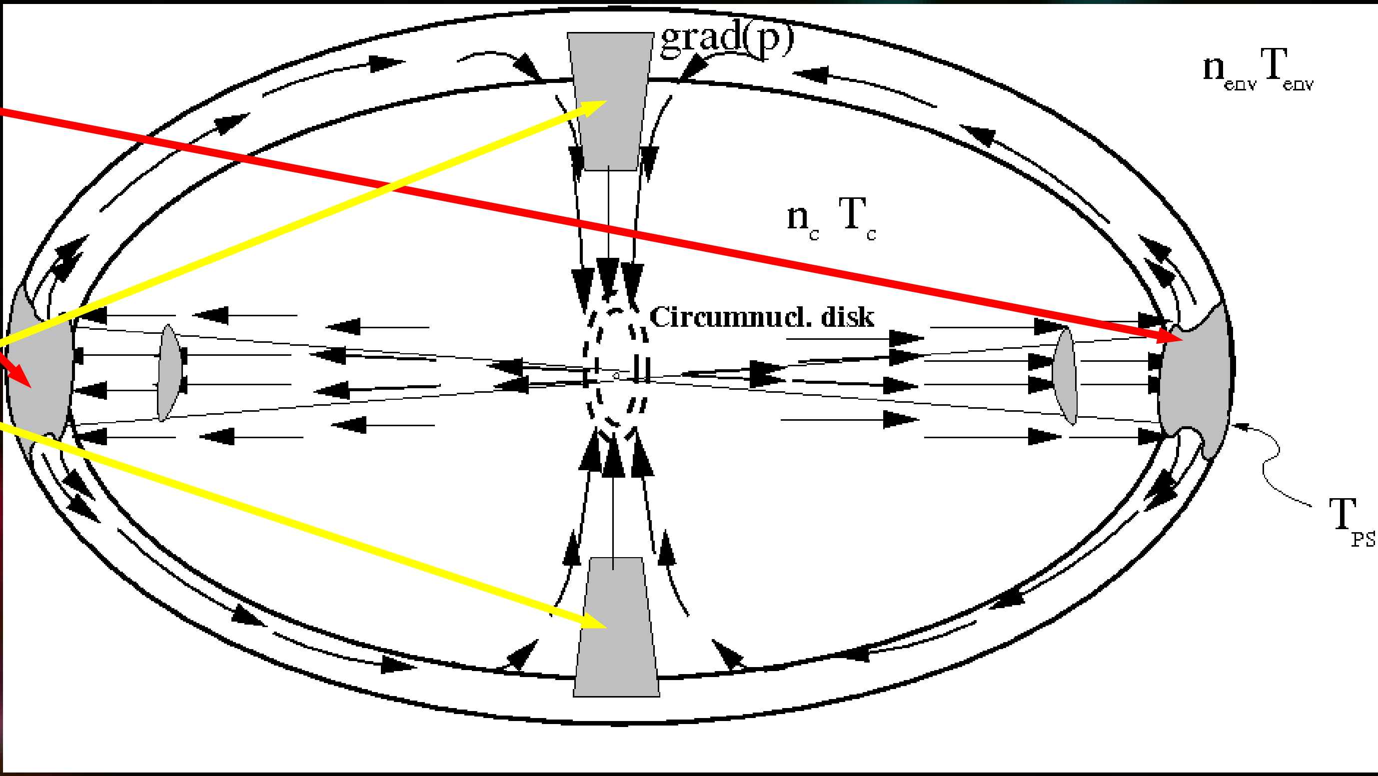
Dynamics of jet propagation into host galaxy



- Jet/cocoon system, $n_{coc} T_{coc} \simeq n_{bs} T_{bs} \gg n_{env} T_{env}$ expands (self-similar model: Falle, 1991)
- Does not determine neither morphology nor internal flows

How does the cocoon expand?

- A *hotspot* develops near the jet's termination
- A *high density region* develops in two spots near the meridional plane



- Backflow from the HS is expected from Crocco theorem

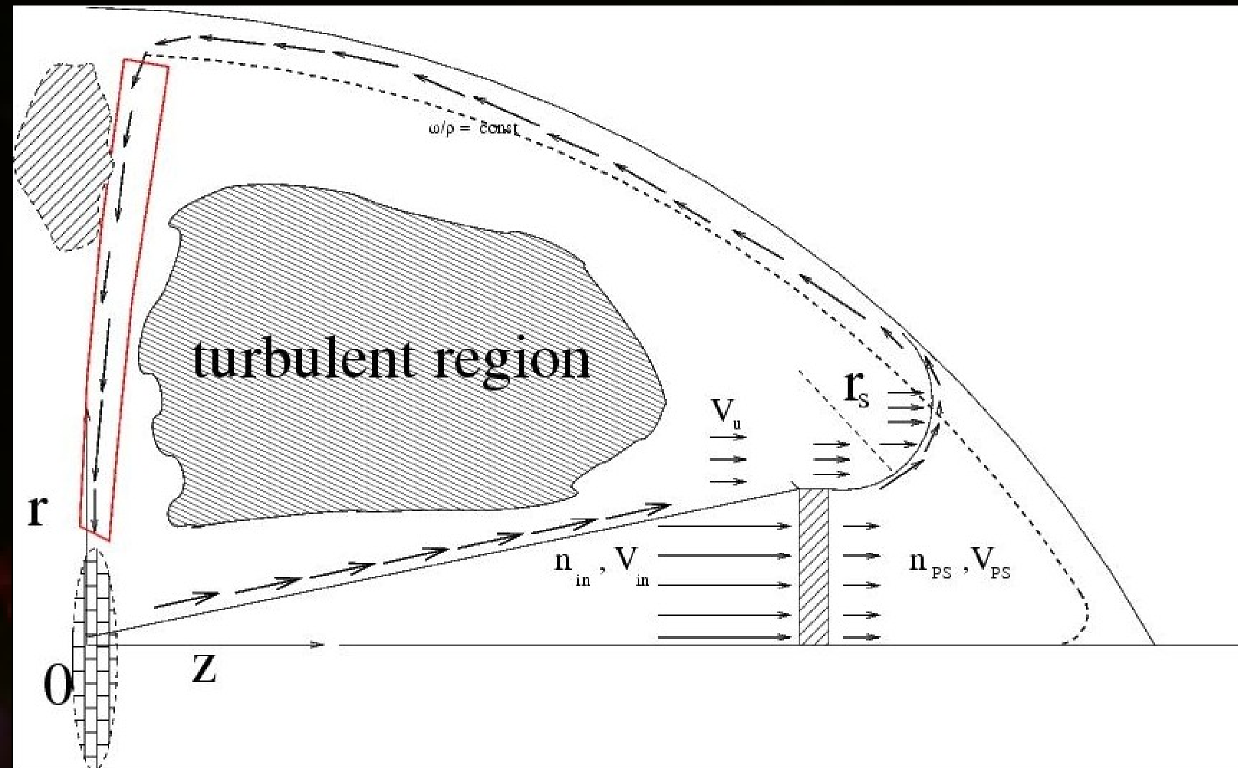
$$\vec{v} \times \mathrm{curl}\,\vec{v} = \nabla h - T \nabla S.$$

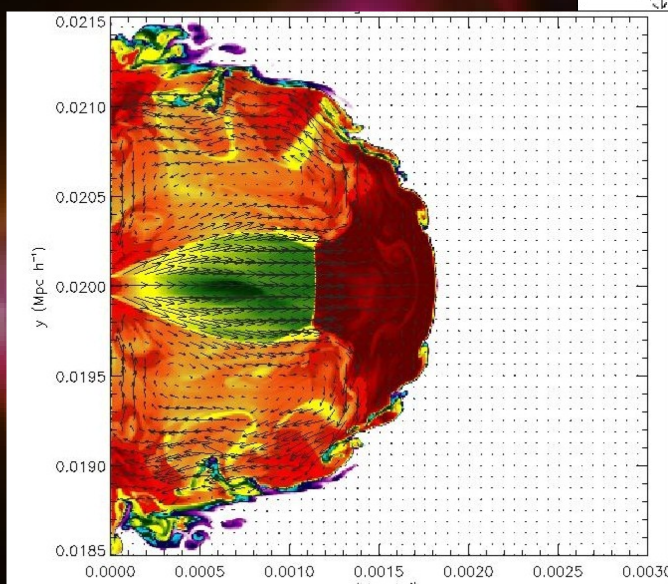
- Shearing gas gains angular momentum when crossing a gradient in h_0 (*specific stagnation enthalpy*) near the meridional spots and the hotspot .

Backflows can feed the AGN

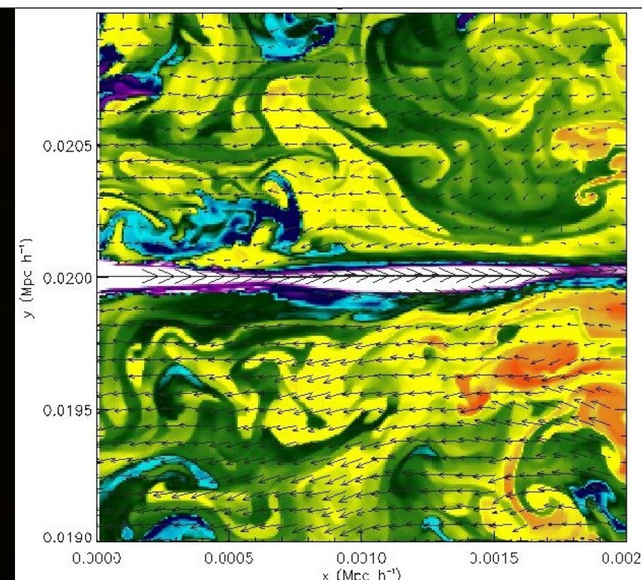
- Backflows can provide ~ 10^{-3} – 1 $M_{\odot}$ yr^{-1} over ~ 10^{7} yrs. → [*Self-feeding AGN*] (V. A.-D. & Silk, 2010)

- 2D Simulations: 2 modes of backflow

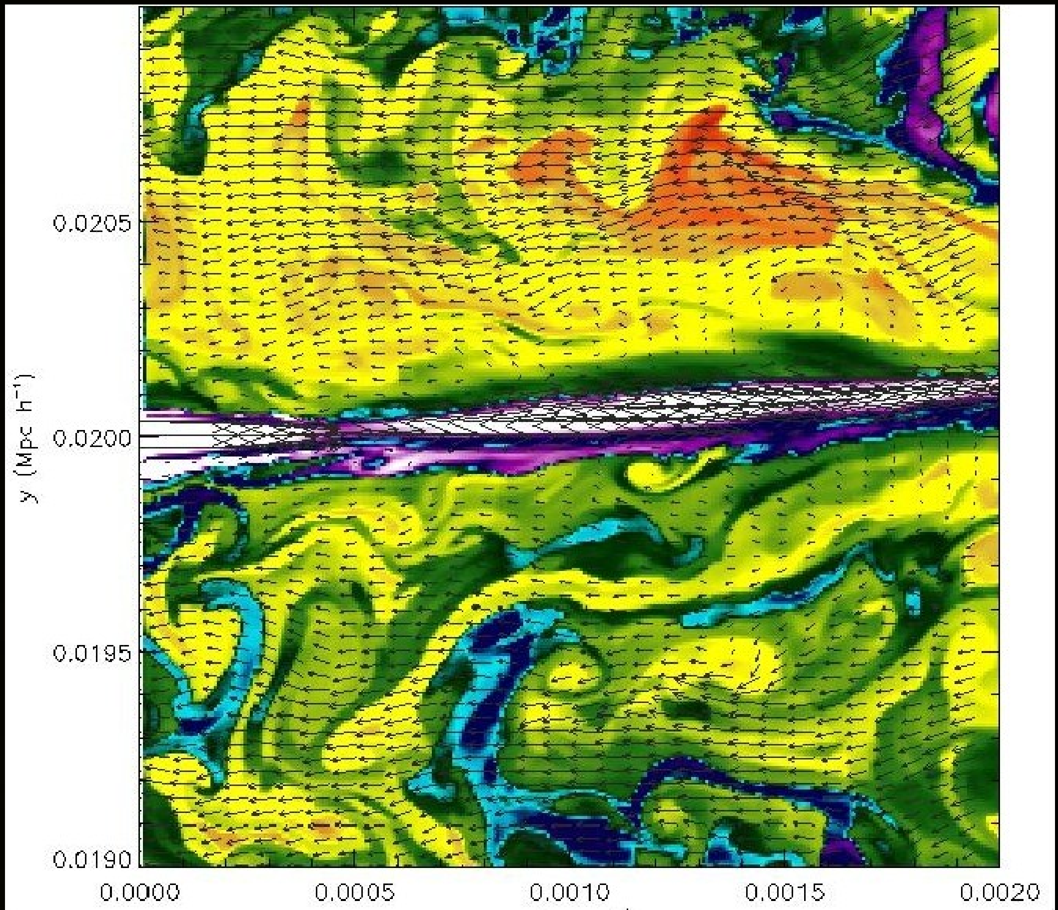




Early: circulation



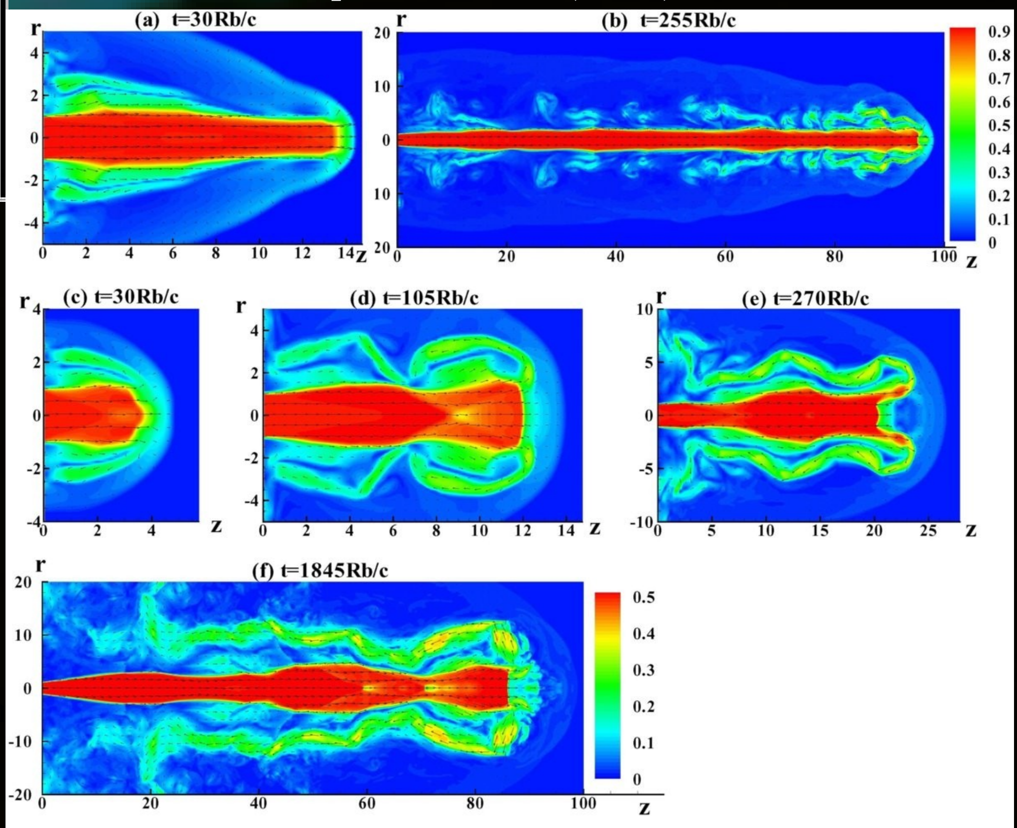
Late: counterjet



- No meridional circulation along the bow shock → flow is directly reflected from the hotspot

- Similar velocity field in simulations of heavy ($\rho_j/\rho_{ism} = 10^{-1}$), adiabatic, relativistic simulations

Mizuta et al., ApJ 709, 1083 (2010)



3D simulations of jet propagation

- FLASH 3.3, Adapt. Mesh Refin.

- Rad. cooling, $Z=0.5\ Z_{\odot}$, incl. pair prod.($10^2 \leq T \leq 10^{12}$ K)
- 8 ref. Levels, L_b = 80 kpc, l_{min} = 78.125 pc

- Rad. field from the central BH (heating)

- Same parameters as in V.A.-D & Silk 2010 (2D)

run	σ_v (km sec^{-1})	M_v ($M_{\odot}$)	ρ_c (cm^{-3})	M_{bh} ($M_{\odot}$)	P_k (ergs· sec^{-1})
s100av	100	$3.23 \cdot 10^{11}$	2.20	8.92×10^6	2.29×10^{44}
s100p1	"	$6.44 \cdot 10^{11}$	7.74	"	"
s100m1	"	$1.62 \cdot 10^{11}$	0.746	"	"
s200av	200	$2.57 \cdot 10^{12}$	2.68	1.2×10^8	1.06×10^{45}
s200p1	"	$5.68 \cdot 10^{12}$	13.26	"	"
s200m1	"	$1.16 \cdot 10^{12}$	0.893	"	"
s300av	300	$8.62 \cdot 10^{12}$	3.04	5.49×10^8	2.61×10^{45}
s300p1	"	$2.03 \cdot 10^{13}$	11.69	"	"
s300m1	"	$3.66 \cdot 10^{12}$	0.98	"	"

run	$\langle v \rangle_j$ (km·sec^{-1})	Γ_j -	β_j -	M_{nr} -	h_j (10^{-16} ergs· sec^{-1})	P_h/P_k -
s100av	217.4	6.74	0.989	11.55	22.83	0.029
s100p1	143.0	3.65	0.961	6.08	80.1	0.108
s100m1	312.8	11.55	0.996	19.93	7.72	0.010
s200av	340.7	13.11	0.997	22.65	27.74	0.008
s200p1	199.3	5.93	0.985	10.12	137.3	0.039
s200m1	491.0	22.68	0.999	39.25	9.25	0.003
s300av	439.2	19.27	0.998	33.33	31.44	0.004
s300p1	281.0	9.84	0.994	16.95	121.05	0.014
s300m1	64.1	33.9	0.999	58.72	10.14	0.001

3D simulations of jet propagation

- FLASH 3.3, Adapt. Mesh Refin.
- DM halo: σ_v = 150 – 250 km/sec, NFW profile
- Hot ISM: $T = 10^7$ K, $\Omega_b = 0.013$, truncated isothermal distribution

- Jet/ISM/Halo/BH: parameters determined by observed scaling distributions (as in V.A.-D & Silk, 2010)

Initial setup:

Scaling relations between M_{BH}, P_{jet}, $\rho_{0|ism}$

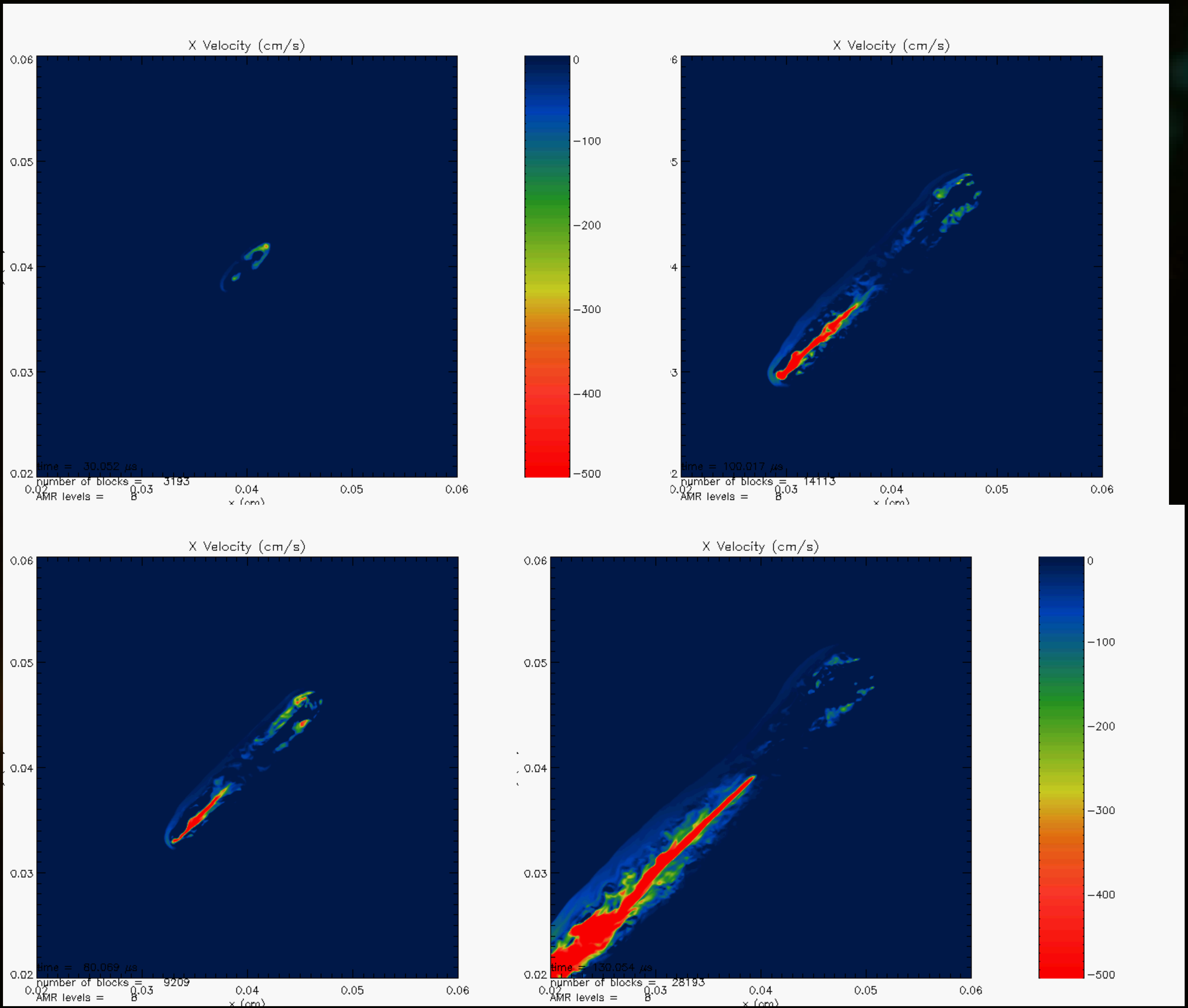
- DM halo virial mass $M_v \propto \sigma_{200}^{2.99\pm0.15}$ (*Lintott, Ferreras & Lahav, 2006*)

- $M_{BH} = (1.2\pm0.2)*10^8 \sigma_{200}^{3.57\pm0.15}\ M_\odot$ (*Ferrarese & Merritt 2000*)

- $\log(P_{jet}) = -0.22 + 0.59*\log(M_{BH}) + 40.48$ (cgs, *Liu, Jiang and Gu 2006*)

  - Diffuse Interstellar Medium: embed an isothermal halo in (appr.) equilibrium within the host NFW halo (central density $\rho_{0|ism}$, *Hester, 2006*)

- North-west: backflow (v opposite to jet) ➣ green/yellow/red
- South-east: contours trace jet



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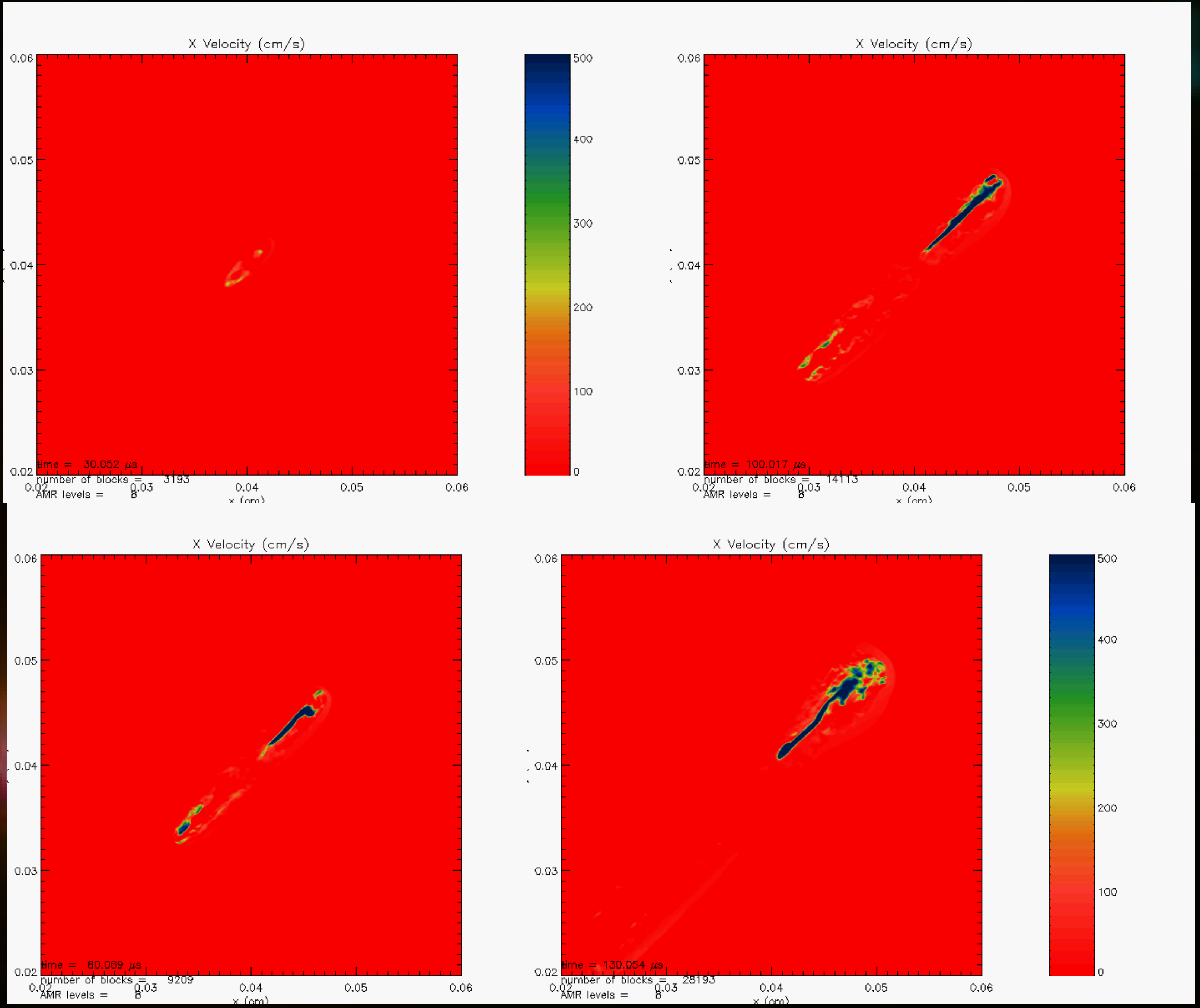
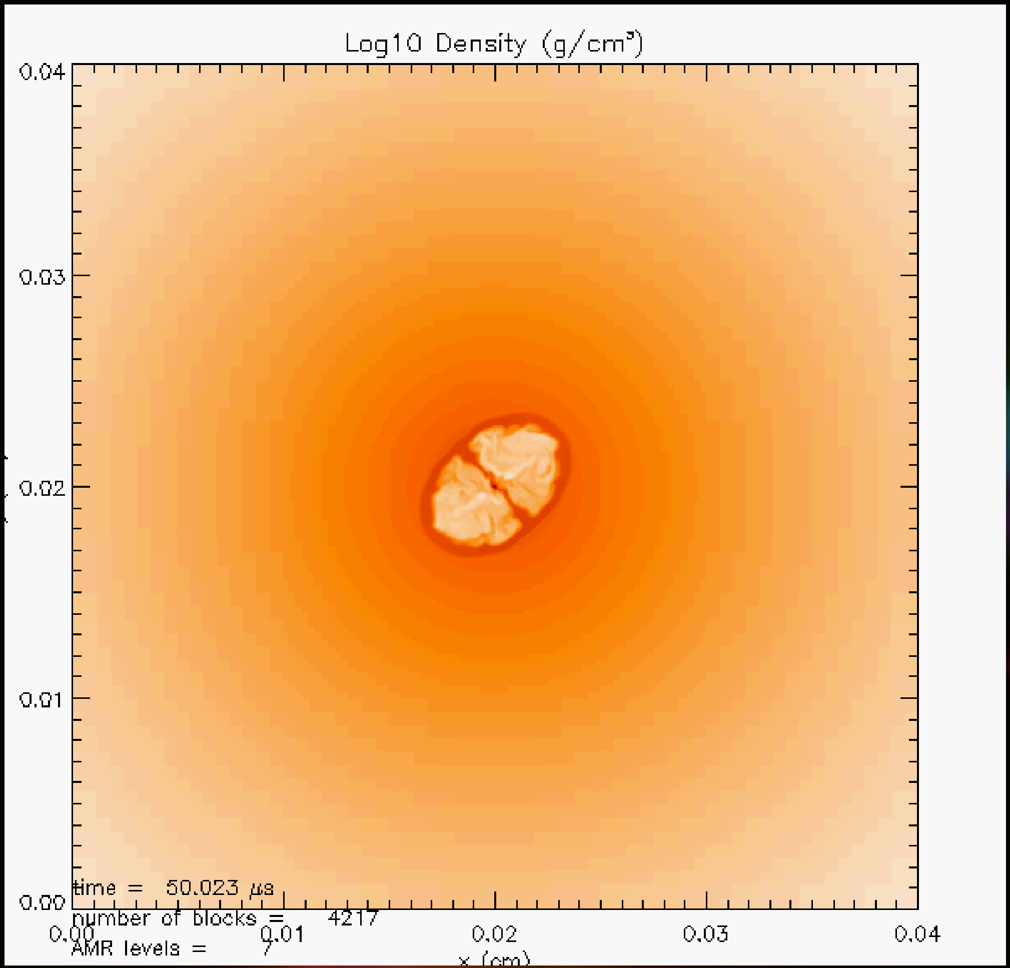
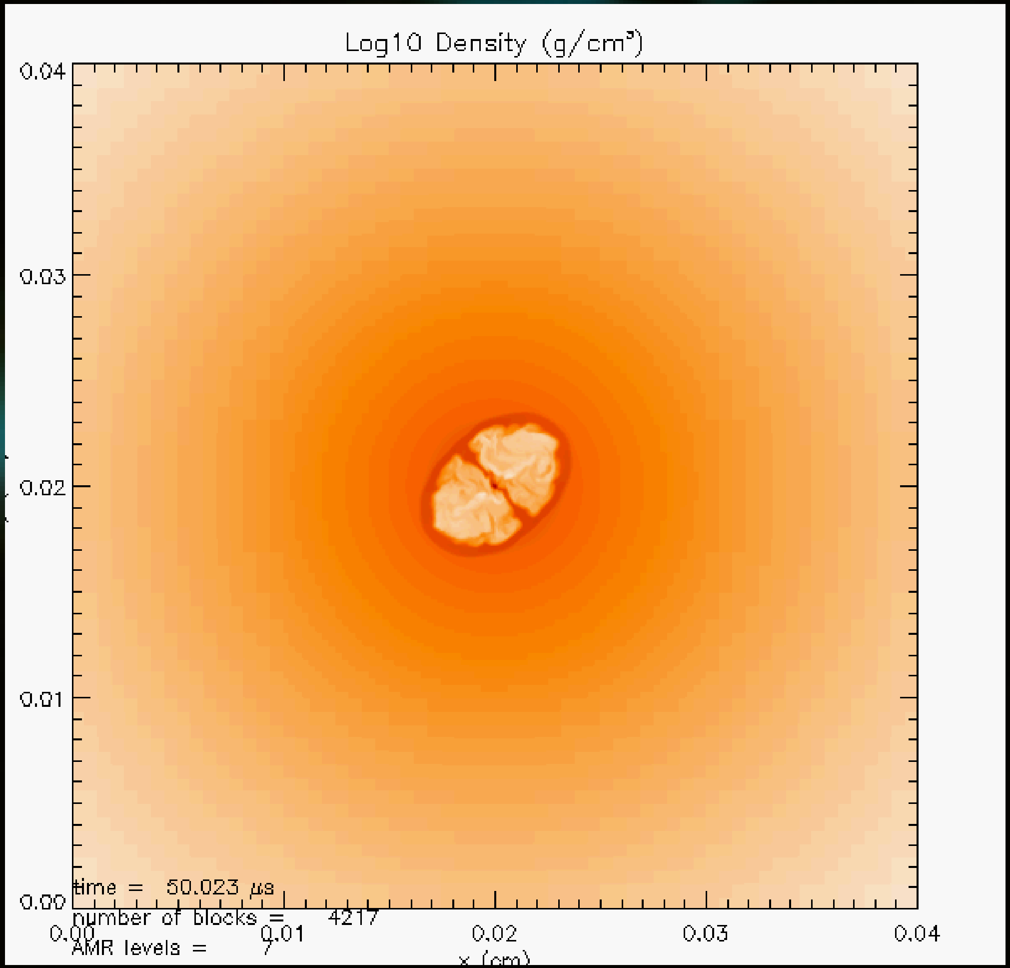


Circumnuclear disc from early backflow

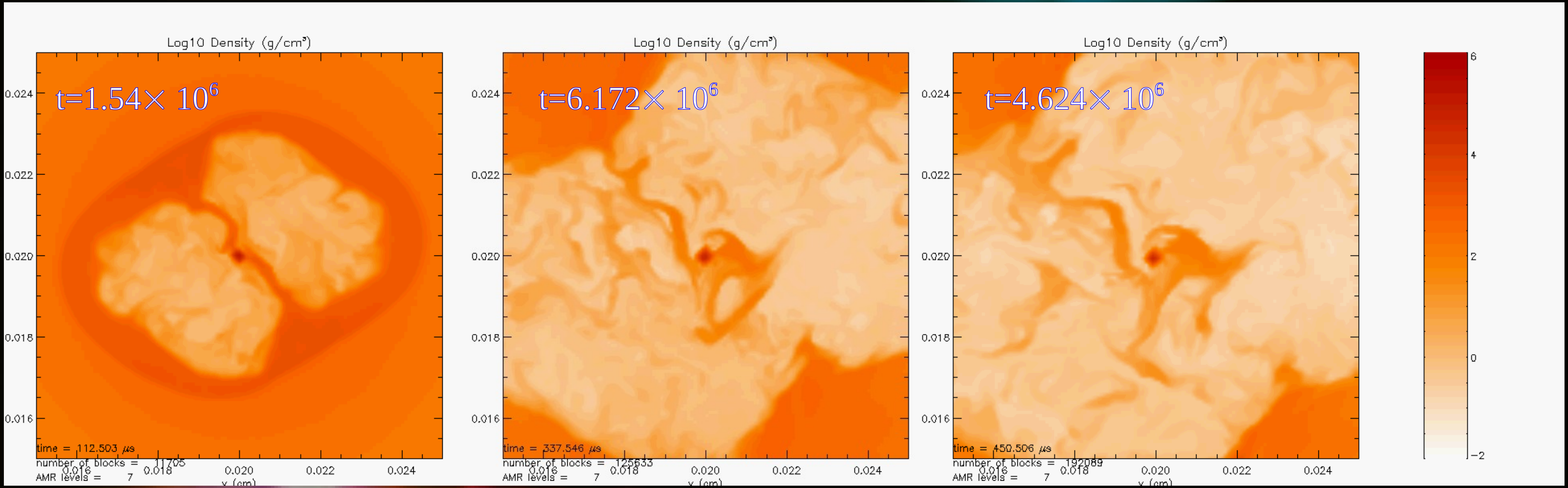




Lobe's expansion drives matter towards the meridional plane

Formation of a circumnuclear disc

- A circumnuclear, high density gaseous disc forms from the combination of backflow and compression from the lobes



Circumnuclear discs are very frequent in nearby Ellipticals (Kawata, Cen and Ho, 2007; Davies et al., 2007)

CN discs can thus arise also in absence of mergers, due to the cocoon dynamics

Formation of a circumnuclear disc

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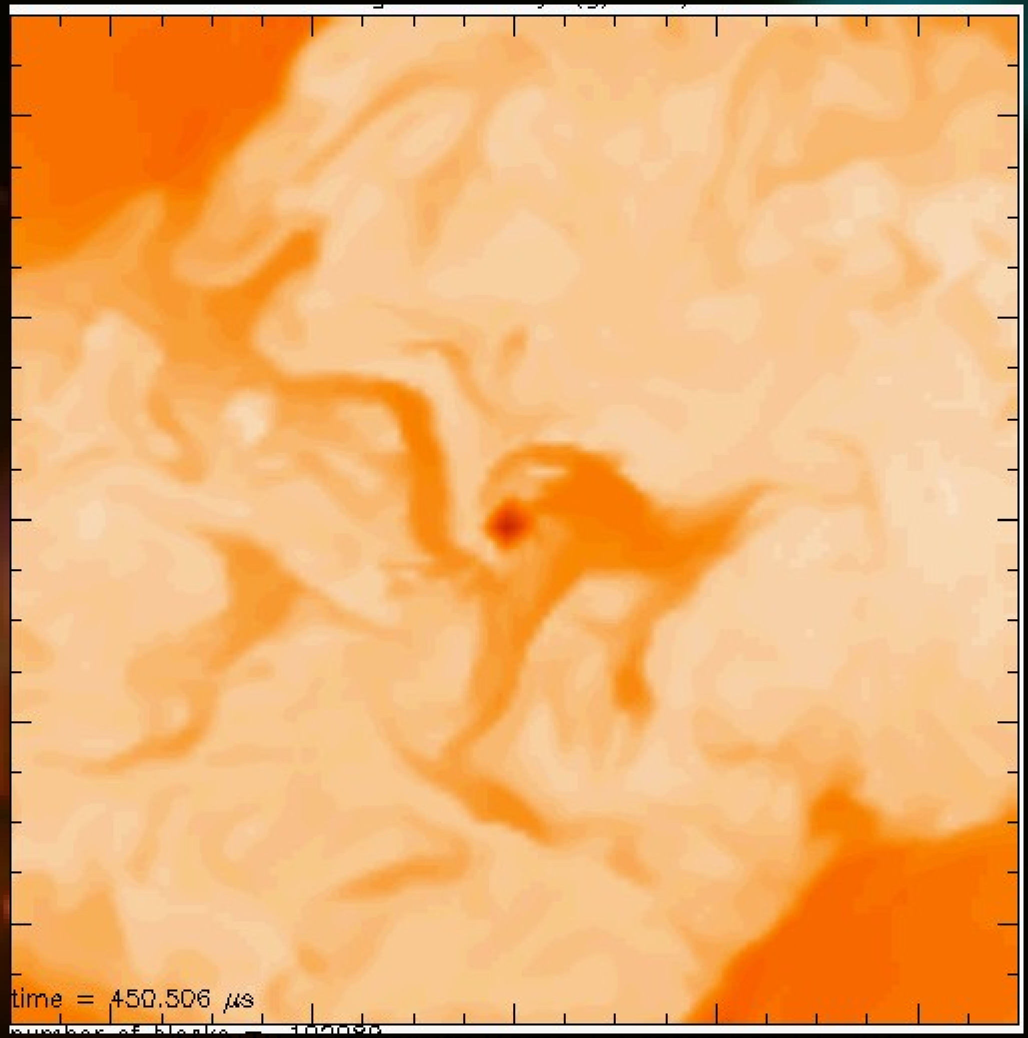


Enhanced SF within circumnuclear discs

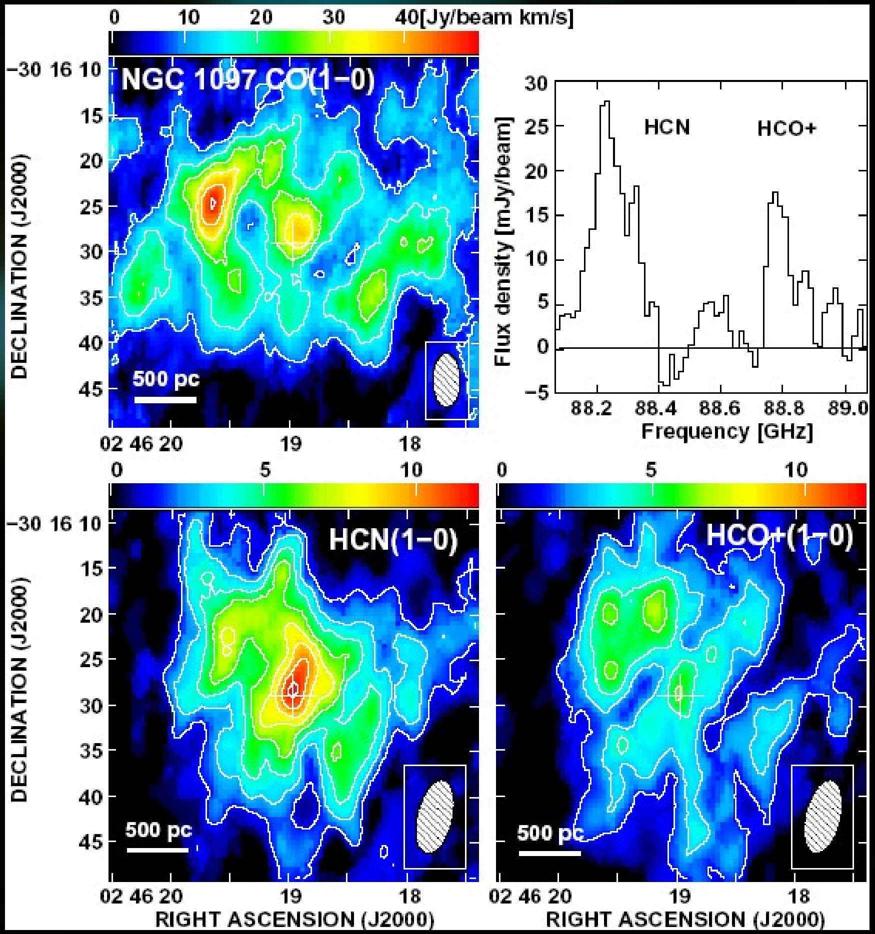
- HCN gas around AGNs' nuclei (Hsieh et al., 2008; Kohno et al., 2008)

Scales larger than Davies et al.: ~ 500 pc

- Line ratios:

$R_{HCN/CO} = 0.39$

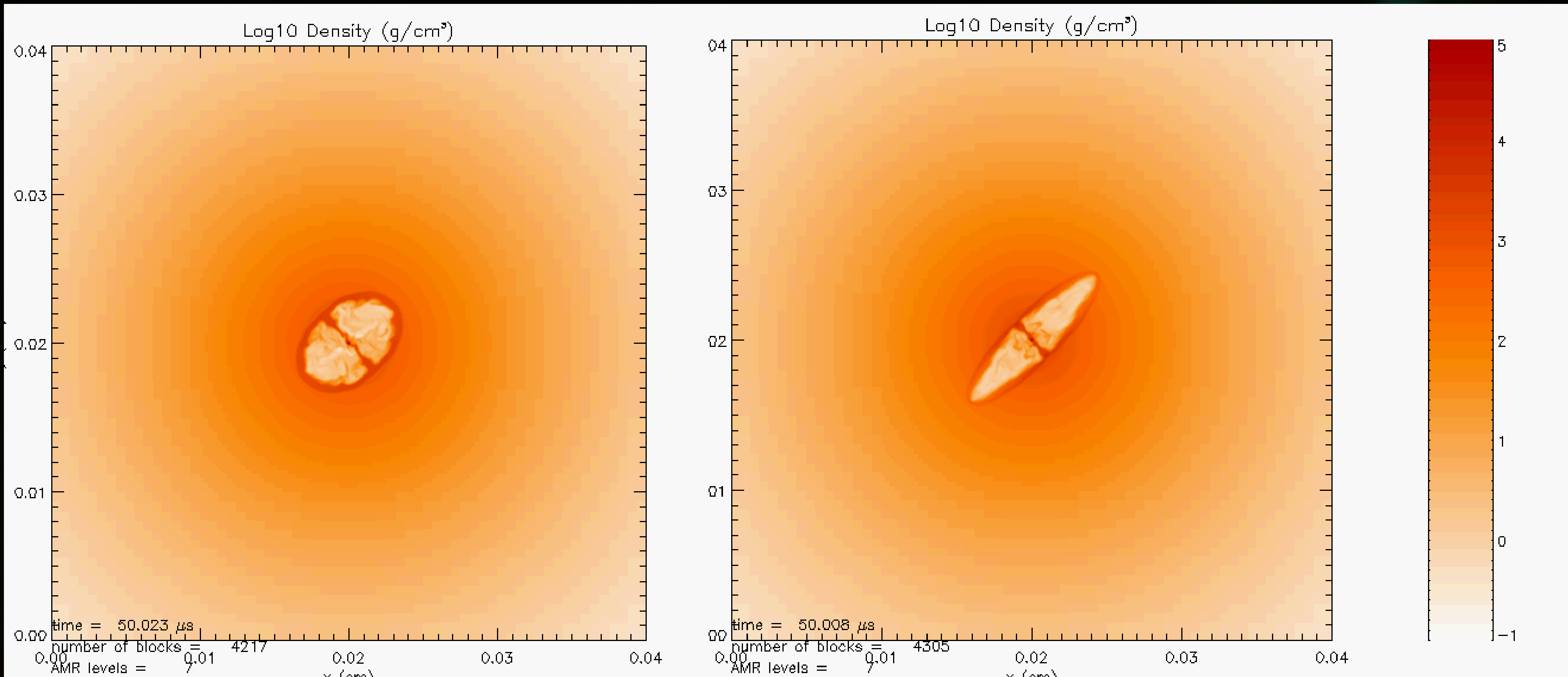
$R_{HCN/HCO+} = 1.9$ enhanced SF?



- Excess gas around the circumnucl. region → <u>accreting gas, in excess of hydr. equil.</u> (Hsieh et al. 2008)
- This gas can only be accounted for if it is **<u>accreting from a larger region</u>**

- Evidence for backflowing gas within the cocoon

What controls cocoon's morphology?

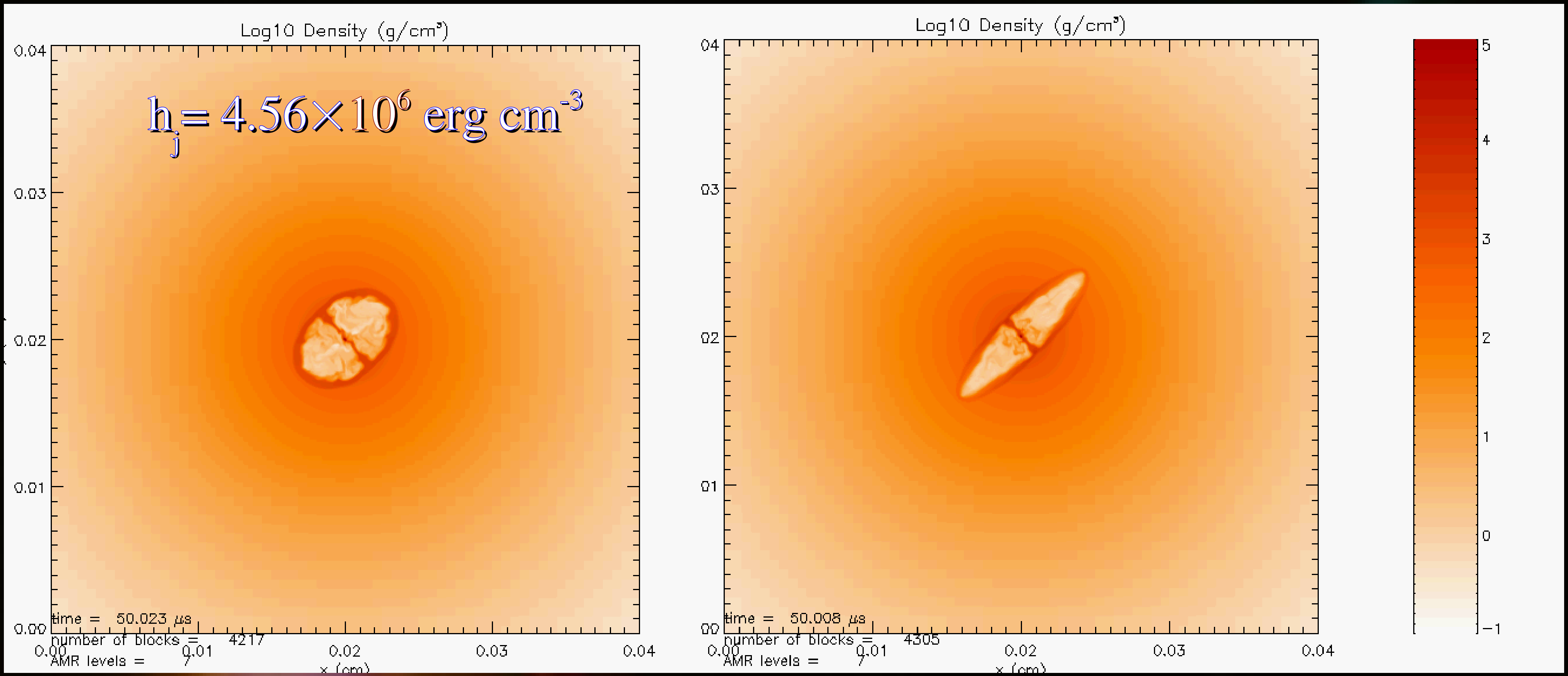


Both runs:

$P_j = 10^{45}$ erg/sec, $M_{gas} \simeq 2.34\times10^{11}\ M_\odot$, $T_g = 10^7$ K

In which parameter do they differ?

What controls cocoon's morphology?



Both runs:

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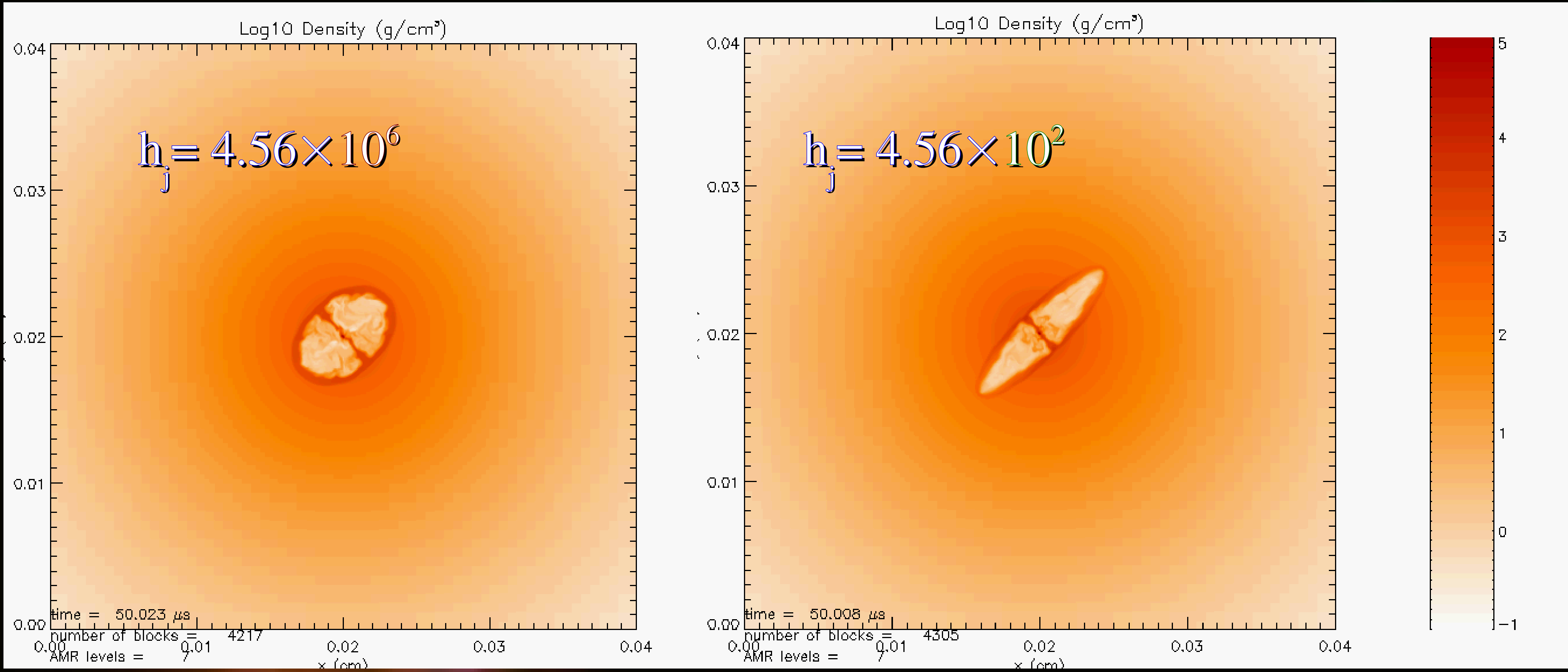
$h_j = e + p + v^2/2$ *stagnation enthalpy*

What controls cocoon's morphology?

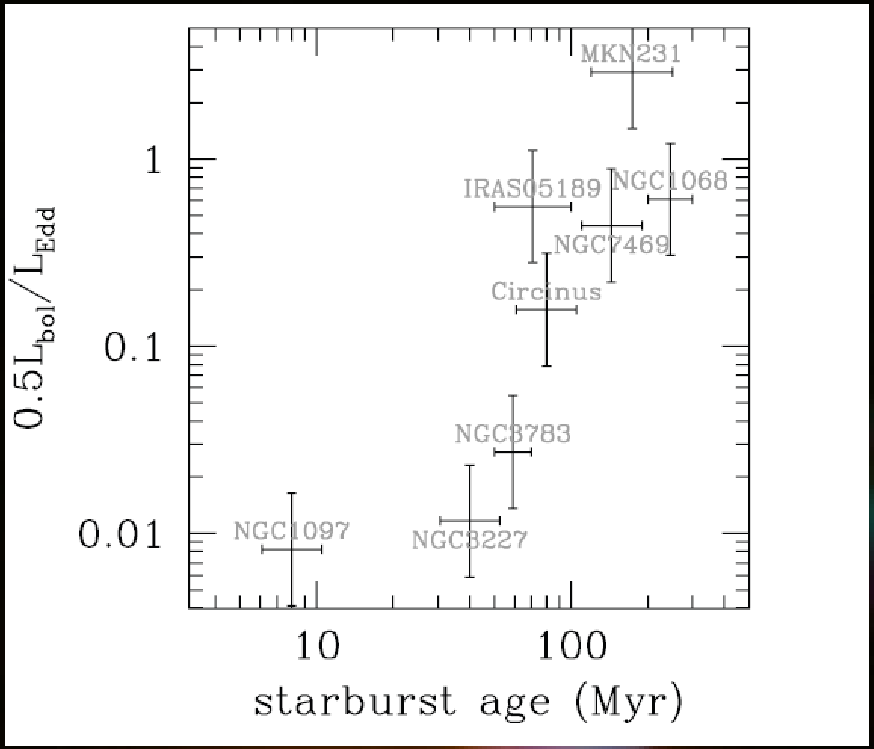


Both runs:

$P_j = 10^{45}$ erg/sec, $M_{gas} \simeq 2.34\times10^{11}\ M_\odot$, $T_g = 10^7$ K

FRI morphology → low internal enthalpy →
Diagnostics of the accretion/BAL region

A possible explanation of the L_{bol}/L_{Edd} – age connection



Davies et al. (2007): older starburst are associated with brighter AGNs

- Model: high $P_j \rightarrow$ higher $p_{bck} \rightarrow$ faster suppression of SF in the disc AND higher $T_{disc} \rightarrow$ higher L_{bol}

- Detailed modelling of gas+stellar discs with external backflow

$$\Omega(r) = \Omega_K(r) = \left(\frac{GM_{BH}}{r^3} + \frac{2\sigma^2}{r^2}\right)^{1/2},$$

$$\dot{\Sigma}_\star = \Sigma_g \Omega \eta,$$

$$p_{gas} + \epsilon \dot{\Sigma}_\star c\left(\tfrac{1}{2}\tau_V + \xi\right) = \rho h^2 \Omega^2,$$

$$p_{gas} = \rho k_B T / m_p,$$

$$T^4 = \frac{3}{4} T_{eff}^4 \left(\tau_V + \frac{2}{3\tau_V} + \frac{4}{3}\right),$$

$$\tau_V = \kappa \Sigma_g / 2,$$

$$\Sigma_g = 2\rho h,$$

$$\dot{M} = 4\pi R h \rho V_r = 4\pi R h \rho m c_s = 4\pi R h^2 \rho \Omega m,$$

$$\dot{M} = \dot{M}_{out} - \int_{R_{out}}^{r} 2\pi r \dot{\Sigma}_\star \, dr.$$

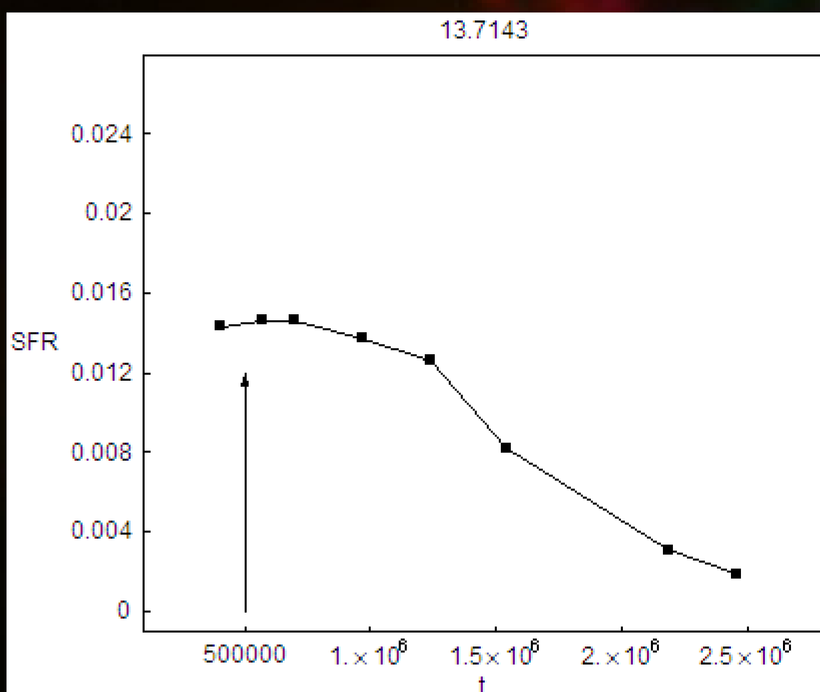
Future work

- VHE γ's from Hot Spot

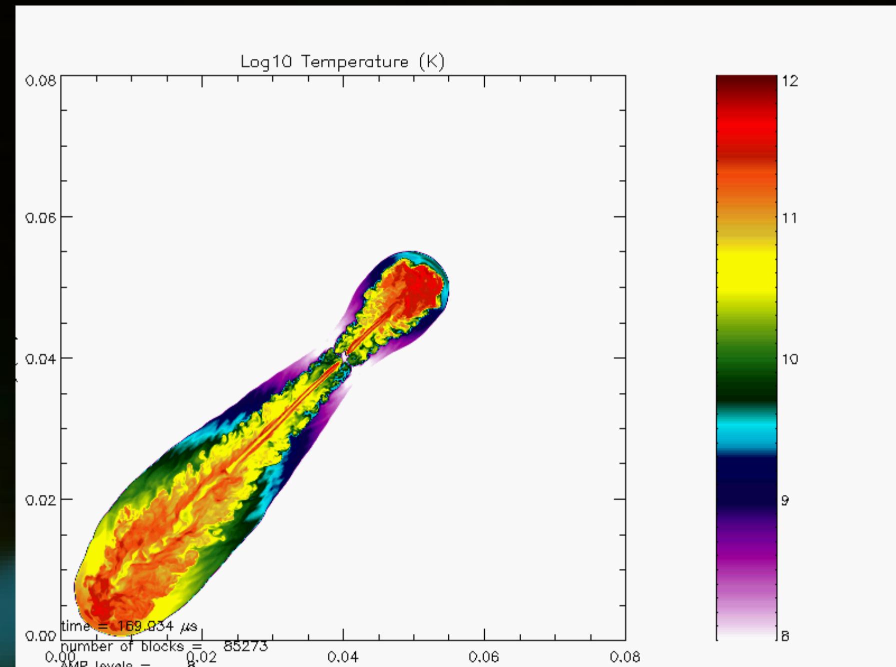
$T > 10^{11}$ K, $n_{hs} \sim 10^{-3} - 10^{-1}$ cm^{-3}

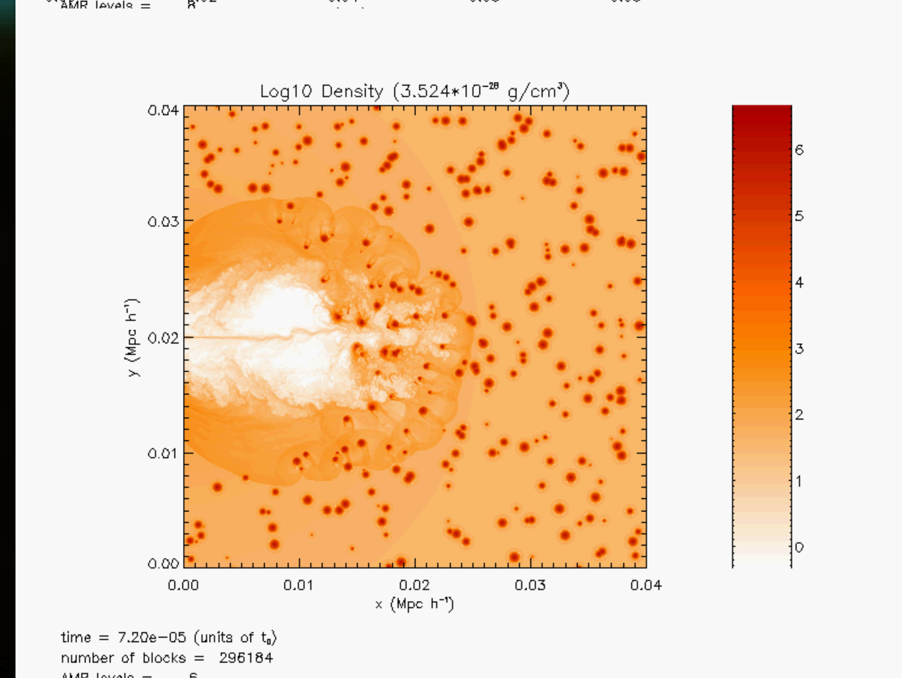
- Cold, star-forming clouds

Feedback: positive vs. negative

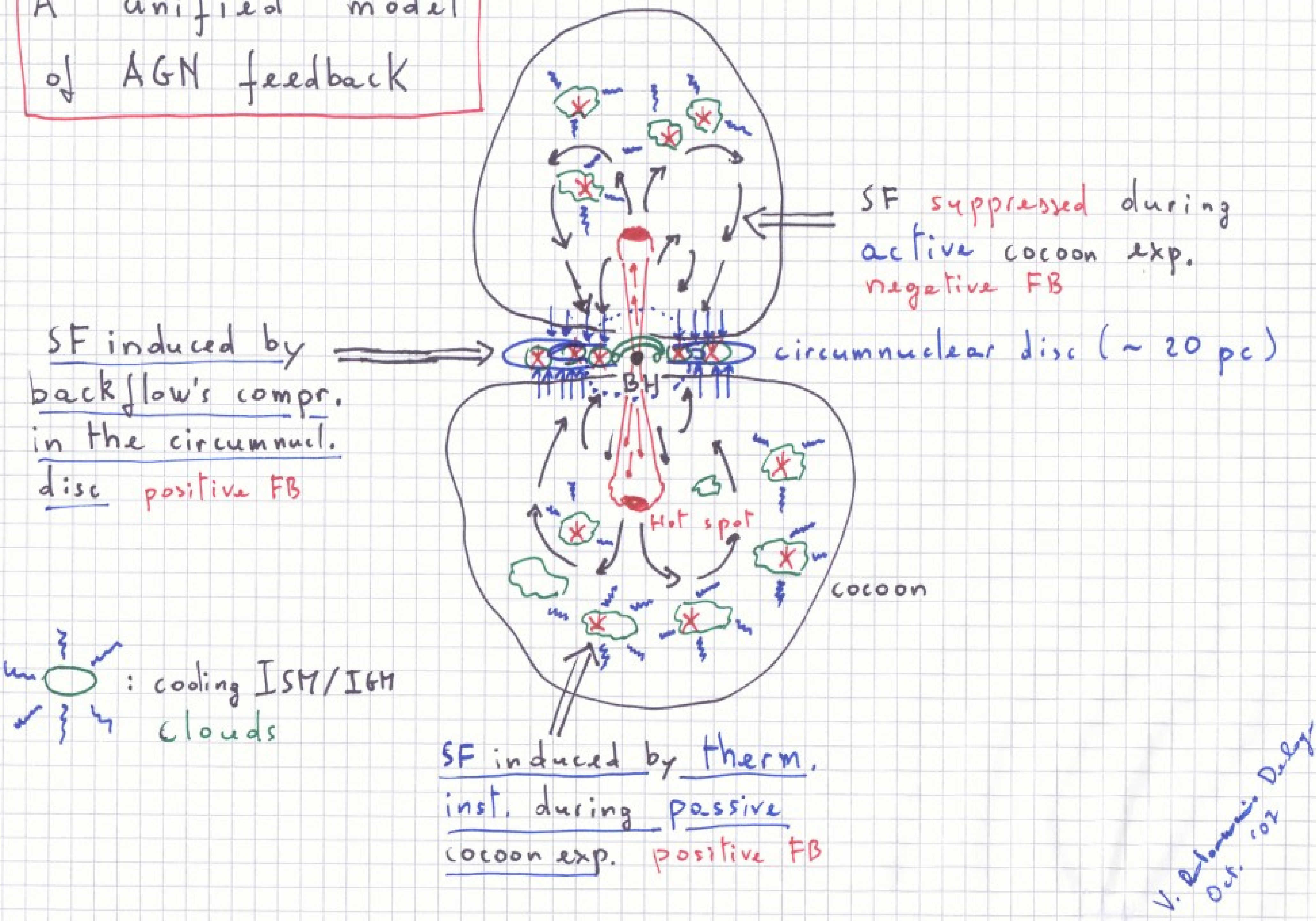


Toetota et al., et al., MNRAS 417, 2789 (2011)

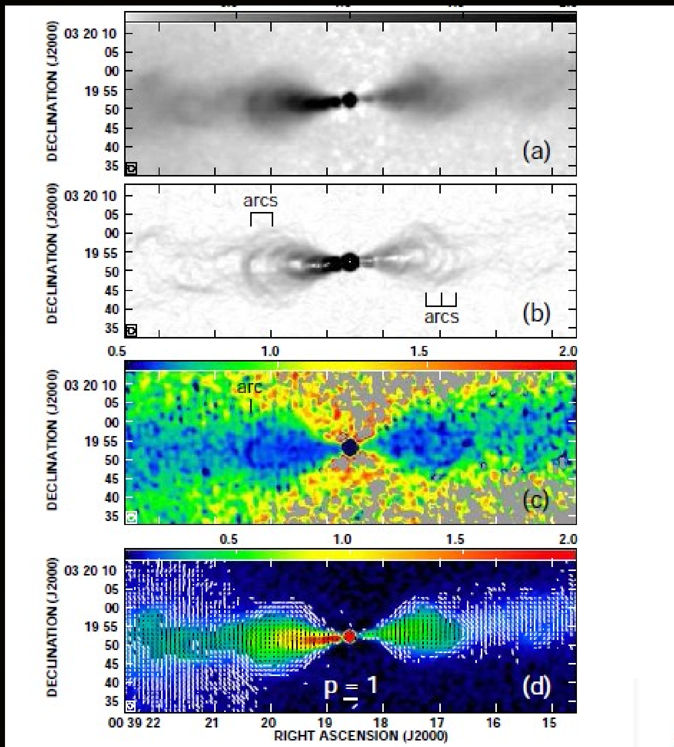




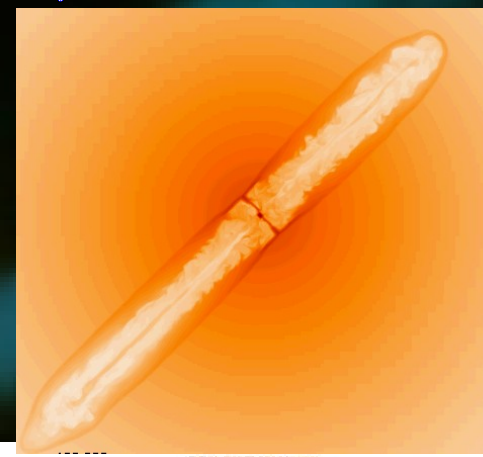
A "unified" model of AGN feedback



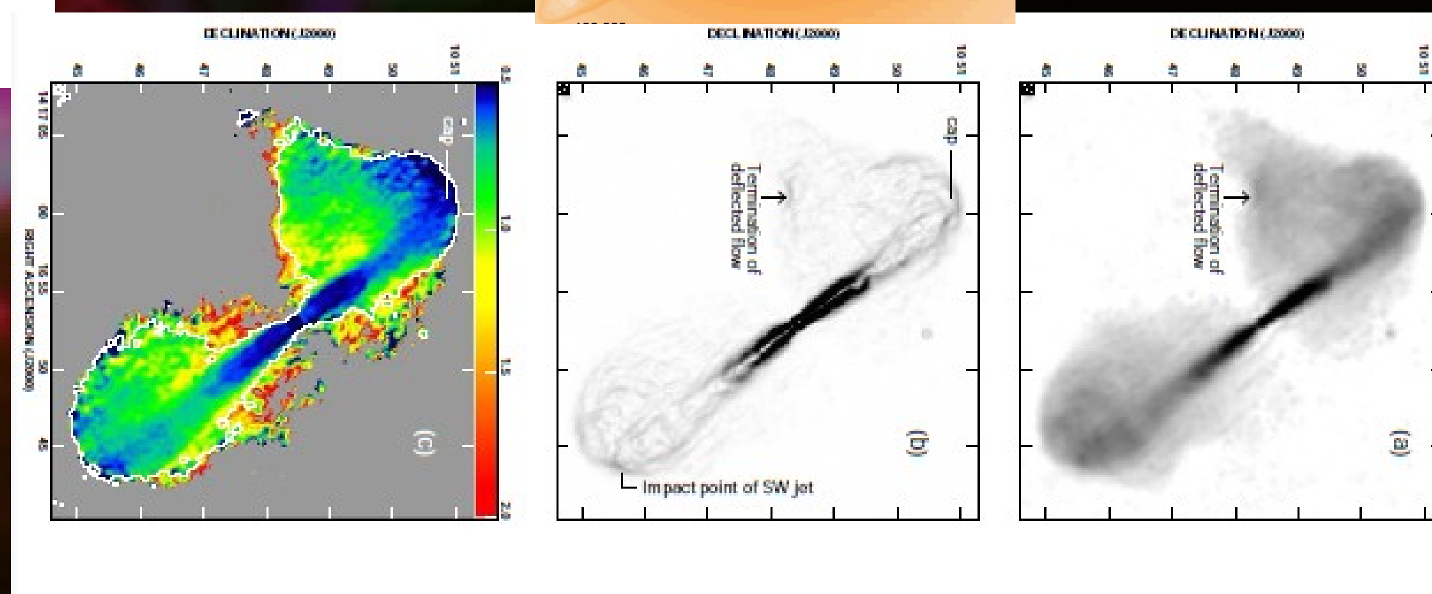
Morphology of FRI radio galaxies



$P_j = 10^{45}$ *erg/sec*



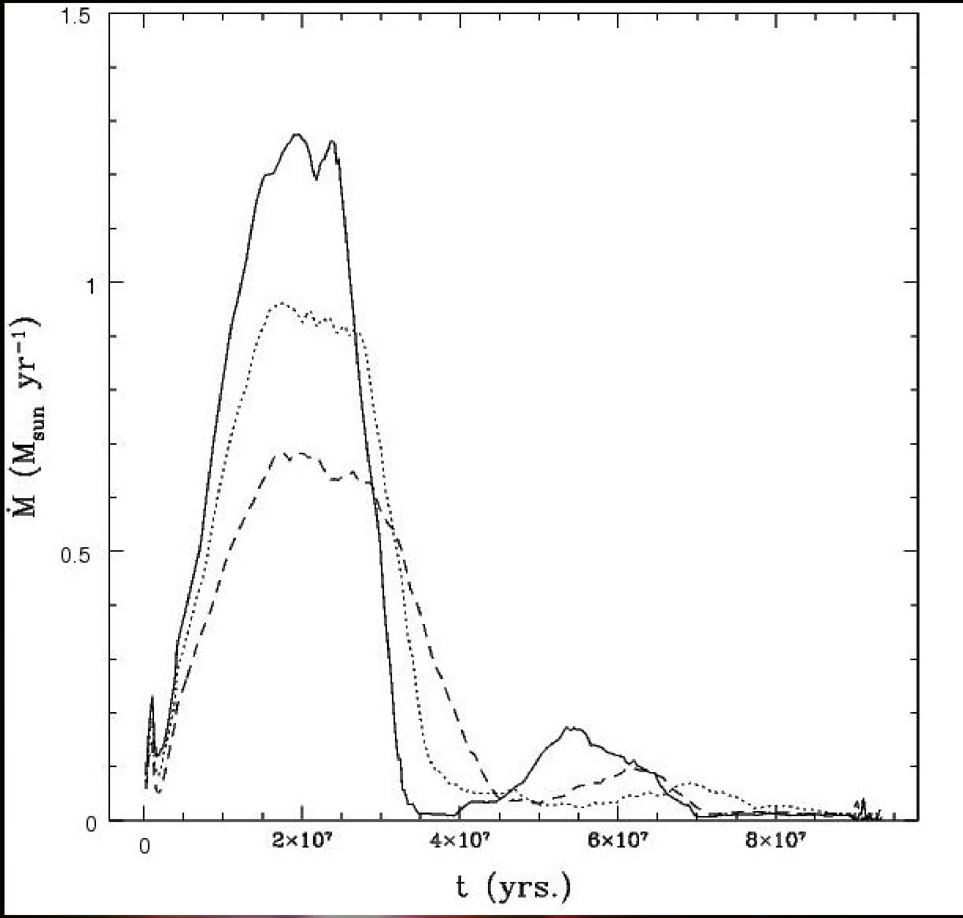
0206+35"

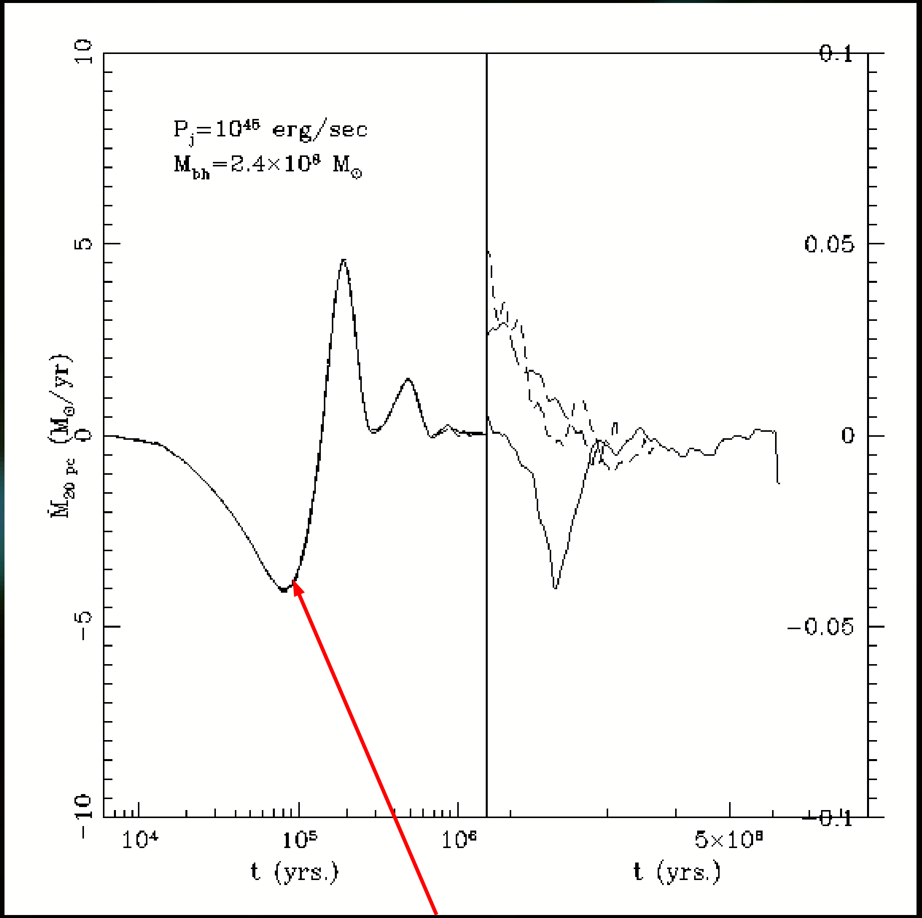


Laing et al., MNRAS 417, 2789 (2011)

Backflows: 2D vs. 3D

Mass flow around a 20pc sphere centred on the BH

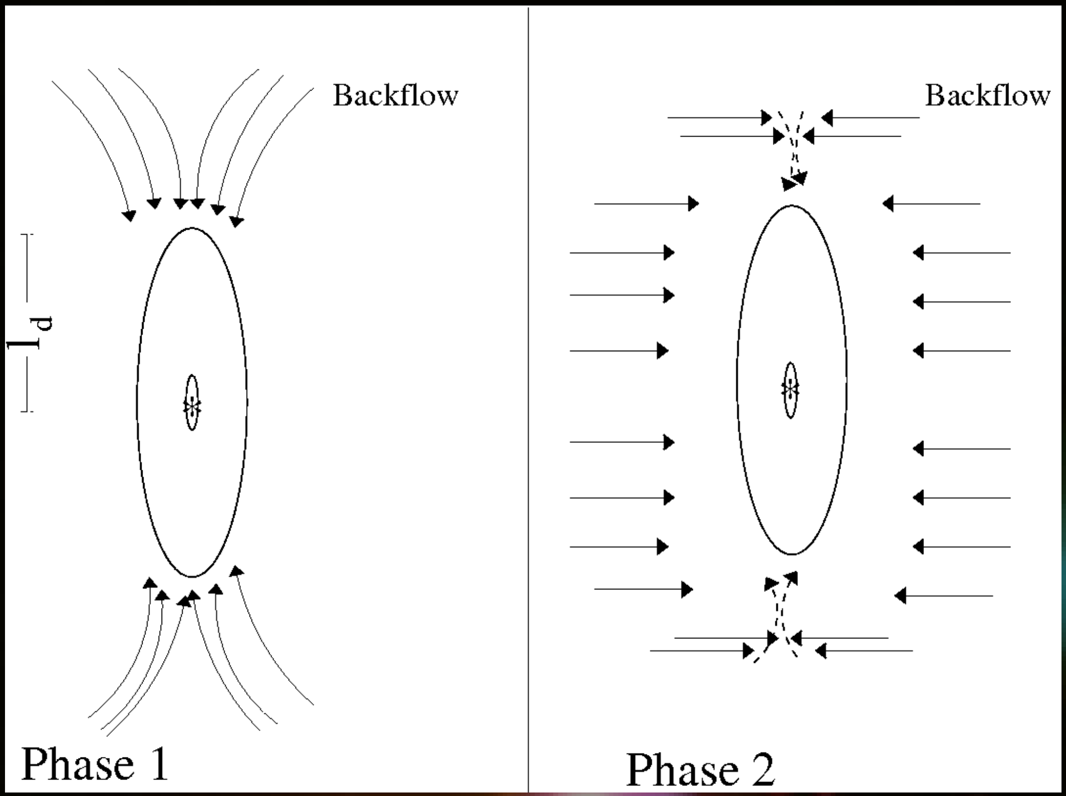




2D: T~2×10⁷ yrs., independent of central density

- *3D: early inflow phase (T~ 10^5 yrs.), indep. of central density*

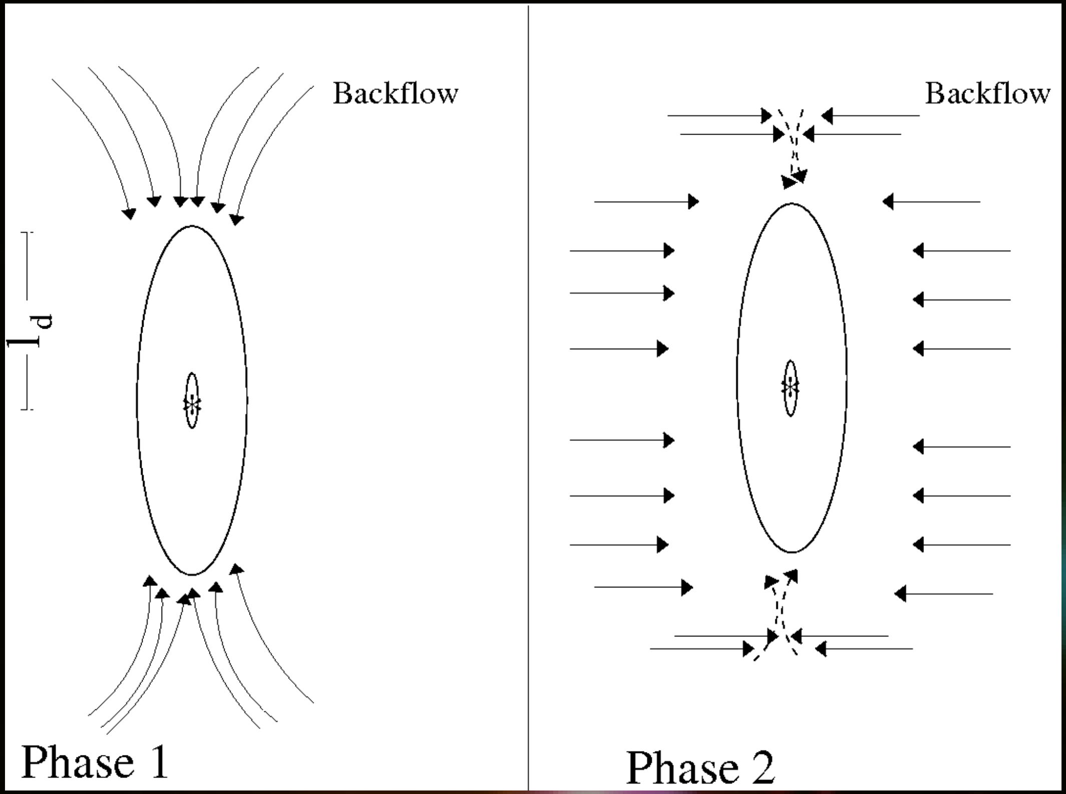
Backflow and the circumnuclear disk



Effects of backflow on CN/accr. discs:

- Stores $L_z \sim 0$ gas into the accr. disc $\rightarrow$ higher P_j
- High enthalpy gas $\rightarrow$ enhanced circumnuclear SF

Backflow and the circumnuclear disk



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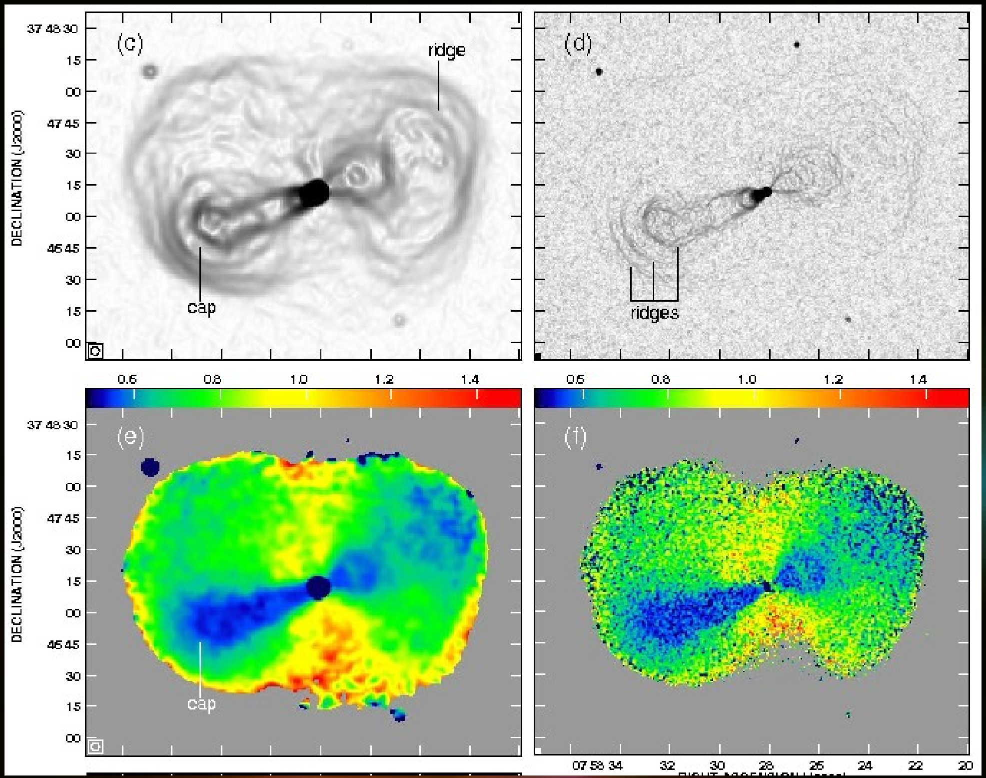
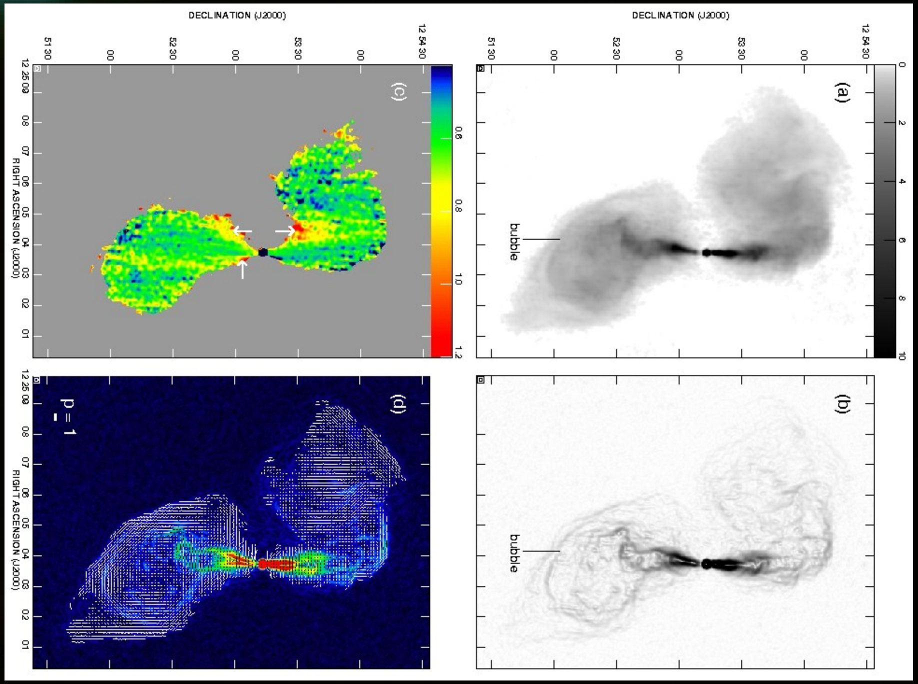
Morphology of FRI radio galaxies

0755+37, 4"/1.3"



Laing et al., MNRAS 417, 2789 (2011)

- Jets are relativistic (β>0.92), even far away (~few kpc) from the BH



M84,

Morphology of FRI radio galaxies

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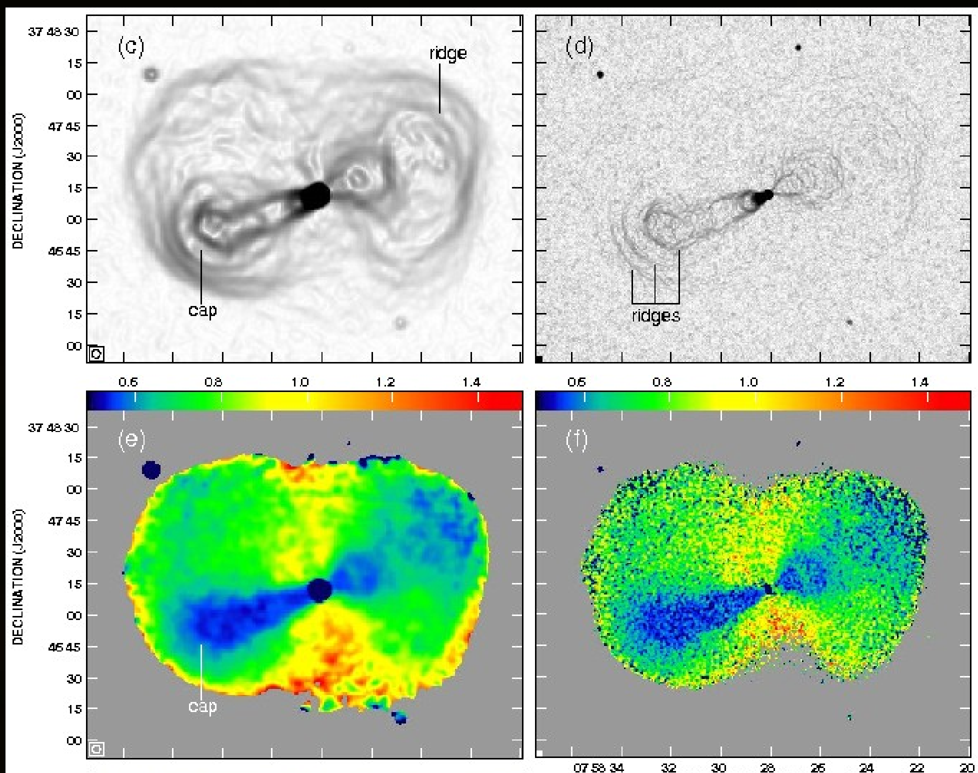

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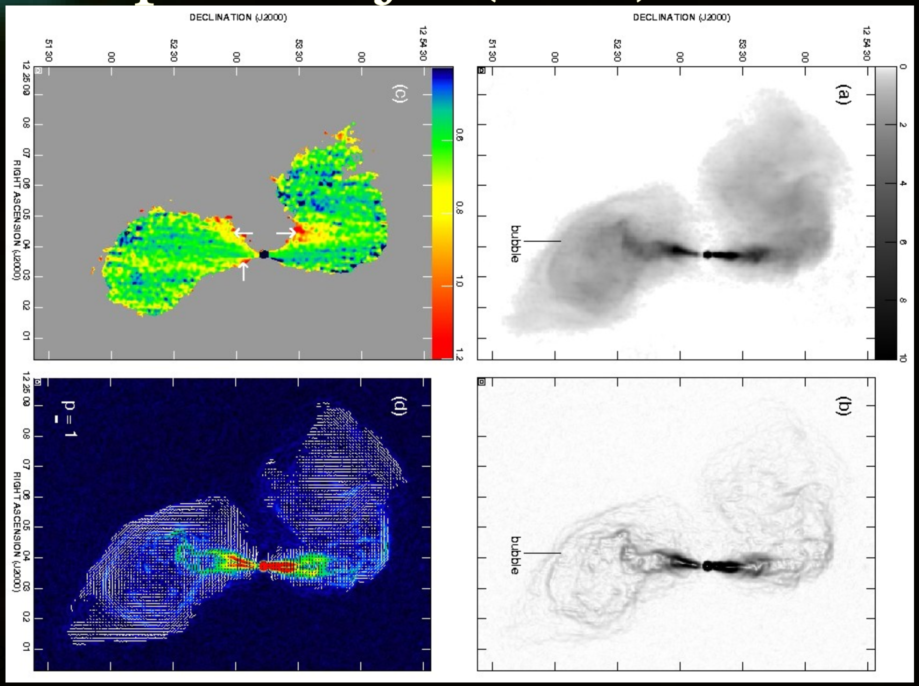
- Jets are relativistic (β>0.92), even far away (~few kpcs) from the BH

- $P_\nu \sim \nu^{-\alpha}$, spectral index α steepens ⊥ jet (MFs)



M84, 4"/1.3"

Morphology of FRI radio galaxies

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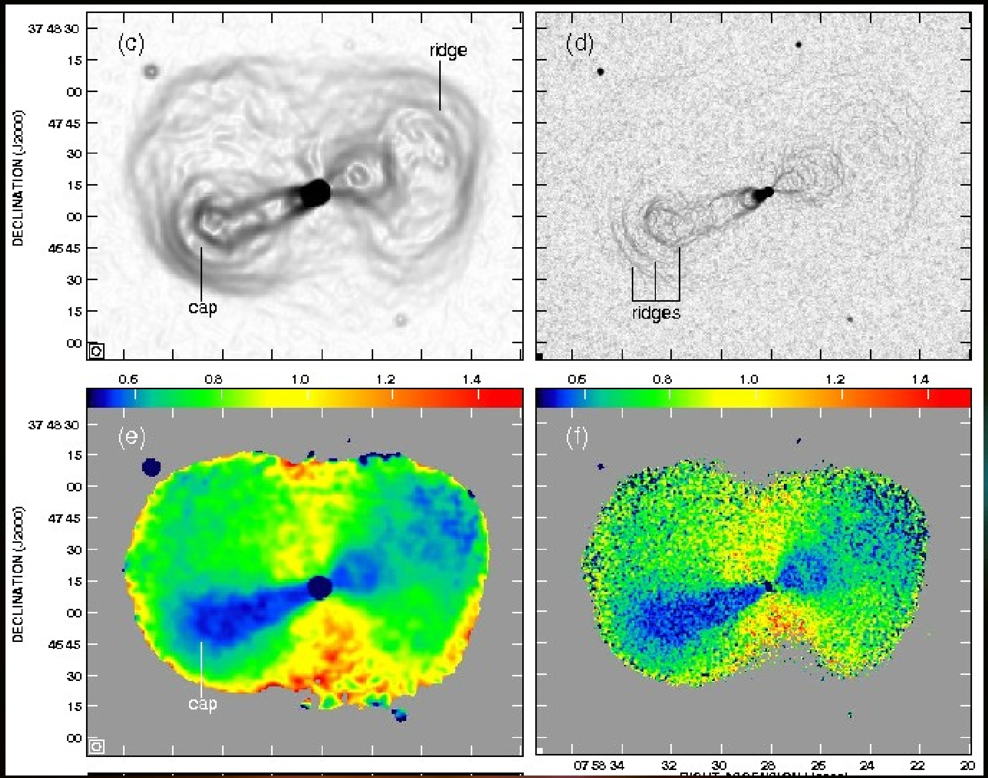

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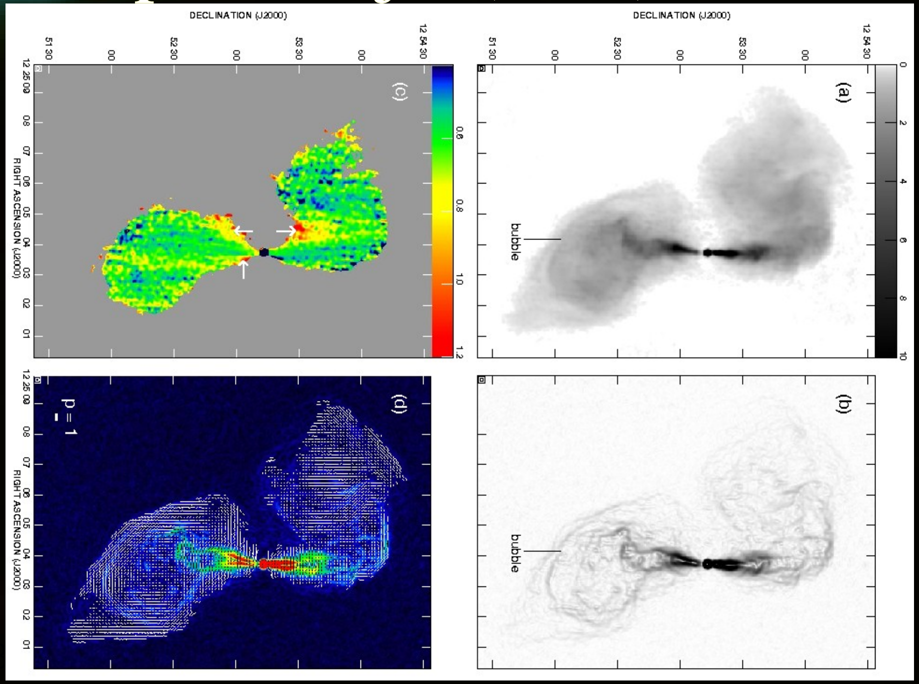
- $P_\nu \sim \nu^{-\alpha}$, spectral index α steepens $\perp$ jet (MFs)

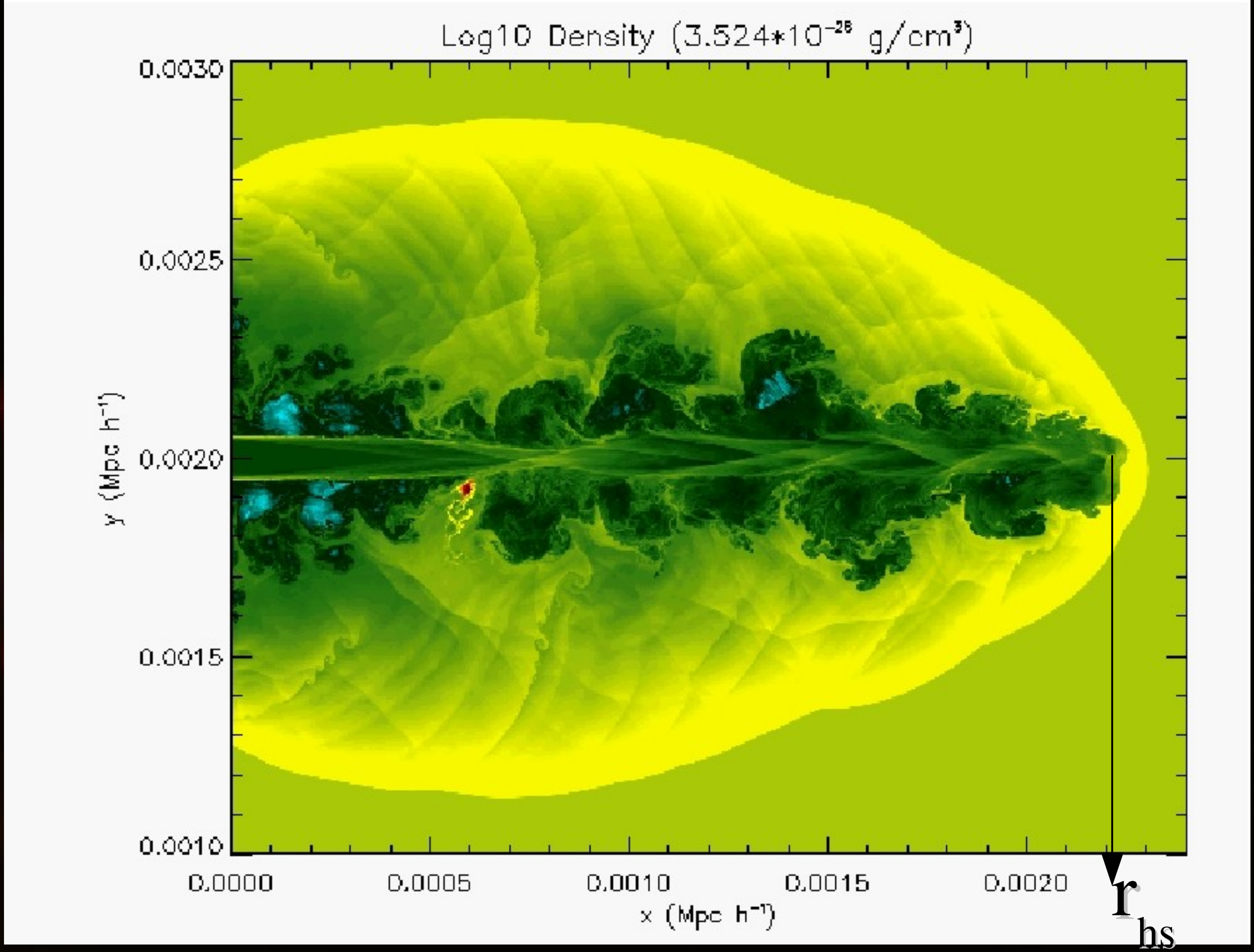


M84, 4"/1.3"

- α flat along the jet, and clumpy (jet-cold clouds interactions?)

- Relativistic jet propagating into the ISM: $\langle n_e \rangle \sim 10^{-3}$-1 cm^{-3}, T ~ 10^8-$2*10^{11}$ K *cocoon*

V. A.-D. and J. Silk, MN 389, 1750 (2008)



- Global expansion: self-similar, $r_{hs} \sim t^{2/5}$ (Falle, 1991)

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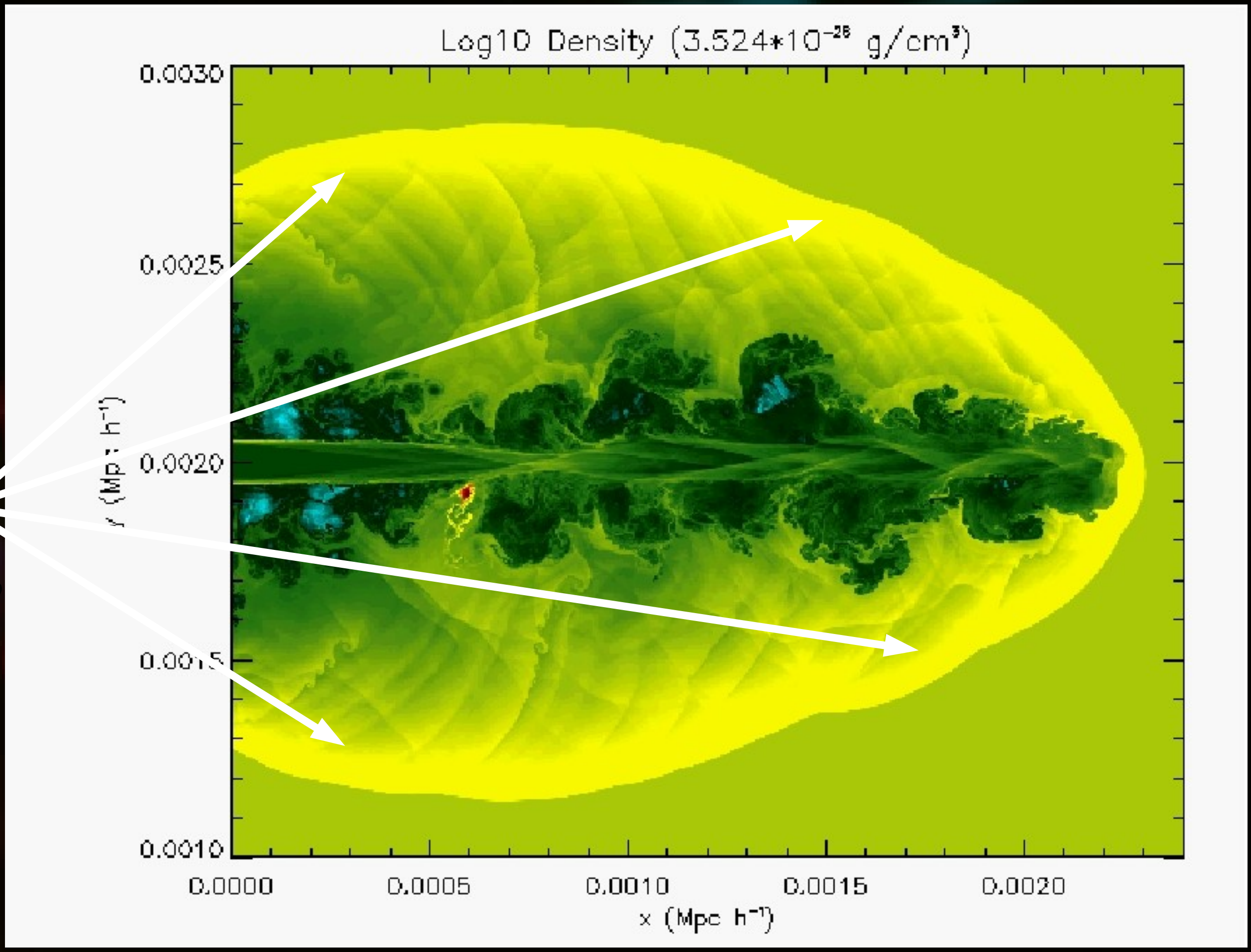
V. A.-D. and J. Silk, MN 389, 1750 (2008)

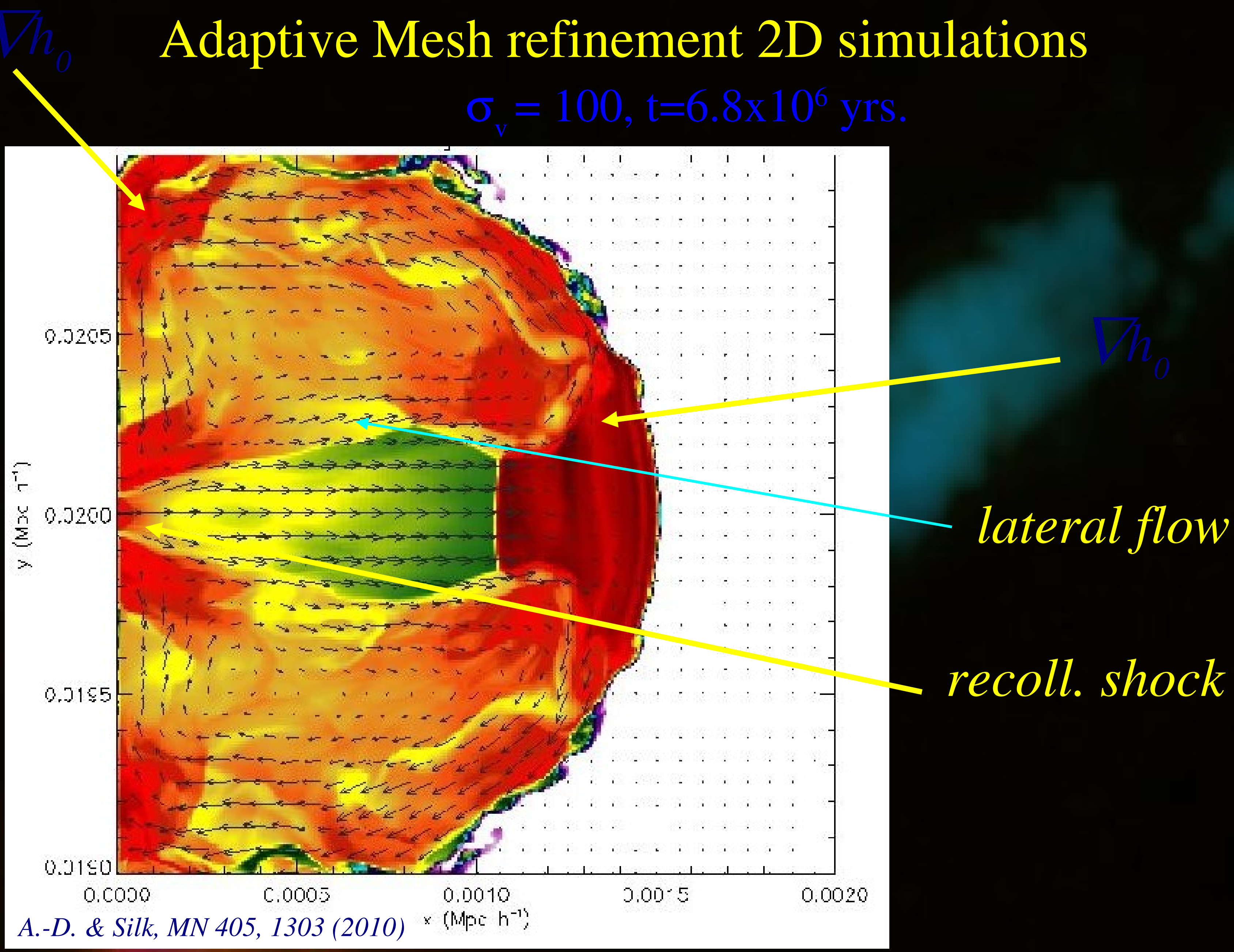
- Global expansion: self-similar, $r_{hs} \sim t^{2/5}$ (Falle, 1991)

- A global circulation arises within the cocoon, with a regular flow along the bow shock



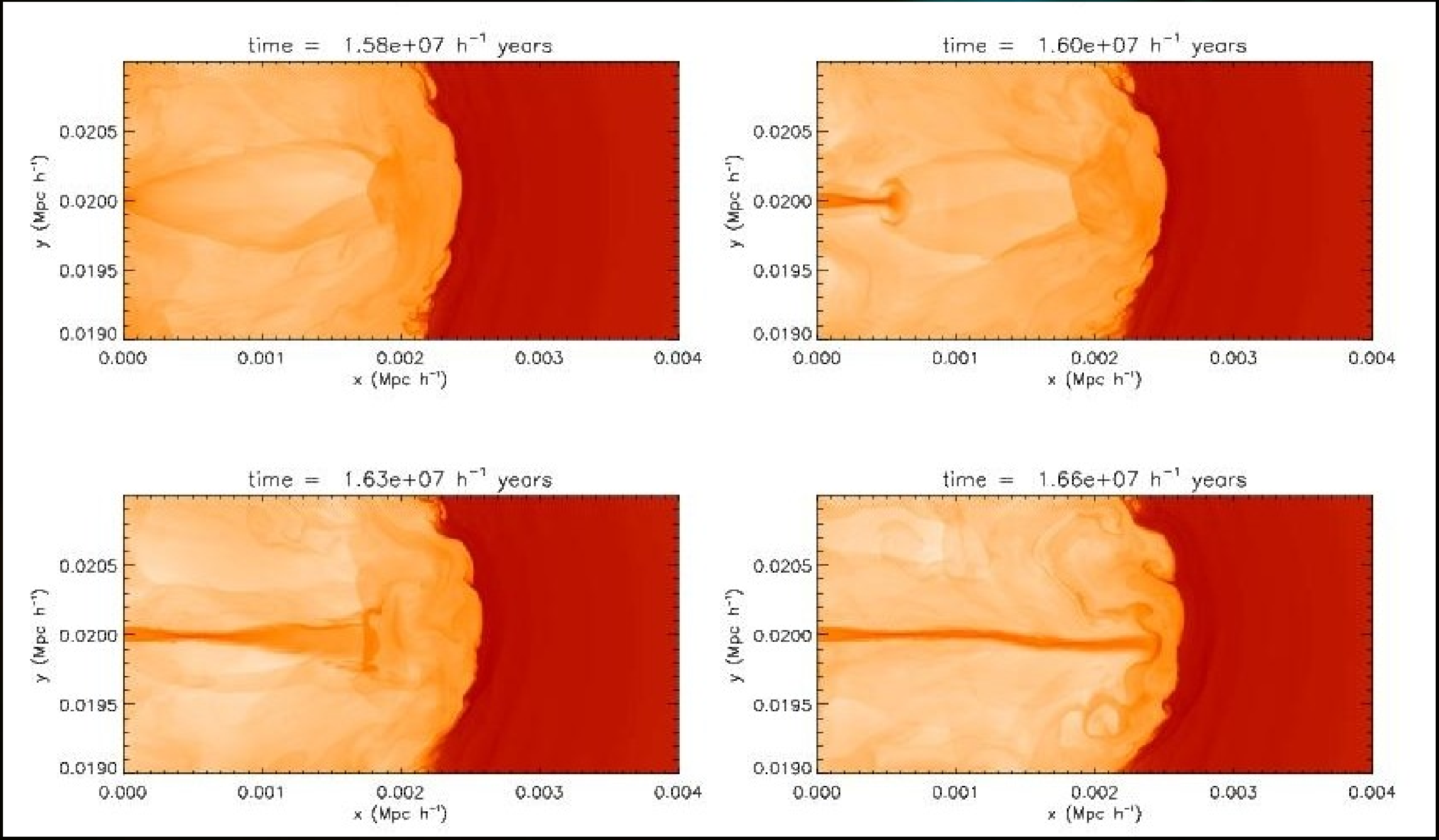
Adaptive Mesh refinement 2D simulations

$\sigma_v = 100$, $t=6.8\times10^6$ yrs.



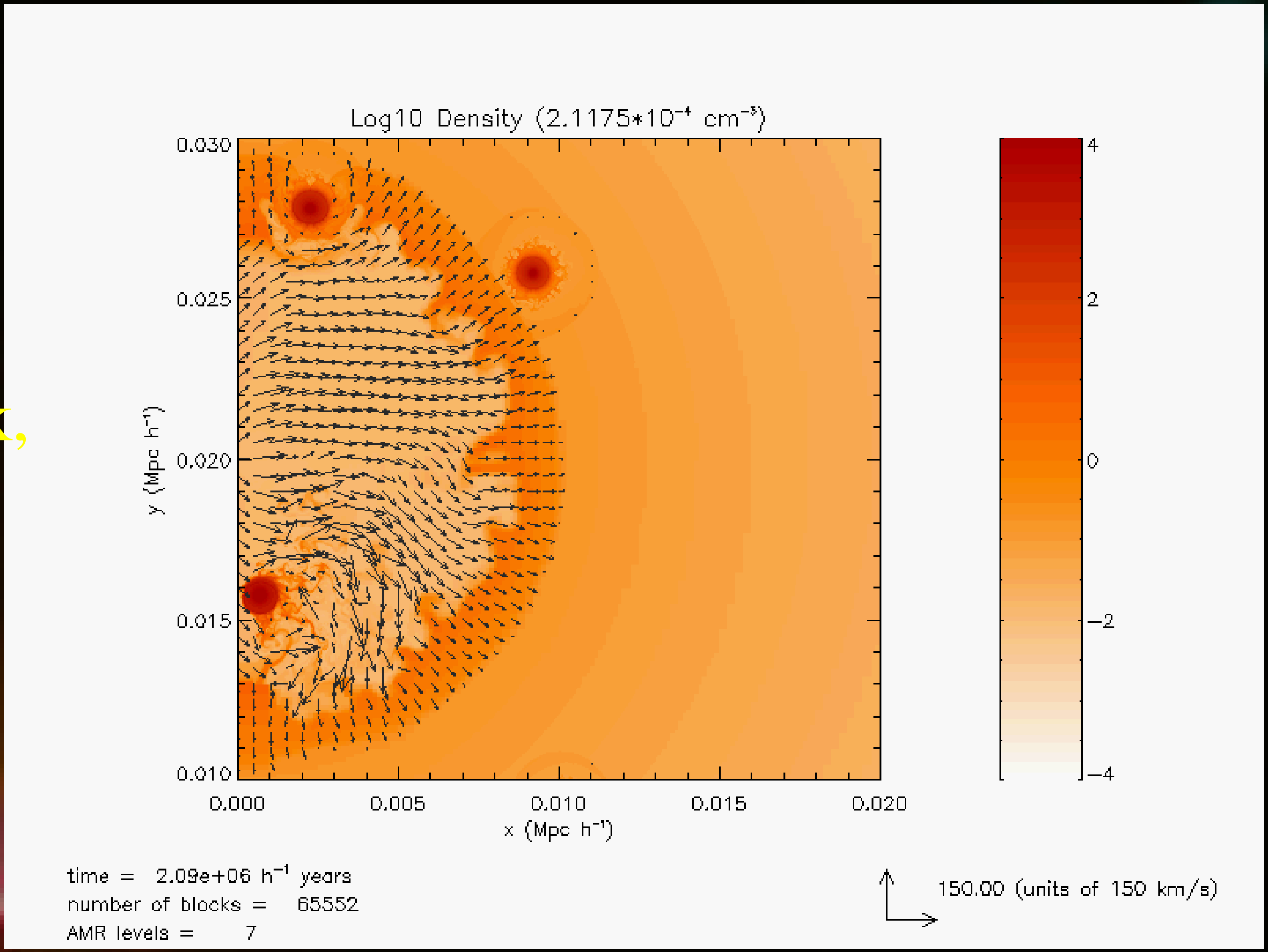
- A *global backflow circulation* develops – abt. ~ 2-10*10^{-3} of the jet's gas flows back towards the BH

- At t~1.6x10^7 yrs. the recoll. shock is destroyed $\rightarrow$ the meridional circulation disappears



- If P_{jet} exceeds by 2σ the fiducial value the backflow disappears

$P_{jet} = 2.3*10^{45}$ erg s^{-1} in model sm200av (V. A.-D. & Silk, 2010)



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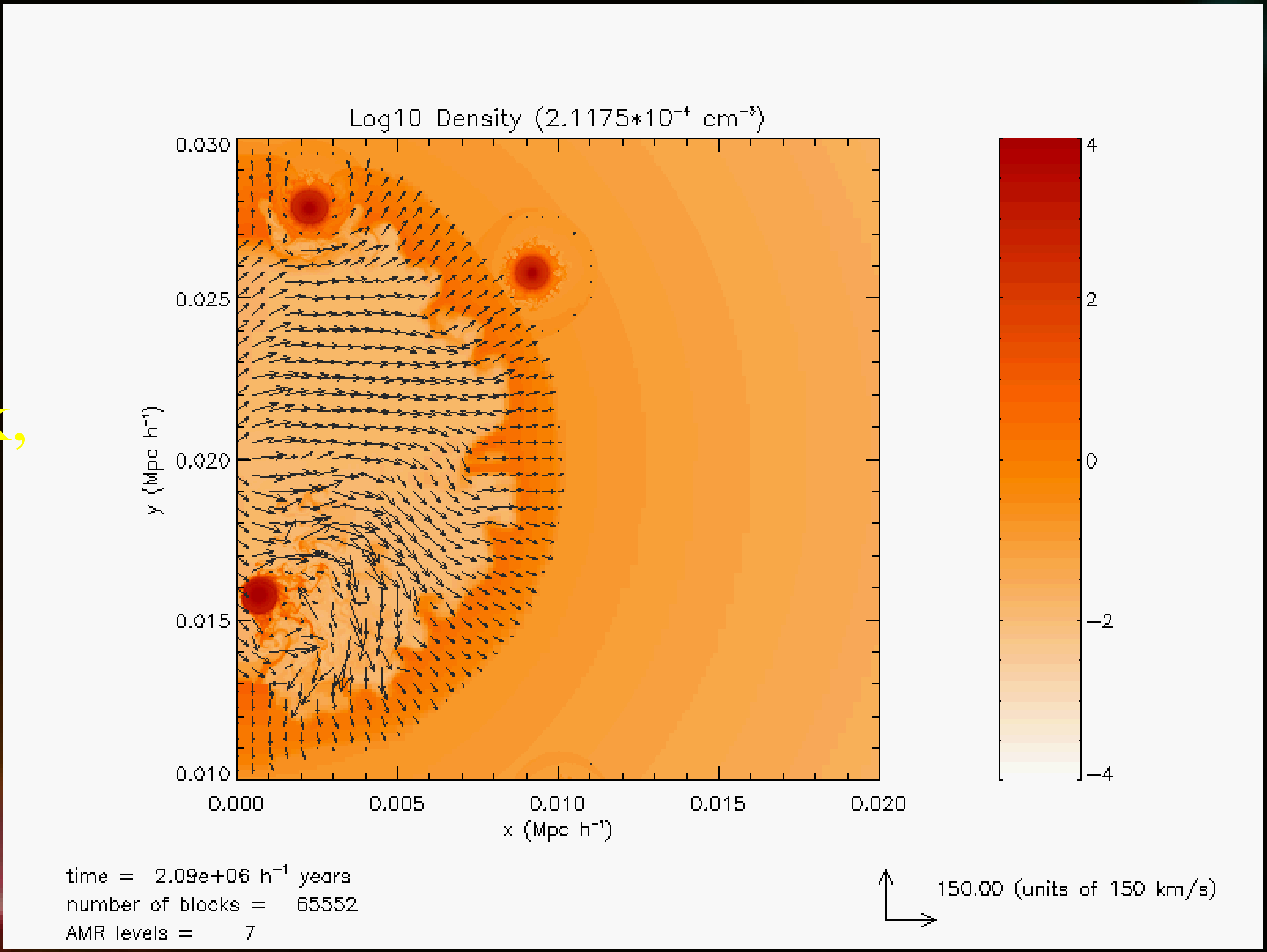
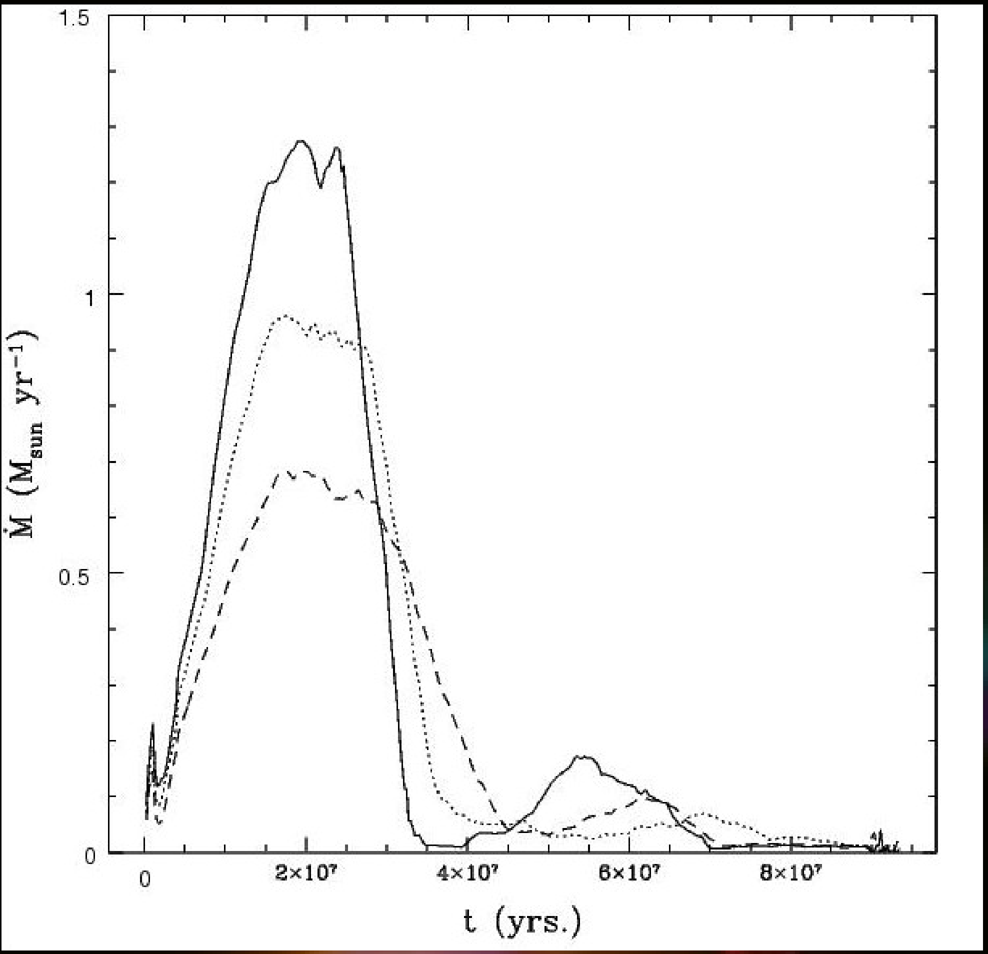
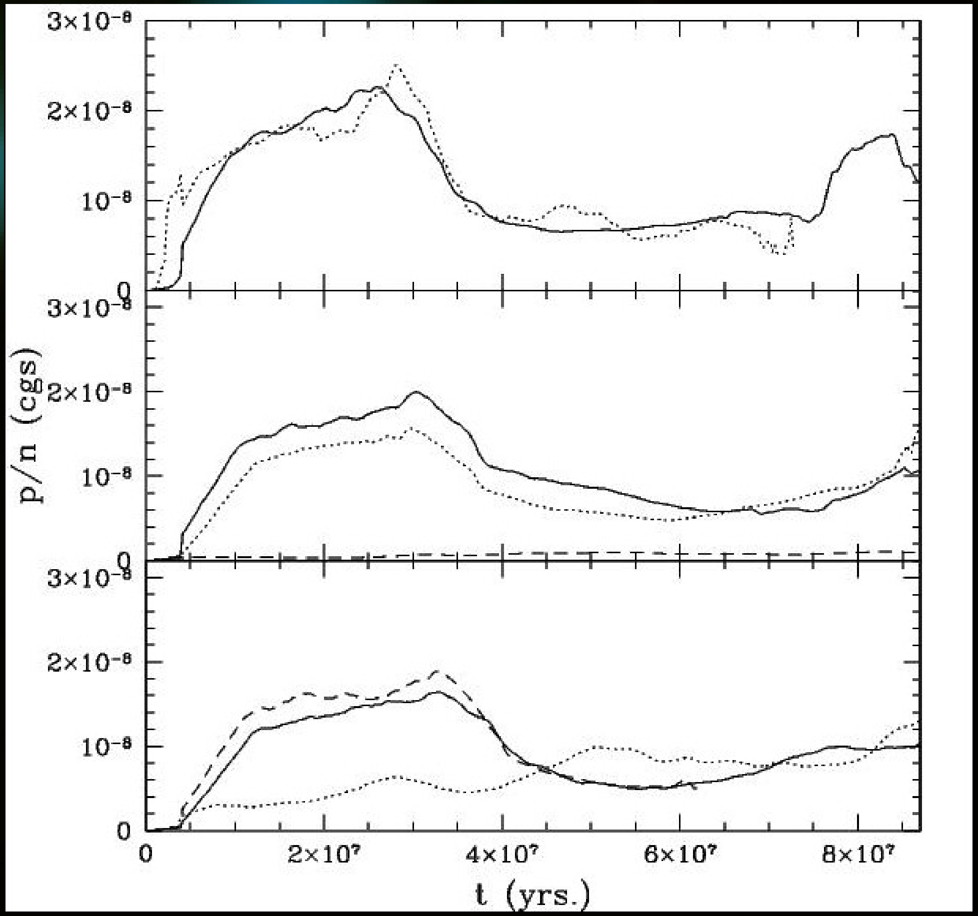


- The environment "seen" by the relativistic jet when it first enters ISM has an impact on the global structure of the cocoon

Mass backflow in a 10 pc circumnuclear region

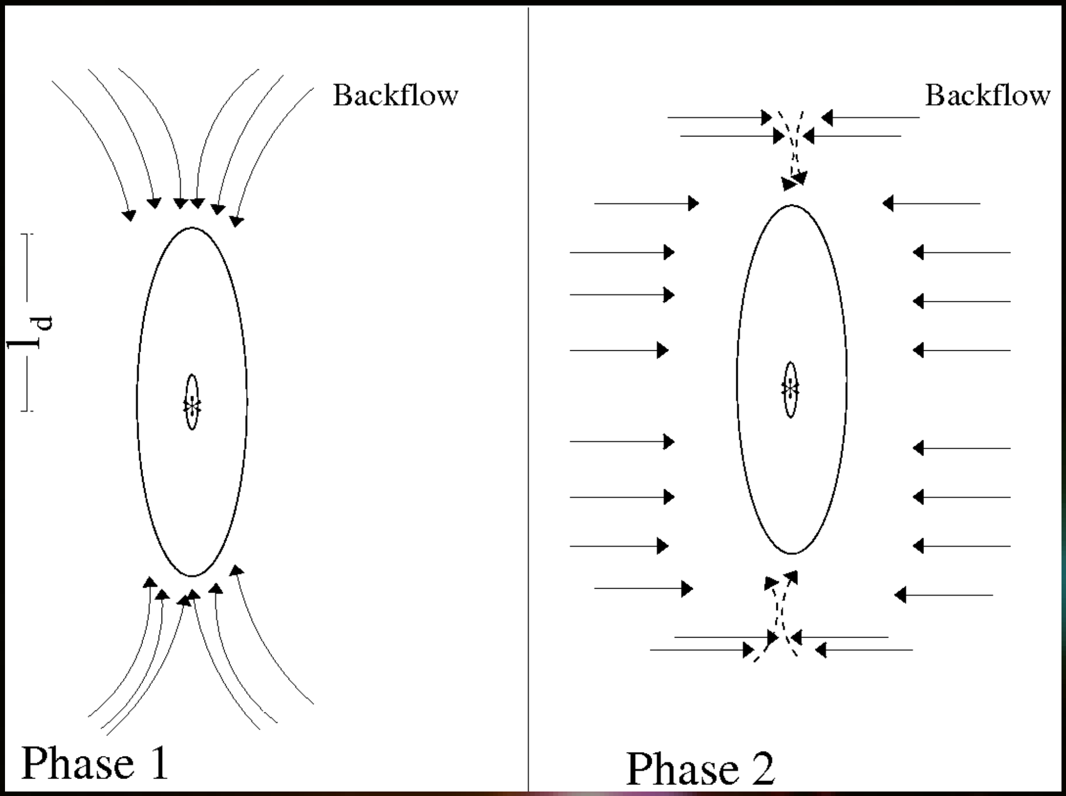


- $dM/dt \sim 0.32 – 0.76\ M_{sun}\ yr^{-1}$, peak values $\sim 0.6 – 1.3\ M_{sun}\ yr^{-1}$
- $\tau \sim 2\text{-}4 \times 10^{7}$ yrs.
- Accr. rates needed: $10^{-5}\ M_{sun}\ yr^{-1}$



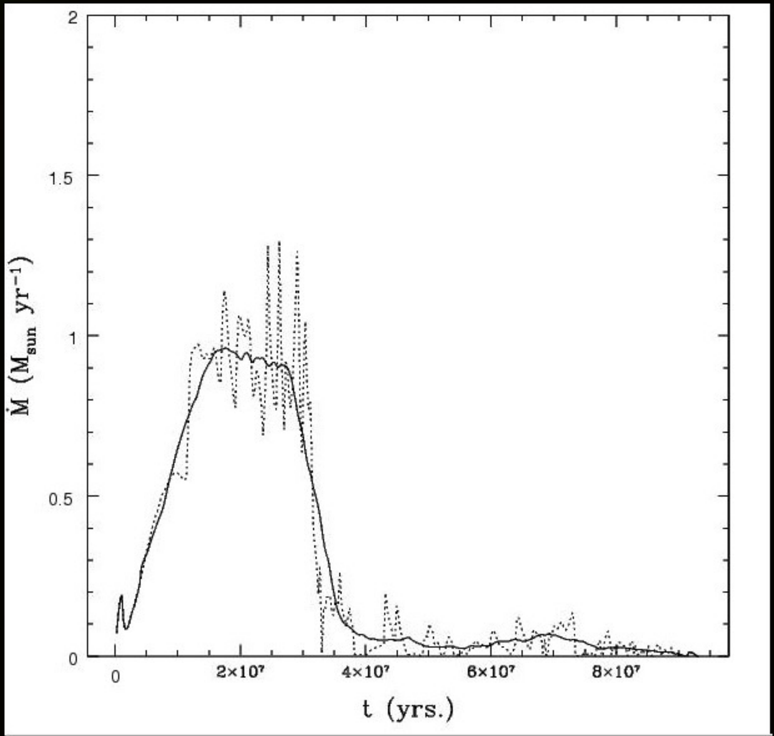
- For given P_j, n_{ism} strongly affects mass flow rates and backflow energetics compression → starburst
- NOTE: the backflow contributes $L_z \sim 0$ gas

Feedback of the backflow ON the circumnuclear disk



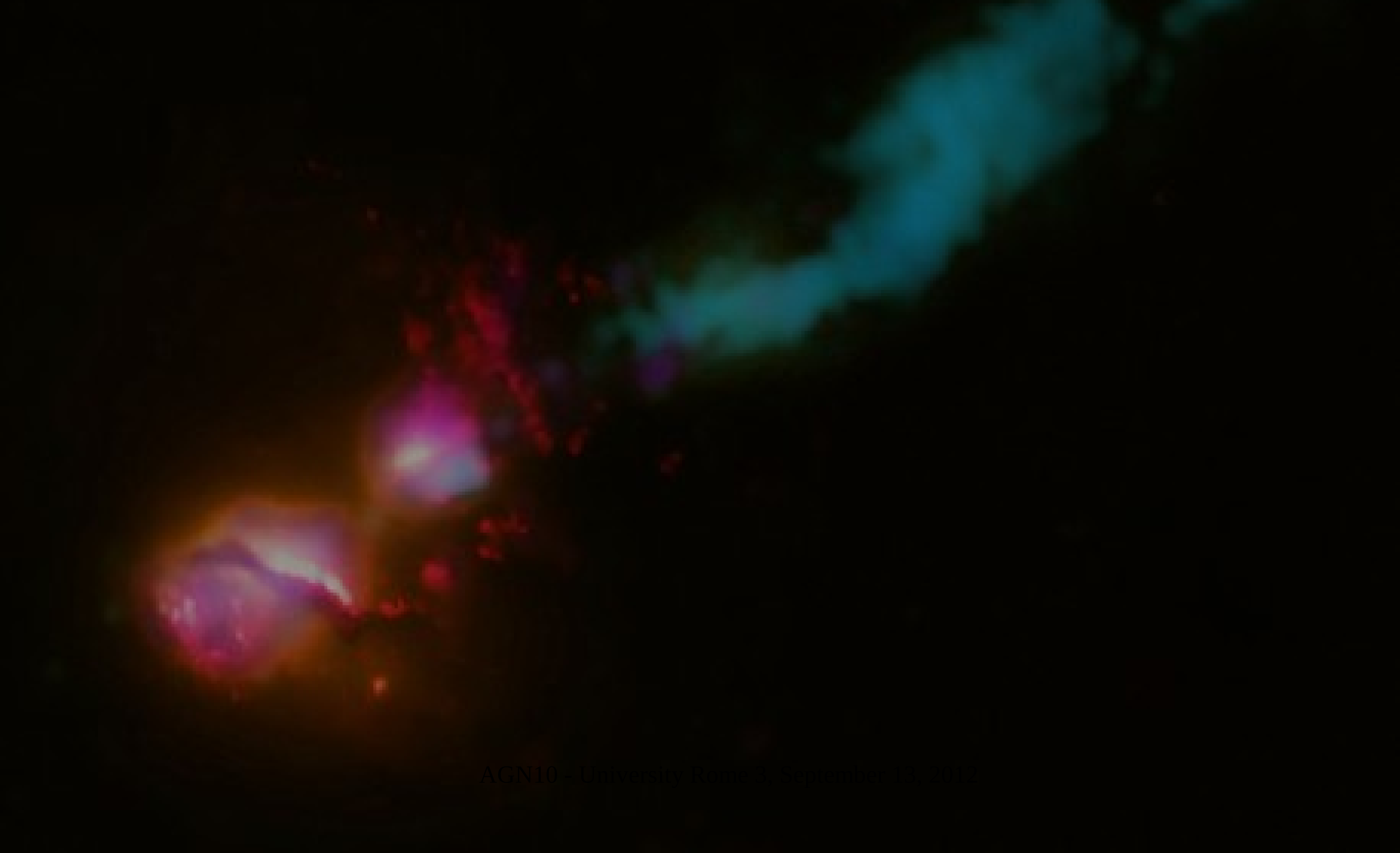
Effects of feedback from backflow:

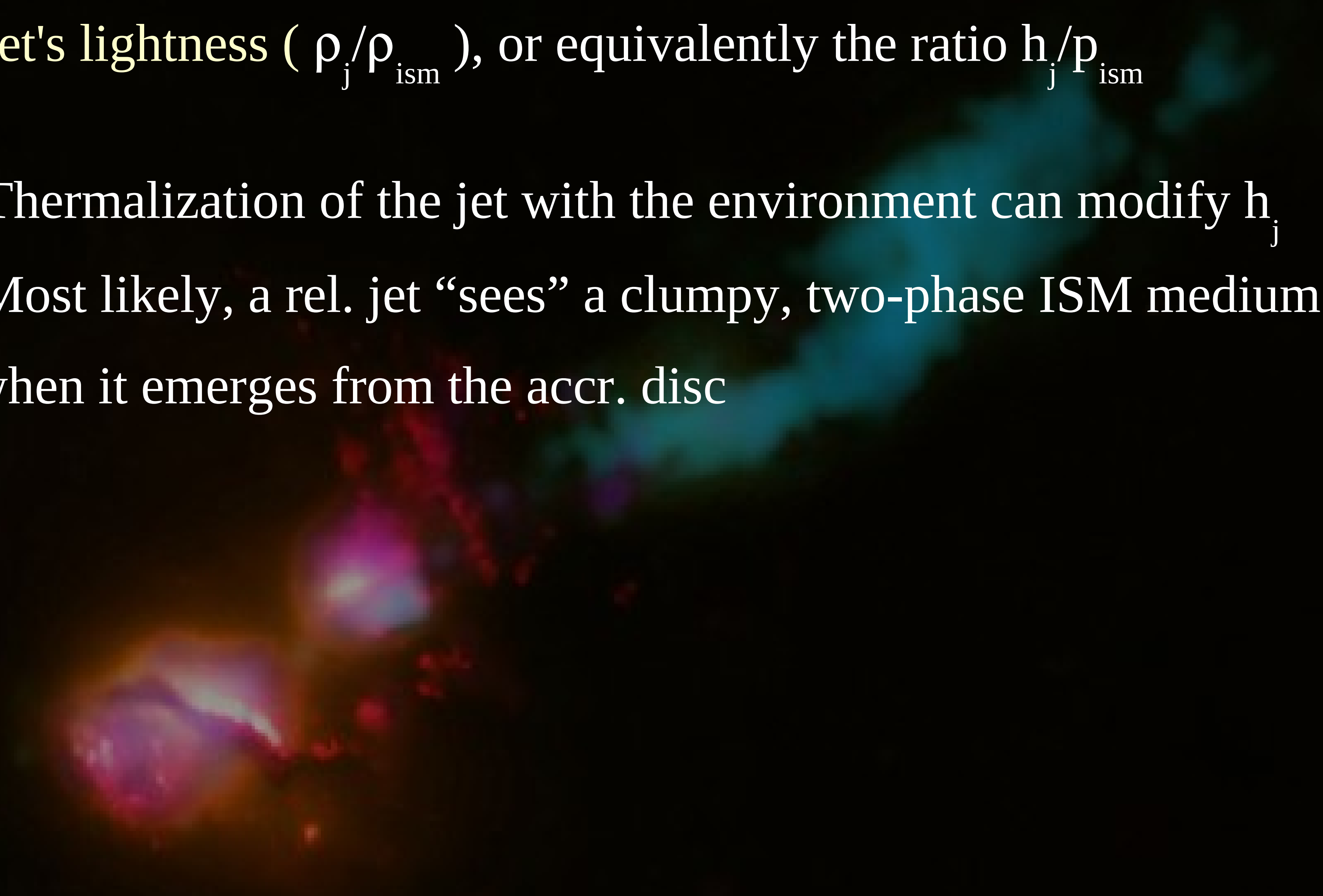
- Stores more $L_z \sim 0$ gas into the accr. disc $\rightarrow$ higher P_j
- Line indices: shocks + starbursts (*Mazzuca et al., 2006; Sarzi et al., 2007*)



- Enhanced SF from compression
- Intermittency $\rightarrow$ Series of SF episodes

- Morphology of the cocoon in simulations is seen to depend on jet's lightness (ρ_j/ρ_{ism}), or equivalently the ratio h_j/p_{ism}





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Most likely, a rel. jet "sees" a clumpy, two-phase ISM medium when it emerges from the accr. disc

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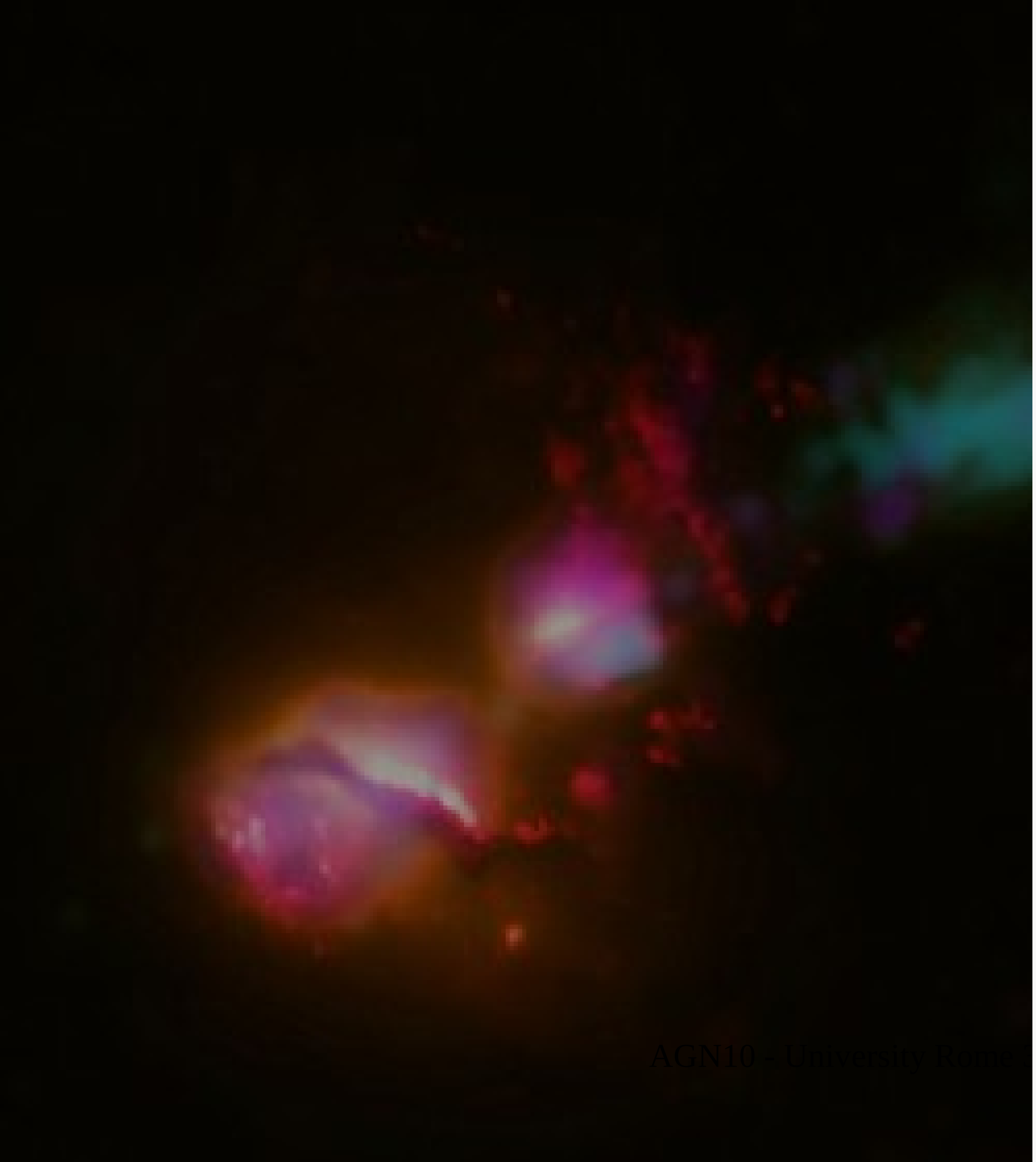
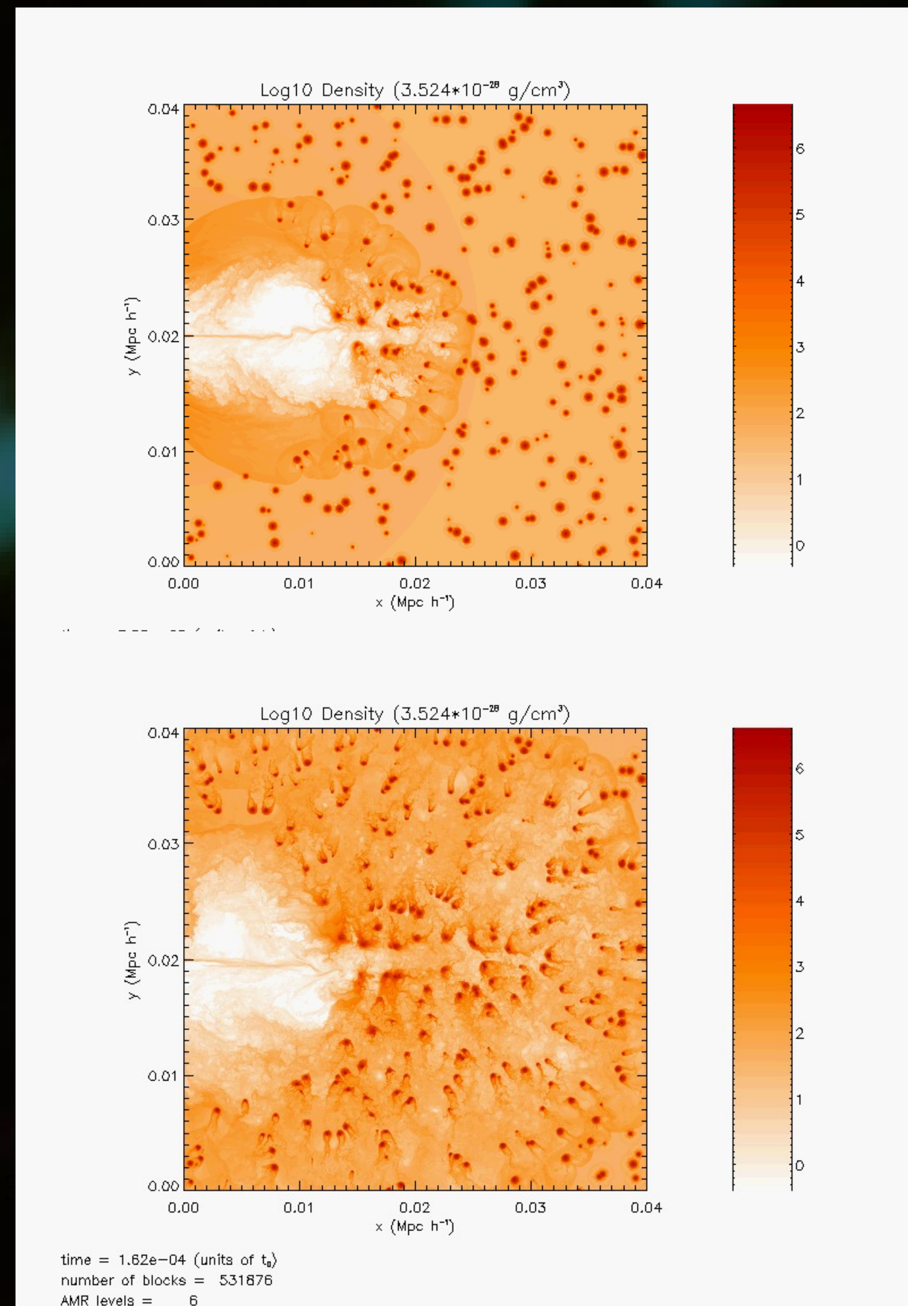
- Jet-clouds interaction can modify also the cocoon

- Thermal energy of the cocoon can be utilized in evaporating cold, star-forming clouds → SF feedback

Tortora et al., MN 396, 61 (2009)





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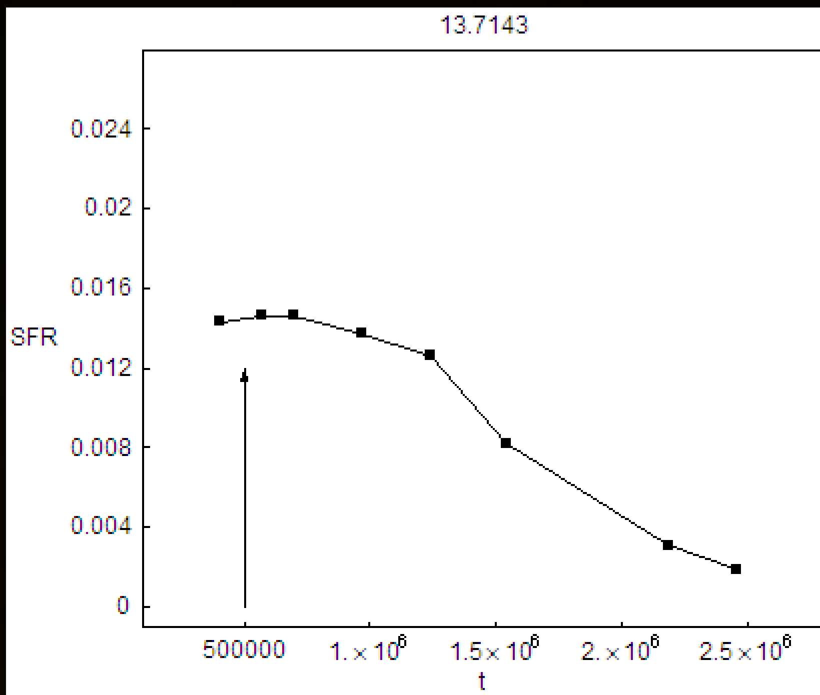

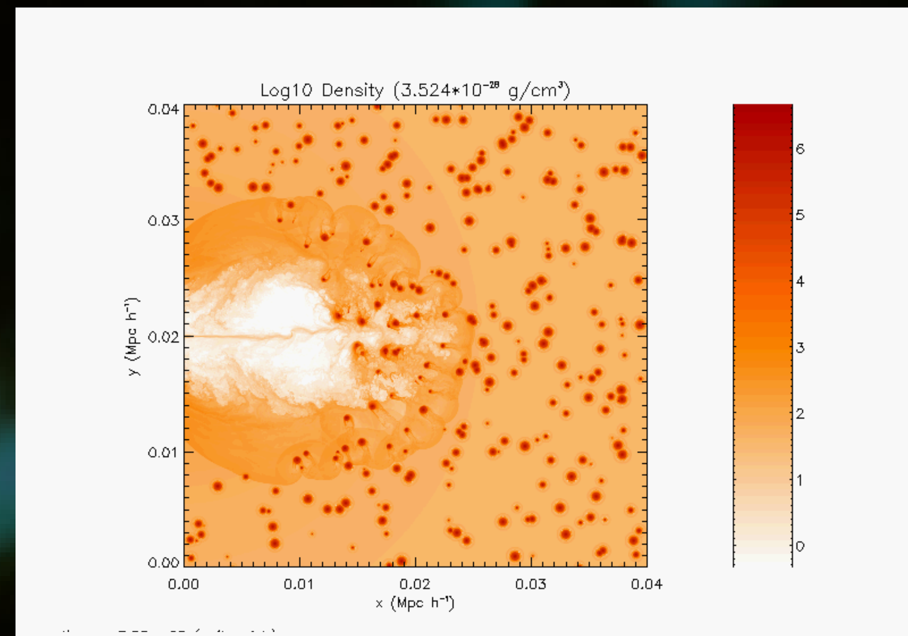

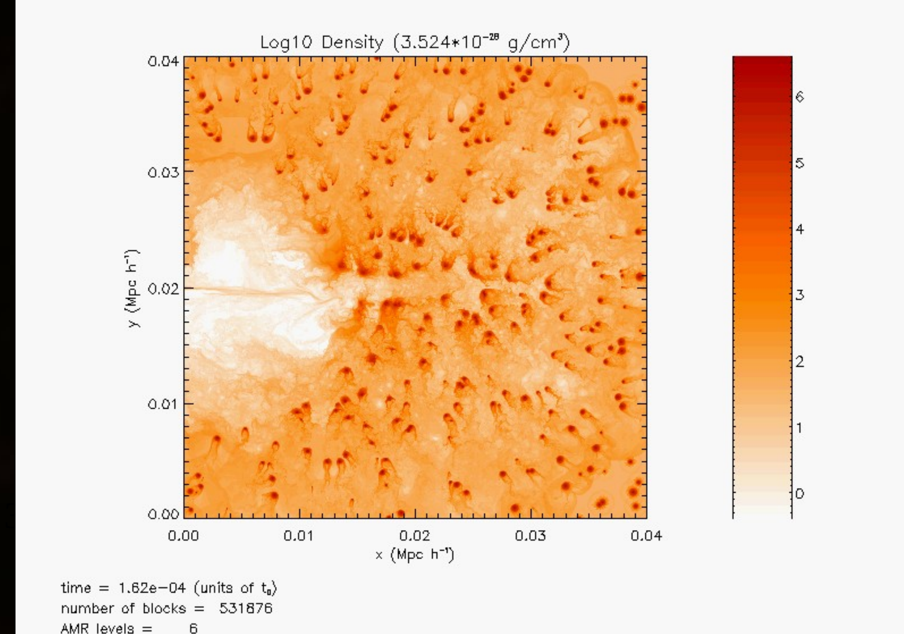

- SFR is damped, AFTER an initial enhancement (positive feedback)

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- Thermalization of the jet with the environment can modify h_j

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- Jet-clouds interaction can modify also the cocoon

- Thermalization of the jet affects the evolution of the cocoon

Thermodynamics of a relativistic jet

- Rel. jet-ISM/stars interactions: Relativistic Thermodynamics
  How does T transform in Special Relativity?
  Ans: depends on the thermometer (Touschek, 1968; Israel, 1986; van Kampen, 1974, ...)
- Non ideal relativistic fluid: T^{ab} explicitly includes thermal conduction (Eckart, 1974):

$$T^{ab} = e u^a u^b + \Pi^{ab}_{(0)} + q^a u^b + u^a q^b - \frac{q^a q^b}{e + p}$$

where:

$$\Pi^{ab}_{(0)} = p(g^{ab} + u^a u^b).$$

Thermodynamics of a relativistic jet (contd.)

- Here q^a is the *thermal conduction 4-vector* ($q^a u_a = 0$)

Relation with T (Vàn, arxiv:0712.1437, eq. 26):

$$q^a = -\frac{\eta}{T^2}\Delta^{ab}\left(\partial_b T + T\dot{u}_b + \frac{T\dot{q}_b}{e}\right)$$

where:

$\Delta^{ab} = \delta^{ab} - u^a u^b$ projection operator

$\dot{q}_b = u^a \nabla_a q^b$ convective derivative

- Rel. thermal conduction law:

$$\frac{1}{T}\left(\frac{\eta}{e}\right)\dot{q}^a + q^a = -\frac{\eta}{T^2}\Delta^{ab}\left(\partial_b T + T\dot{u}_b\right)$$

Thermodynamics of a relativistic jet (contd.)

- Define $$\lambda^a = q^a + eu^a$$

Energy-momentum conservation: $$\nabla_a T^{ab} = 0$$

We get: $$\partial_a \lambda^a + p\partial_a u^a - q^a \dot{u}_a - \frac{q^a q^b}{e+p}\partial_b u_a = 0$$

Note that: $$q^a \dot{u}_a = (q^a u_a)^{\dot{}} - u_a \dot{q}^a =$$

$$= -u_a \left[-\frac{e}{\eta} T q^a - \frac{e}{T} \Delta^{ab} \left(\partial_b T + T \dot{u}_b \right) \right] = 0$$

Thermodynamics of a relativistic jet (contd.)

- Thus:

$$\partial_a \lambda^a + p\partial_a u^a - \frac{q^a q^b}{e+p}\partial_b u_a = 0$$

- Jet flow line interacting with a cloud in pressure equilibrium: $p_j = p_c$, thus:

We get:

$$\partial_a(\lambda^a + p u^a) - \frac{q^a q^b}{e+p}\partial_b u_a = 0$$

From here, imposing the *thermal equilibrium* condition between Jet an cloud, we get: $T_j \approx \beta\gamma\, T_c$

- If instead the jet is in eq. With a photon gas (hot stellar corona we get (Israel, 1982): $T_j \approx \gamma\, T_\star$

Thermodynamics of a relativistic jet (end)

- Thus, the temperature structure of a jet emerging from the BH is strongly affected by the local environment within the first few pcs of the AGN

- Detailed modeling requires also to take into account the effects of the backflow on the thermal structure of the jet

Conclusion

- The thermal structure (enthalpy, lightness) of a relativistic AGN jet is important to determine the structure/morphology of the cocoon

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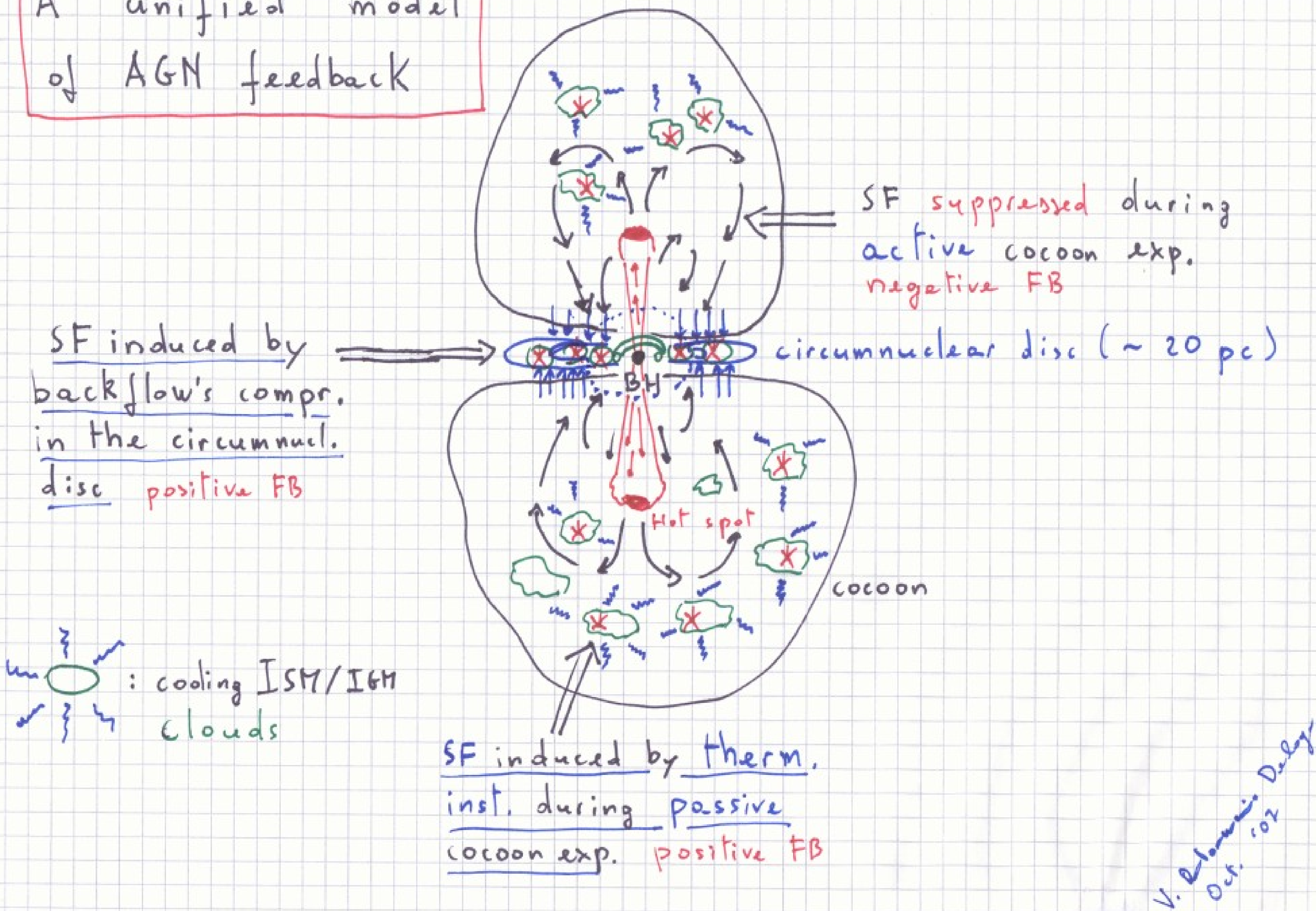
Conclusion

- The thermal structure (enthalpy, lightness) of a relativistic AGN jet is important to determine the structure/morphology of the cocoon
- Unknown composition of the jet can be constrained by a relativistic thermodynamic of the jet-cloud-coronae interaction
- At higher z (~1-4) more gas-rich AGN hosts, thus the impact of jet's thermalization is potentially higher than in the nearby Universe

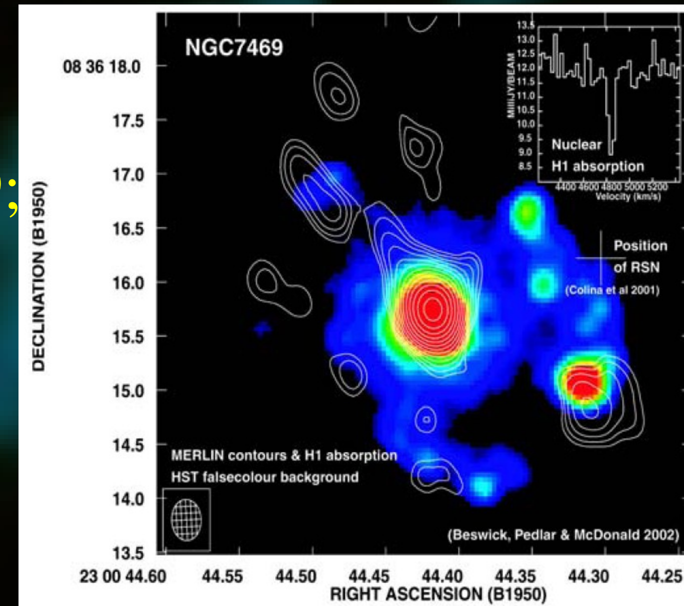
A "unified" model of AGN feedback

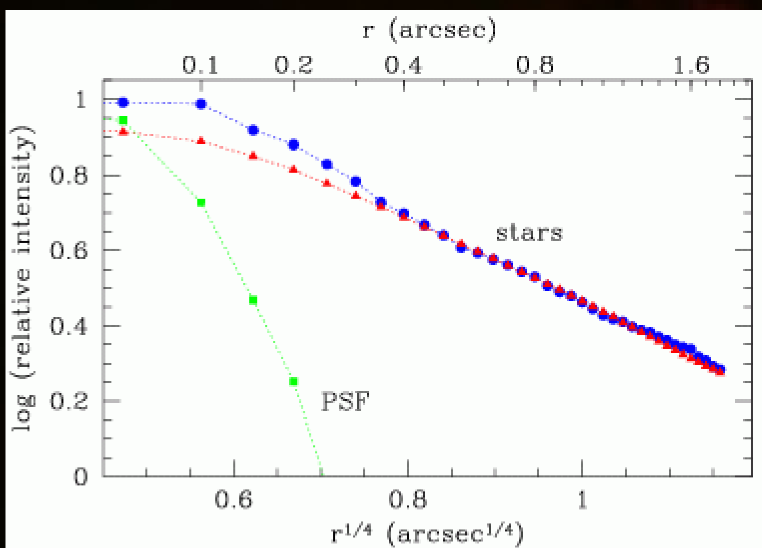


Discs around AGNs are ALSO stellar

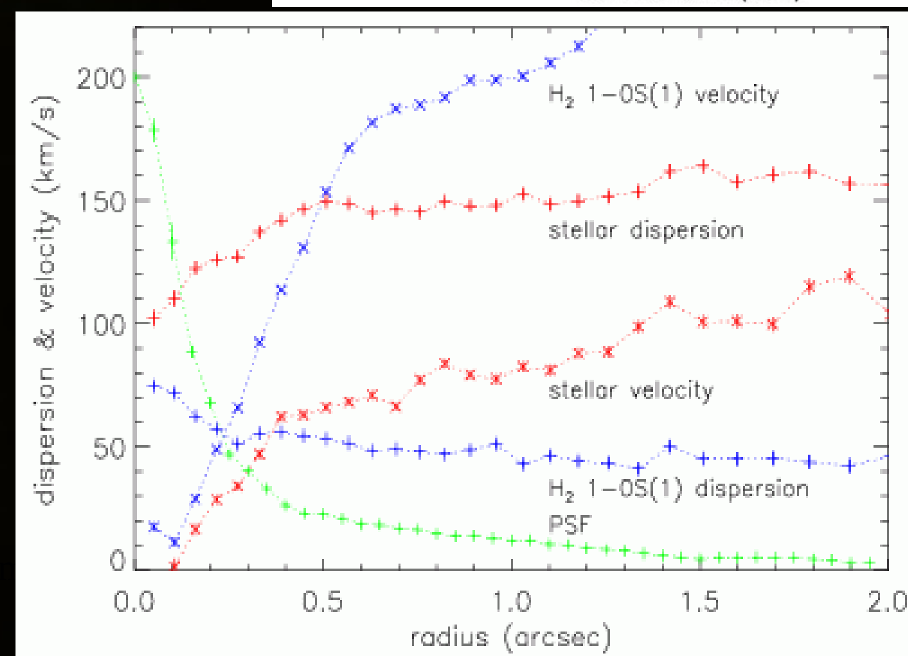
- Evidence for stellar discs on scales 1-100 pc in Sey 2 (SIMFONI, Davies et al., 2007)
- Circumnuclear disc are at most ***marginally stable*** to grav. inst. (Sloshman & Begelman, 1989; Collin & Zahn, 2008)

1'' = 330 pc





1'' = 80 pc



- NGC 1097 (Davies et al., 2007): HI disc less massive than stellar
- $M_{bh} = 1.25 \times 10^8 M_{sun}$

Enhanced SF within circumnuclear discs

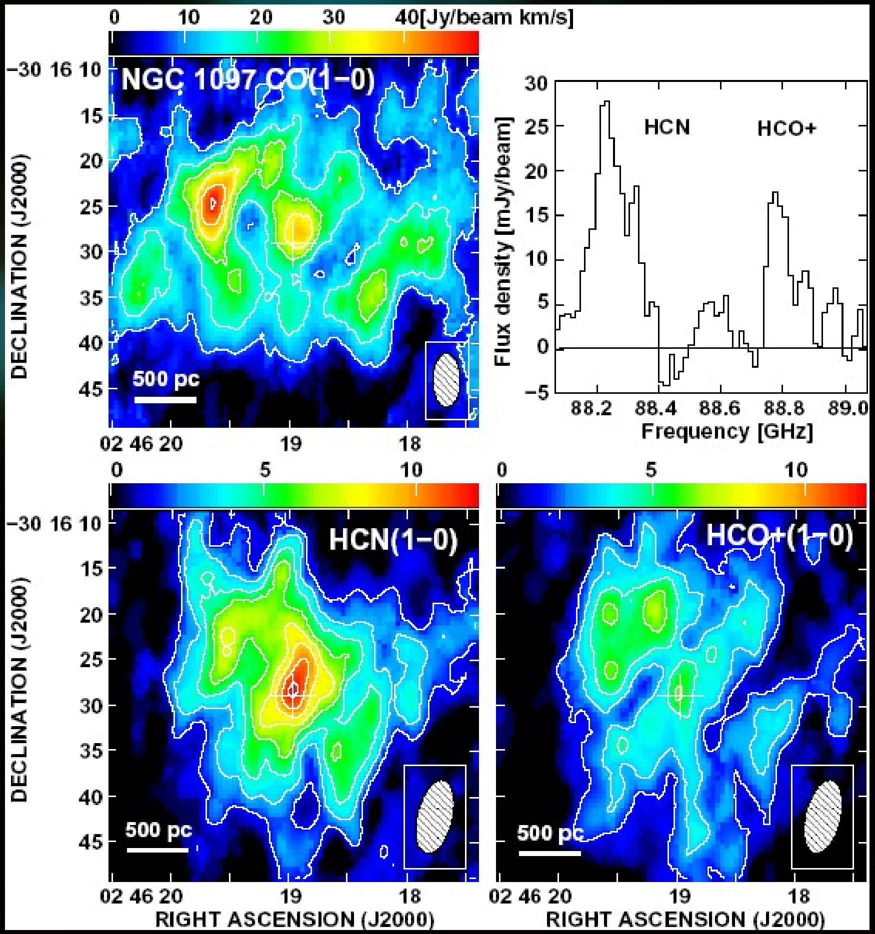
- HCN gas around AGNs' nuclei (Hsieh et al., 2008; Kohno et al., 2008)

 Scales larger than Davies et al.: ~ 500 pc

- Line ratios:

$R_{HCN/CO} = 0.39$

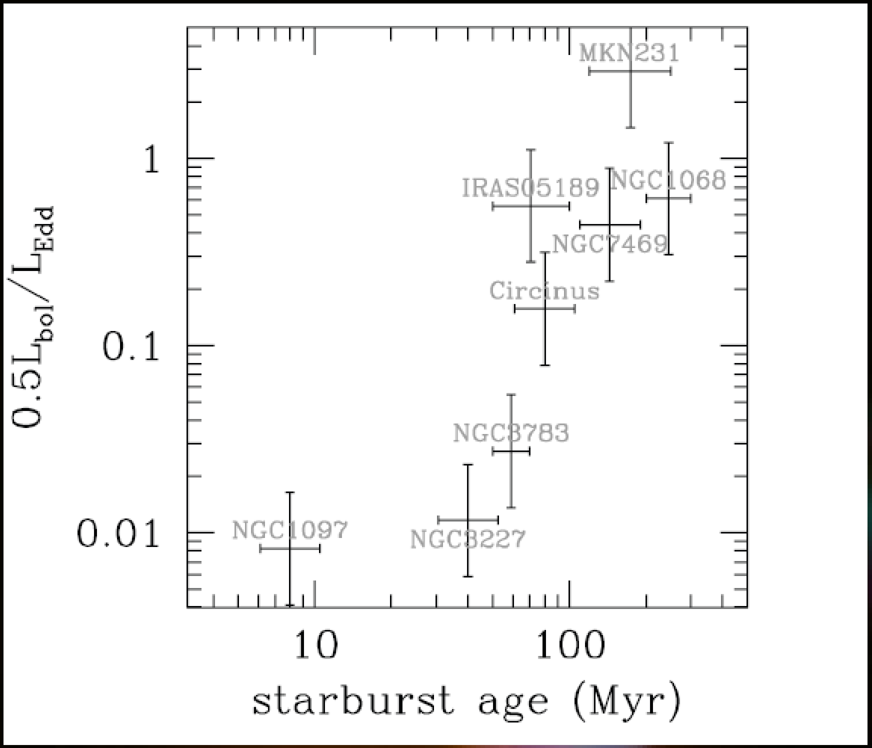
$R_{HCN/HCO+} = 1.9$ enhanced SF?



- Excess gas around the circumnucl. region → <u>accreting gas, in excess of hydr. equil.</u> (Hsieh et al. 2008)
- This gas can only be accounted for if it is <u>**accreting from a larger region**</u>

- Evidence for backflowing gas within the cocoon

A possible explanation of the L_{bol}/L_{Edd} – age connection



Davies et al. (2007): older starburst are associated with brighter AGNs

- Model: high $P_j \rightarrow$ higher $p_{bck} \rightarrow$ faster suppression of SF in the disc AND higher $T_{disc} \rightarrow$ higher L_{bol}

- Detailed modelling of gas+stellar discs with external backflow

$$\Omega(r) = \Omega_K(r) = \left(\frac{GM_{BH}}{r^3} + \frac{2\sigma^2}{r^2}\right)^{1/2},$$

$$\dot{\Sigma}_\star = \Sigma_g \Omega \eta,$$

$$p_{gas} + \epsilon \dot{\Sigma}_\star c\left(\tfrac{1}{2}\tau_V + \xi\right) = \rho h^2 \Omega^2,$$

$$p_{gas} = \rho k_B T / m_p,$$

$$T^4 = \frac{3}{4} T_{eff}^4 \left(\tau_V + \frac{2}{3\tau_V} + \frac{4}{3}\right),$$

$$\tau_V = \kappa \Sigma_g / 2,$$

$$\Sigma_g = 2\rho h,$$

$$\dot{M} = 4\pi R h \rho V_r = 4\pi R h \rho m c_s = 4\pi R h^2 \rho \Omega m,$$

$$\dot{M} = \dot{M}_{out} - \int_{R_{out}}^{r} 2\pi r \dot{\Sigma}_\star \, dr.$$