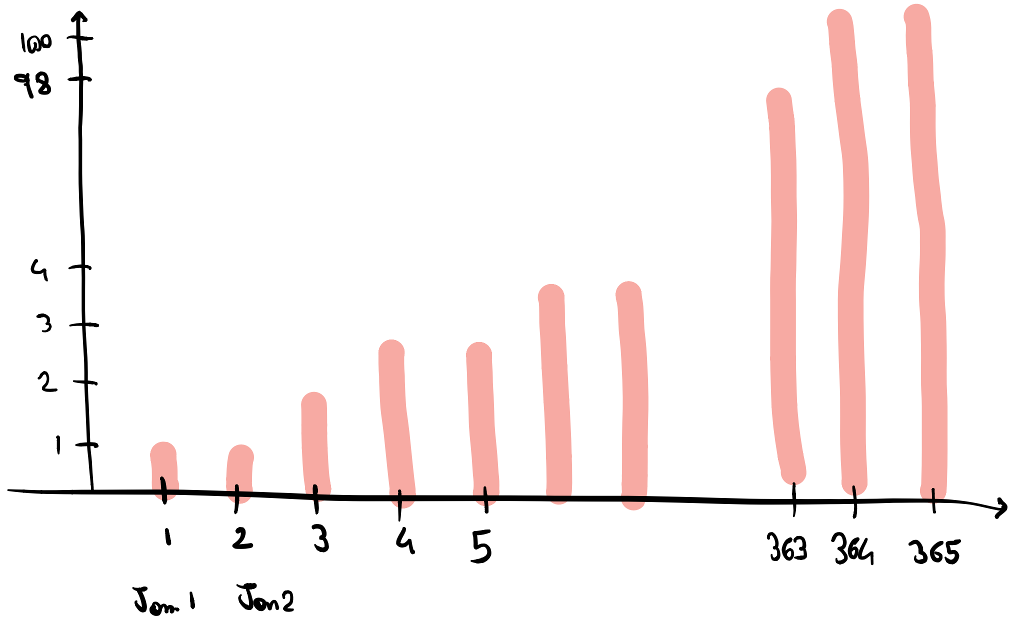
Birthdays in your village since the beginning of the year



$$
C(d) = \begin{cases}
1 & d = 1 \quad \text{Jan 1} \\
1 & d = 2 \quad \text{Jan 2} \\
2 & d = 3 \quad \text{Jan 3} \\
3 & d = 4 \\
3 & d = 5 \\
 & \vdots \\
98 & d = 363 \\
100 & d = 364 \\
100 & d = 365
\end{cases}
$$

1 person born on Jan 2

1 person born on Jan 3

2 people born on Dec 30^{th}

C expresses the cumulative number of birthdays (events) that happened already from the beginning of the year

$$p(x) \equiv \frac{d}{dx} C(x) \qquad \int_1^d p(x)\, dx = C(d)$$

$p(x)$ is the change in the cumulative $C(x)$ as a function of x

$p(x)$ is the probability that an event happens with x in the range $[x, x+\Delta x]$

$f(x)$ averaged over $p(x)$

$$\sum_i f(x_i) \cdot p(x_i) = \langle f \rangle$$

$$\langle f \rangle = \int_{x_{inf}}^{x_{sup}} f(x)\, p(x)\, dx$$

$f(x) = x$ $\qquad E[x] =: \bar{x}$

momento

$f(x) = x^n$ $\qquad E[x^n] = \mathcal{M}^{(n)}$ $\qquad \mathcal{M}^{(1)} = \bar{x}$

linearità dell'integrale

$E[x] = \bar{x}$ $\qquad \underbrace{E[x - \bar{x}]}_{\int x f(x) - \int f(x) \cdot \bar{x}\, dx \,=\, \bar{x} - \bar{x}} = 0$

$$E[(x-\bar{x})^2] = \int f(x)(x^2 + \bar{x}^2 - 2x\bar{x})\, dx \neq 0$$

2 - momento "centralizzato"

$$\mathcal{M}^{(n)}_{\mu} = E[(x-\mu)^n]$$

$n \simeq 1$: all x weighs more or less the same

$n \gg 1$: enhances $x \neq \mu$

since it is most interesting to know what happens near μ usually a $p(x)$ is characterized by $\mathcal{M}^{(n)}$ $n \simeq 1, 2, \dots$ few

$\mathcal{M}^{(2)}_{\bar{x}}$ è la deviazione standard σ^2

$$z(x) \equiv \frac{f(x)}{1-C(x)}$$

probabilità di avere x rispetto alla probabilità di tutto ciò che sta prima di x (p.es. un tempo precedente)

- $f(x) = \text{const}$

tutti gli eventi, tutte le possibili x sono equi-probabili

$\Rightarrow C(x)$ è lineare

se $x \in [a, b]$

$$\int_a^b f(x)\, dx = 1 \qquad \text{const} = \frac{1}{|a-b|}$$

- $z(x) = \text{const}$

$$1 - \text{const} \int_0^x f(z)\, dz = f(x)$$

$$-\text{const}\, f(x) = f'(x)$$

$$f(x) = e^{-\text{const}\, x}$$

In general $\quad f(x) = \frac{dC(x)}{dx} = z(x)\left[1 - C(x)\right]$

$$\frac{dC(x)}{dx} \cdot \frac{1}{1 - C(x)} = z(x)$$

$$= \frac{-d\left(1 - C(x)\right)}{dx} \quad \frac{1}{1 - C(x)} = z(x)$$

$$= \frac{d\log\left(1 - C(x)\right)}{dx} = z(x)$$

So I can relate all the characteristic function

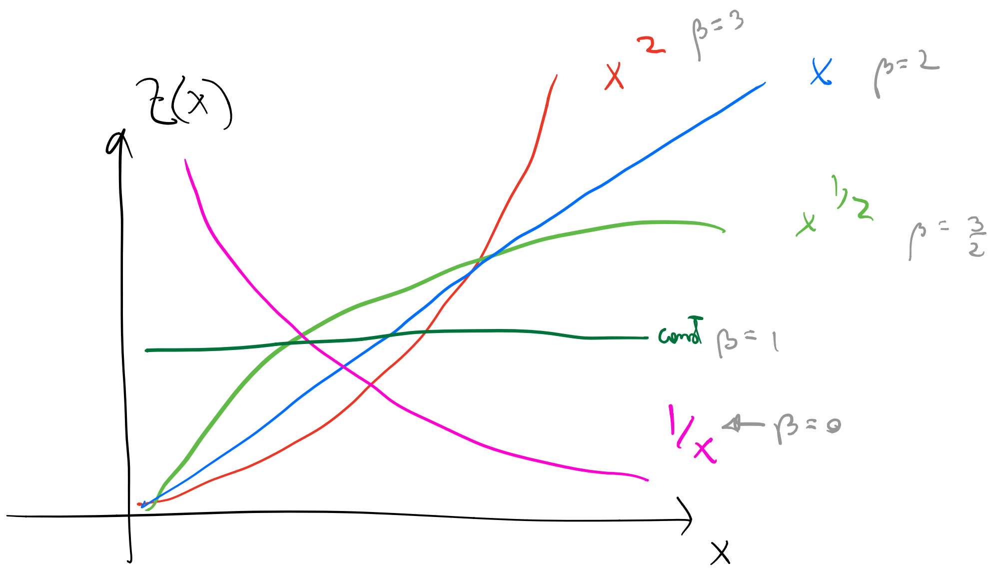
to $z(x)$

$$C(x) = 1 - e^{-\int_0^x z(x')dx'}$$

$$f(x) = z(x) \; e^{-\int_0^x z(x')dx'}$$

$z(x) = \lambda \beta x^{\beta - 1}$ Weibull $\beta > 0$

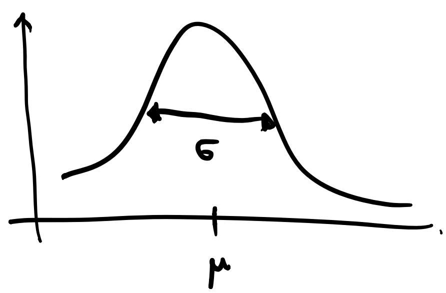
$\beta = 1 \Rightarrow z(x) = \text{const} \Rightarrow \exp$



$$f(x) = \beta \cdot \lambda \, x^{\beta - 1} \, e^{-\lambda x^{\beta}}$$ Weibull

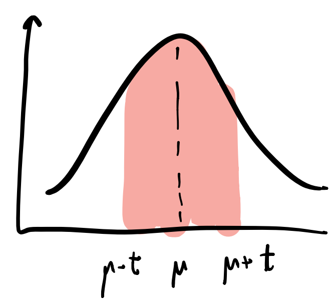
$$C(x) = 1 - e^{-\lambda x^{\beta}}$$

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$ GAUSSIAN



$$M^{(1)} = \mu \qquad M^{(2)}_{\mu} = \sigma$$

$$\int_{\mu-t}^{\mu+t} G(\mu,\sigma)(z)\,dz$$



$C(x)$ is a thing on its own.

$$C(x) = E_2 f(x) = \frac{2}{\sqrt{\pi}} e^{-\int_0^x e^{-t^2} dt}$$

PROB. di eventi multipli

$x = x_1 \qquad p(x_1)$

$x = x_2 \qquad p(x_2)$

$x = x_1$ & $x = x_2 \qquad p(x_1) \cdot p(x_2) = p(x_1 \cap x_2)$

$x = \{x_1 \ldots x_n\}$ in n trials is $\prod_{i=1}^{N} p(x_i)$

if the second event depends on the first

$p(\text{wet if rained}) \neq p(\text{wet for any weather})$

$$P(A \cap B) = P(A)\,P(B) \quad \text{if independent}$$

if dependent:

$$P(A \cap B) = P(A|B)\,P(B) \lessgtr P(A)\,P(B)$$

as well as

$$P(A \cap B) = P(B|A)\,P(A)$$

BAYES' RULE

$$\boxed{p(B|A)\,p(A) = p(A|B)\,p(B)}$$

$$p(A) = \frac{p(A|B)\,p(B)}{p(B|A)}$$

independent combined probabilities

$$e^{a} \cdot e^{b} = e^{a+b}$$

Taylor 5.5

$$p\,(x_1 \cap x_2 \cap \dots x_N) = p\left(x_1 \dots\dots x_N\right) = \frac{1}{\sigma^N} e^{-\sum \frac{(x_i-\mu)^2}{2\sigma^2}}$$

max of probability is $\frac{dp}{d\mu} = 0 = \sum_i (x_i - \mu)$

$$N\mu = \sum x_i \qquad \boxed{\mu = \frac{\sum x_i}{N}}$$

SUM IN QUADRATURE OF UNCERTAINTY

Taylor 5.6

$$p(x) \propto e^{-x^2/2\sigma_x^2}$$

$$p(y) \propto e^{-y^2/2\sigma_y^2}$$

$$p(x)\cdot p(y) \propto \left(e^{-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}} \right) =$$

$$= e^{-\frac{\sigma_y^2 x^2 + \sigma_x^2 y^2}{2\sigma_x^2 \sigma_y^2}} =$$

$$= e^{-\frac{(x+y)^2}{2(\sigma_x^2+\sigma_y^2)} - \frac{1}{2}\cdot\frac{(\sigma_x y - \sigma_y x)^2}{\sigma_x \sigma_y (\sigma_x + \sigma_y)}}$$

the sum is the combination of x and y that I want

$$= \overbrace{e^{-\frac{(x+y)^2}{\underbrace{2(\sigma_x^2+\sigma_y^2)}_{\sigma^2 \text{ of the sum } x+y}}}}^{\tilde{p}(x,y)} \; \overbrace{e^{-z^2(x,y)}}^{p(z)}$$

this is another combination that I am not interested

$$p(x+y, \cancel{x/y}) = \int dz\, e^{-z^2} f(x,y) = \text{const } f(x,y)$$