Interplay of intrinsic and extrinsic spin-orbit coupling in a two-dimensional electron gas

Advanced Workshop: Spin and Charge Properties of Low Dimensional Systems

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Motivations and background

- **Why spin-orbit (SO) is attractive**: spin manipulation by electric fields
- **Describe spin-charge phenomena in diffusive systems**
  - spin Hall effect (SHE)
  - current-induced spin polarization (CISP)
  - spin injection Hall effect (SIHE) (cf. Yungwirth’s talk)
- **Most theoretical activity focused on either intrinsic SO or extrinsic**
  - Inoue et al., PRB **70**, 041303 (2004); Mishchenko et al., PRL **93**, 226602 (2004); Raimondi and Schwab, PRB **71**, 033311(2005)
  - Chalaev and Loss, PRB **71**, 245318 (2005); Dimitrova, PRB **71**, 245327 (2005); Khaetskii, PRL **96**, 056602 (2006)
  - Engel et al., PRL **95**,166005 (2005); Tse and Das Sarma, PRL **96**, 056601 (2006)
- **Interplay of extrinsic and intrinsic SO coupling important for experiments**
  - Wunderlich et al., arXiv:0811.3486 **SIHE**
- **Contrasting views in the present status of theory**
  - Tse and Das Sarma, PRB **74**, 245309 (2006)
  - Hankiewicz and Vignale, PRL **100**, 026602 (2008)
Outline

- Overview of intrinsic and extrinsic SO mechanisms
- Symmetry and spin continuity equation
- Microscopic formulation
- *User-friendly* spin diffusion equation
- Applications to SHE and CISP
- Conclusions
The model: 2DEG with SO coupling

\[ H = \frac{p^2}{2m} - \sigma \times \left( \alpha \hat{e}_z + \frac{\lambda_0^2}{4} \partial_x V(x) \right) \cdot p + V(x) \]

- Dyakonov-Perel spin relaxation
  \[ L^{2}_{SO} = D \tau_{DP} \]
- Side-jump
  \[ \Delta x \sim \lambda_0^2 s \times \Delta k \]
- Skew-scattering
  \[ S = A + k \times k' \cdot \sigma B \]
  \[ \tau_s^{-1} \sim |B|^2 \propto \lambda_0^4 \]

\[ \delta p_F = 2m\alpha \]

\[ l = v_F \tau \]

Mean free path

\[ L_{SO} = \frac{\hbar}{m\alpha} \]

\[ \lambda \]
Spin-dependent vector potential

\[ \tilde{A} = m \sigma \times \left( \alpha \hat{e}_z + \frac{\lambda_0^2}{4} \partial_x V(x) \right) \equiv \tilde{A} = \frac{1}{2} \tilde{A}^a \sigma^a \]

- Spin density along the \( a \)-axis
- Spin current
- Covariant derivative

\[ s^a = \frac{1}{2} \text{Tr} \sigma^a \]

\[ j^a = \frac{1}{4} \text{Tr} \left\{ \sigma^a, \frac{p - \tilde{A}}{m} \right\} \]

\[ \tilde{\partial}_x \cdot j^a = \partial_x \cdot j^a + \epsilon_{abc} \tilde{A}^b \cdot j^c \]

This operator equation cannot be applied directly to disorder averaged quantities because

\[ \tilde{A}^b \cdot j^c \neq \tilde{A}^b \cdot j^c \]
The strategy to deal with disorder in SO coupling

- space-independent vector potential
  \[ \mathbf{A} = \alpha m \mathbf{\sigma} \times \mathbf{\hat{e}}_z \]

- spin-dependent disorder potential
  \[ U(x) = V(x) - \frac{\lambda_0^2}{4} \mathbf{\sigma} \times \partial_x V(x) \cdot \mathbf{p} \]

- normal current
  \[ j^a_0 = \frac{1}{4} \text{Tr}\{\sigma^a, \frac{\mathbf{p} - \mathbf{A}}{m}\} \]

- anomalous current
  \[ j^a_{av} = \frac{1}{4} \text{Tr}\{\sigma^a, \mathbf{\tilde{A}} - \mathbf{A}\} \]

Spin continuity equation with disorder-dependent extra terms

\[ \partial_t \overline{s^a} + \mathbf{\tilde{\partial}}_x \cdot \overline{j^a_0} = ? \]

What is the form of the RHS?
We expect
- spin relaxation
- anomalous current
The technical tool: the Keldysh Green function

Equation of motion (EOM) Raimondi et al., 74, 035340 (2006)

\[ \partial_t \tilde{G} + \tilde{\partial}_x \cdot \frac{1}{2} \left\{ \frac{p - A}{m}, \tilde{G} \right\} = -i \left[ \tilde{\Sigma}, \tilde{G} \right] \]

Technical Details

- Include electric field

\[ \tilde{\partial}_x(\cdot) = (\partial_x - eE\partial_\epsilon)(\cdot) - i[A, (\cdot)] \]

- Wigner representation

\[ \tilde{G}(\epsilon, p, x, t) = \int_{-\infty}^{\infty} d\eta e^{i\epsilon\eta} \int dr e^{-ip\cdot r} \tilde{G}(x + \frac{r}{2}, t + \frac{\eta}{2}; x - \frac{r}{2}, t - \frac{\eta}{2}) \]

- Spin and Keldysh matrix structure

\[ \tilde{G} = \begin{pmatrix} G^R & G \\ 0 & G^A \end{pmatrix} \quad G = G^0\sigma^0 + G^a\sigma^a, \quad a = x, y, z \quad G^\prec = \frac{1}{2}(G - G^R + G^A) \]

Physical observables

\[ s^a(x, t) = -\frac{i}{2} \int \frac{d\epsilon}{2\pi} \sum_p \text{Tr}(\sigma^a G^\prec) \quad j^a_0(x, t) = -\frac{i}{4} \int \frac{d\epsilon}{2\pi} \sum_p \text{Tr} \left( \sigma^a \left\{ \frac{p - A}{m}, G^\prec \right\} \right) \]
The self-energy

Standard white-noise disorder model

\[ V(x_1)V(x_2) = n_i v_0^2 \delta(x_1 - x_2) \]

Scattering time
\[ \sim \frac{1}{\tau} \]

Side-jump

Spin relaxation
\[ \sim \frac{1}{\tau_s} \]

skew-scattering

First correction to Born scattering amplitude

Goal: evaluate the self-energy in Wigner coordinates

\[ \tilde{\Sigma}(p, (x_1 + x_2)/2) = \int d(x_1 - x_2)e^{-ip \cdot (x_1 - x_2)} \tilde{\Sigma}(x_1, x_2) \]
An important example

The scattering time contribution

\[ \tilde{\Sigma}_0(x_1, x_2) = \frac{V(x_1) \tilde{G}(x_1, x_2) V(x_2)}{v_o^2 n_i \delta(x_1 - x_2) \tilde{G}(x_1, x_1)} \]

By Fourier transform

\[ \tilde{\Sigma}_0(p, x) = v_o^2 n_i \sum_{p'} \tilde{G}(p', x) \]

\[ = v_o^2 n_i N_0 \int d\hat{p}' \int d\xi \tilde{G}(p', x) \]

\[ = \frac{i}{2\tau} \frac{(-i)}{\pi} \langle \int d\xi \tilde{G}(p', x) \rangle \]

The microscopic expression of the elastic scattering time

\[ \tau^{-1} = 2\pi N_0 n_i v_o^2 \]
The quasiclassical approximation

The quasiclassical Green function describes length scales \( L \gg \lambda_F \)

\[
\tilde{g}(\epsilon, \hat{p}, \mathbf{x}, t) = \frac{i}{\pi} \int \frac{d \xi}{\xi} \tilde{G}(\epsilon, \mathbf{p}, \mathbf{x}, t) \quad \xi = \frac{p^2}{2m} - \mu
\]

The Eilenberger EOM for \( \lambda_0 = 0 \)

\[
\partial_t \tilde{g} + \tilde{\partial}_x \left\{ \frac{1}{2} \left( \frac{\mathbf{p} - A}{m}, \tilde{g} \right) \right\} = -\frac{1}{2T} [\langle \tilde{g} \rangle, \tilde{g}] 
\]

How to go

1. Keldysh "12" component \( \Rightarrow \) kinetic equation
2. Project on \( \sigma^0 \) \( \Rightarrow \) charge equation \( \rho = \frac{eN_0}{2} \int d\epsilon \langle g^0 \rangle \)
3. Project on \( \sigma^a \) \( \Rightarrow \) spin equation \( s^a = -\frac{N_0}{4} \int d\epsilon \langle g^a \rangle \)
The kinetic equation (including $\lambda_0$)

$$\partial_t g + \partial_{x} \cdot \frac{1}{2} \{ \frac{p - A}{m}, g \} = \frac{1}{\tau} \left( \frac{1}{2} \left\{ 1 + \frac{A \cdot \hat{p}}{2p_F}, \langle g \rangle \right\} - g \right)$$

+ $\frac{\lambda_0^2 p_F}{8\tau} \epsilon_{abc} \partial_a \left\{ (\langle \hat{p}_c g \rangle - \hat{p}_c \langle g \rangle), \sigma^b \right\}$

+ $\frac{(\lambda_0 p_F)^2}{8\tau} 2\pi N_0 v_0 \varepsilon_{abc} \partial_a \left\{ \langle \hat{p}_b g \rangle, \sigma^c \right\}$

- $\frac{1}{\tau_s} \frac{1}{2} \left( \langle g \rangle - \sigma^z \langle g \rangle \sigma^z \right)$

- momentum relaxation and D-P spin relaxation
- side-jump
- skew-scattering
- E-Y spin relaxation

A first consequence: the anomalous current

$$j^a_{av,i} = \varepsilon_{iaj} \frac{N_0}{4} \int d\epsilon \epsilon \frac{\lambda_0^2 p_F}{4\tau} \langle \hat{p}_j g^0 \rangle = \varepsilon_{iaj} \frac{1}{2} \epsilon \frac{\lambda_0^2}{4} nE_j \equiv \varepsilon_{iaj} \frac{1}{2} \sigma^{SH}_j E_j$$

- anomalous velocity contribution as in AHE
  (Nozieres and Lewiner, J. Phys. (Paris) 34, 901 (1973))

- Similar contribution in SHE in the absence of Rashba
  (Engel, Halperin, Rashba, PRL 95, 166605 (2005); Tse and Das Sarma, PRL 96, 056601 (2006))

- This term is unaffected by the presence of the Rashba term
More consequences

The second bit of the side-jump

Notice the covariant derivative acting on the second term of the side-jump contribution

\[ \hat{\partial}_a \langle g^0 \rangle \equiv (\partial_a - eE_a \partial_\epsilon)\langle g^0 \rangle \]

- In the absence of Rashba this leads to a contribution identical to the previous one
- With Rashba this term is modified

The skew scattering

- In the absence of Rashba

\[ \sigma_{SS}^{SH} = \frac{1}{4} (p_F l) (2\pi N_0 v_0) \sigma_{SJ}^{SH} \]

- In the presence of Rashba this term is modified

E-Y spin-relaxation time

This term was not considered in previous analyses

\[ \tau_s^{-1} = \tau^{-1} \left( \lambda_0 p_F / 2 \right)^4 \]
User friendly spin-density diffusion equation

Physical assumptions

\[ \tau \ll T \quad v_{FT} \ll L \quad v_{FT} \ll L_{so} \Rightarrow \alpha p_{FT} \ll 1 \]

Spin current and continuity equation

\[ j^a_i = \mu s^a E_i - D \tilde{\partial}_i s^a + (\sigma_{SJ}^{SH} + \sigma_{SS}^{SH}) \varepsilon_{ab} E_b + \frac{e}{8 \pi} (\tau D) \varepsilon_{ikj} B^a_k E_j \]

\[ \partial_t s^a + \tilde{\partial}_x \cdot j^a - \frac{1}{2} \varepsilon_{abc} A^b \cdot j^c_{SJ} + \frac{1}{\tau_s} s^a = 0 \]

- Covariant derivative in the diffusion term Kalevich, Korenev, Merkulov, Solid State Commun. 73, 559 (1994); PRB 74, 041308 (2006)
- New source of spin current in the last term due to \( SU(2) \) magnetic field

\[ B^a = \tilde{\partial}_x \times A^a \quad \Rightarrow \quad \sigma_{int}^{SH} = \frac{e}{8 \pi} \tau DB^z_z \]
The SHE

Main formula

\[ \sigma^{sH} = \frac{1/\tau_s}{1/\tau_s + 1/\tau_{DP}} \left( \sigma^{sH}_{\text{int}} + \sigma^{sH}_{SS} + \frac{1}{2} \sigma^{sH}_{SJ} \right) + \frac{1}{2} \sigma^{sH}_{SJ} \]

1. Is analytic in both limits \( \lambda_0 \to 0 \) and \( \alpha \to 0 \)
   - Inoue et al. PRB 70, 041308 (2004); Raimondi and Schwab, 71, 033311 (2005)
   - Mishchenko et al. PRL 93, 226602 (2004)
   - Engel, Halperin, Rashba, PRL 95, 166605 (2005); Tse and Das Sarma, PRL 96, 056601 (2006)

2. \( \sigma^{sH}_{\text{int}} \) is the diffusive limit of the bubble contribution of the diagrammatic language

3. The ratio \( \tau_s/\tau_{DP} \) acts as a tuner of the spin Hall effect

4. Since \( 1/\tau_s \propto \lambda_0^4 \) and \( 1/\tau_{DP} \propto \alpha^2 \), it would be unphysical to take the limit \( \alpha \to 0 \), with \( 1/\tau_s = 0 \)

5. In appropriate limits agrees with previous analyses
   - Tse and Das Sarma, PRB 74, 245309 (2006)
   - Huang and Hu, PRB 73, 235314 (2006)
   - Hankiewicz and Vignale, PRL 100, 026602 (2008)
GaAs: input parameters  Sih et al., Nature Phys. 1, 31 (2005)

\[ n_s = 10^{12}\text{cm}^{-2}, \mu = 10^3\text{cm}^2\text{Vs}, \lambda_0 = 4.7 \times 10^{-8}\text{cm}, \alpha = 10^{-12}\text{eVm}, N_0v_o = -\frac{1}{2} \]

Estimates

\[ e\sigma_{SJ}^{sH} = 1.3 \times 10^{-7}\Omega^{-1}, \quad e\sigma_{SS}^{sH} = -4.3 \times 10^{-7}\Omega^{-1}, \quad e\sigma_{int}^{sH} = 8.2 \times 10^{-9}\Omega^{-1} \]

Spin relaxation times

\[ \tau_s = 8.4 \times 10^3\text{ps} \gg \tau_{DP} = 90\text{ps} \]

By having \( \alpha \) smaller one may reach the interesting region \( \tau_s \approx \tau_{DP} \)
CISP: in-plane vs out-of-plane

The classical Edelstein result \((\text{Solid State Commun. 73, 233 (1990)})\)

\[ s^y = -m\mu \alpha N_0 E, \quad \lambda_0 = 0 \]

The effect of extrinsic SO

\[ s_E = -\frac{2m\alpha}{1/\tau_s + 1/\tau_{DP}} \left( \sigma_{\text{int}}^{sH} + \sigma_{SS}^{sH} + \frac{1}{2} \sigma_{SJ}^{sH} \right) E. \]

Origin of out-of-plane CISP with in-plane magnetic field

\[ \partial_t s^y = -\left( \frac{1}{\tau_{DP}} + \frac{1}{\tau_s} \right) (s^y - s_E) + b_x s^z \]
\[ \partial_t s^z = -\frac{2}{\tau_{DP}} s^z - b_x s^y + m\alpha \mu N_0 b_x E \]

- With \(\lambda_0 = 0\) no out-of-plane CISP unless with angle-dependent scattering \((\text{Milletari et al. EPL 82, 67005 (2008)})\) or energy-dependent scattering and non parabolic dispersion \((\text{Engel et al. PRL 98, 036602 (2007)})\)

- In general \(s^z \sim b_x (m\mu \alpha N_0 E - s_E)\)
Out-of-plane CISP without magnetic field

Linear Dresselhaus SO for (110) QW (Hankiewicz, Vignale, Flatté, PRL 97, 266601 (2006))

\[ A_D = -m\beta \sigma^z \hat{e}_z \quad B_D^a = \tilde{\partial}_x \times A_D^a = 0 \]

For electric-field induced spin polarization both Rashba and Dresselhaus are necessary!

\[ A \rightarrow A + A_D \]

Electric-field orientation dependence

\[ s^z = E_y 2m\beta \frac{\sigma_{SS}^{\text{SH}} + \frac{1}{2} \sigma_{SJ}^{\text{SH}} + \frac{\tau e}{4\pi} (\tau_s^{-1} + 4m^2 D(2\alpha^2 + \beta^2))}{4m^2 D(2\alpha^2 + \beta^2) + 2\tau_s^{-1}} \]

- interplay of Rashba and linear Dresselhaus provides a mechanism for out-of-plane CISP
- CISP \( s^z \propto E_y \) shows a strong dependence on the orientation of the applied electric field
Comparison with measured CISP

- (110) GaAs quantum well (Sih et al., Nature Phys. 1, 31 (2005))
- Electric field along various crystal directions
- Out-of-plane CISP measured via Kerr spectroscopy

**Experimental findings**

1. For $E \parallel [0, 0, 1]$ CISP only at edges and no in the bulk $\Rightarrow$ SHE
2. CISP in the bulk without magnetic field for $E \parallel [1, \bar{1}, 0]$, $[1, \bar{1}, 1]$ and $[\bar{1}, 1, 2]$ in agreement with theory
3. Surprisingly CISP has the same sign for $E \parallel [1, \bar{1}, 0]$ and $[\bar{1}, 1, 2]$, which is opposite for $[1, \bar{1}, 1]$, not explained by the theory
4. Cubic Dresselhaus $\sim k_y(k_y^2/2 - k_x^2)$ and non-linear effects may be important
Conclusions

- Theory for both, intrinsic and extrinsic, SO mechanisms
- *User friendly* spin diffusion equation
- SHE: non-analytic puzzle settled
- SHE: intrinsic mechanism as *tuner* of the extrinsic-driven effect
- CISP: due to interplay of intrinsic and extrinsic, even without magnetic field
Why the electric field does not contribute to the side-jump?

Consider the Hamiltonian

$$H = \frac{k^2}{2m} - eE \cdot x + \lambda eE \cdot k \times s$$

and the associated velocity operator and spin current

$$v = \frac{k}{m} + \lambda es \times E \quad j^a = \frac{1}{2} \{ v, s^a \}$$

To first order in $E$, the first SJ term is due to the anomalous velocity

$$\delta j^a = \operatorname{Tr} \sum_k f(k) \lambda e \frac{1}{2} \{ s \times E, s^a \} = \frac{1}{4} \lambda en \ \hat{e}_a \times E$$

The second SJ is due to a change of the distribution function

$$j_0^a = \operatorname{Tr} \sum_k \frac{k}{m} s^a \frac{\partial f}{\partial \epsilon} \lambda eE \cdot k \times s = -\frac{1}{4} \lambda en \ \hat{e}_a \times E$$

The cancellation is similar to that for the diamagnetic term and only exists at zero frequency.
What is the connections with diagrams? SJ terms

\[ \Gamma^x = -\alpha \frac{\tau}{\tau_s} \sigma_y \frac{1}{\frac{\tau_l}{\tau_{DP}} + \frac{\tau}{\tau_s}} \]

\[ \Gamma^{yz} = \frac{v_F}{4} \frac{2\alpha p_F \tau}{1 + (2\alpha p_F \tau)^2} \left(1 - \frac{\tau}{\tau_s}\right) \sigma_y \frac{1}{\frac{\tau_l}{\tau_{DP}} + \frac{\tau}{\tau_s}} \]

For \( \tau_s \to \infty \), \( \Gamma^x = 0 \) so that \( C + D = 0 \). Also \( \Gamma^{yz} = (v_F/4\alpha p_F \tau) \sigma^y \) which leads to the cancellation \( E + F + G + H = 0 \). Only \( A + B \) survives which is half the SJ and is not changed by Rashba.