Levitation

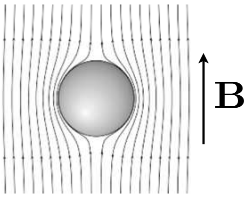
an elementary treatment

Main reference:
Orlando, Delin:
Foundations of Applied Superconductivity,
Addison Wesley, 1991

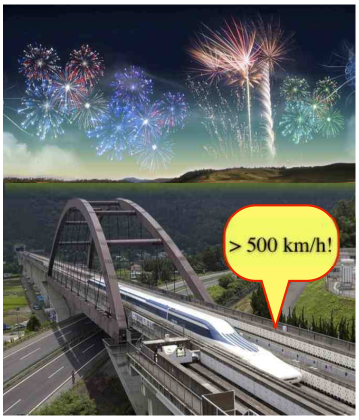
The usual presentation slide....

Meissner effect:
Inside a superconductor

$$\mathbf{B} = 0$$



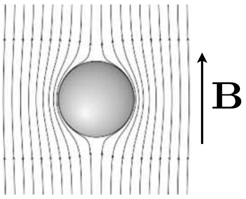
$\Rightarrow$



Maybe it’s worth some more words...

Meissner effect

Inside a superconductor $\mathbf{B} = 0$



Repulsive forces develop between a
magnet and a superconductor
(in the Meissner state)

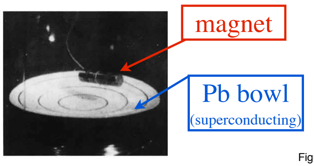


Figure from
Buckel, Kleiner,
Superconductivity - Fundamentals and Applications

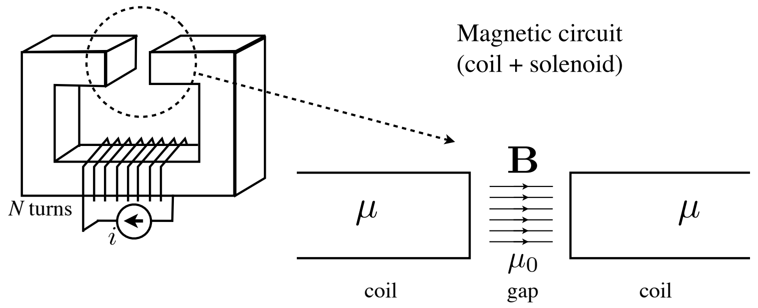
Note:
True Meissner effect (flux *expulsion*) is rather difficult to observe.
Most “demonstrations” (search Youtube...) with HTS materials are full or partial flux *trapping* instead.

The basics:
magnetic material repelled/attracted by a coil (and superconductor levitating above a magnet).

Key: constant-current-source.

Some recall from basic magnetism

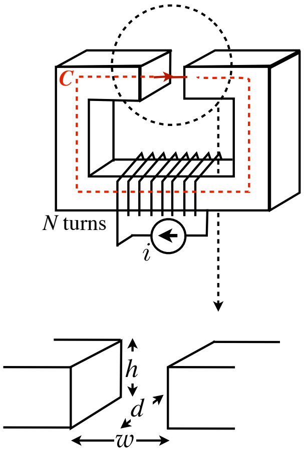
Magnetic circuit
(coil + solenoid)



Continuity of $B_\perp$
+ $B = \mu H$

- B approx. uniform across the gap/coil
- $H_{gap} \gg H_{coil}$

Magnetic field in the gap



$$\oint_C \mathbf{H}\cdot d\mathbf{l} = \int_{S(C)} \mathbf{J}\cdot d\mathbf{s} = I_C$$

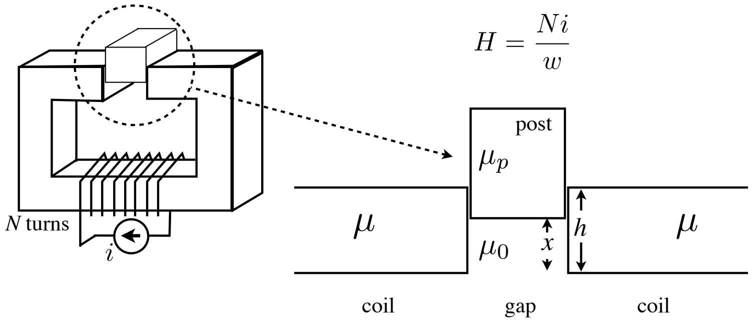
$$\oint_C \mathbf{H}\cdot d\mathbf{l} \approx Hw \qquad \int_S \mathbf{J}\cdot d\mathbf{s} = Ni$$

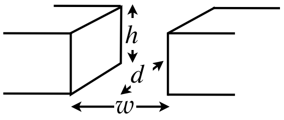
$$\boxed{H_{gap} \gg H_{coil}}$$

$$H = \frac{Ni}{w}$$

Post in the gap: fields

$$H = \frac{Ni}{w}$$



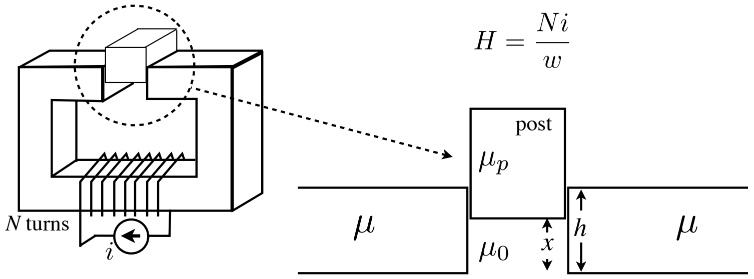


$$B_{post} = \mu_p \frac{Ni}{w}$$

$$B_{gap} = \mu_0 \frac{Ni}{w}$$

Post in the gap: energy

$$H = \frac{Ni}{w}$$



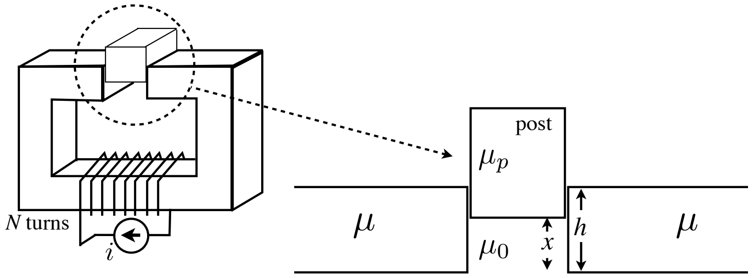
Circuit with inductance *L* at <u>constant current</u> *i*.
When the post moves by *dx*, the flux changes by *d*Φ.
Work spent by the generator: $dW_{gen} = id\Phi = i^2 dL$

Inductance *L*: by definition, $\Phi = Li$.

This work goes in:

- change in the magnetic energy stored $dW_{mag} = d\left(\frac{1}{2}Li^2\right) = \frac{1}{2}i^2 dL$
- mechanical work on the post $dW_{mech} = F dx$

Post in the gap: force



Circuit with inductance *L* at <u>constant current</u> *i*.

$$dW_{gen} = dW_{mag} + dW_{mech}$$

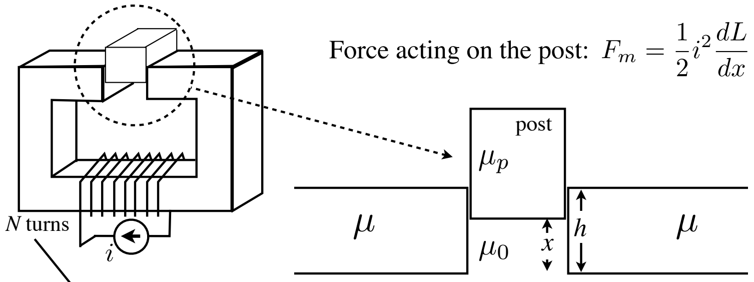
$$i^2 dL = \frac{1}{2}i^2 dL + F_m dx$$

Force acting on the post: $F_m = \frac{1}{2}i^2 \frac{dL}{dx}$

We need to calculate the inductance *L*

Post in the gap: inductance

Force acting on the post: $F_m = \frac{1}{2} i^2 \frac{dL}{dx}$



Inductance *L*: by definition, Φ(**B**)= *Li*.

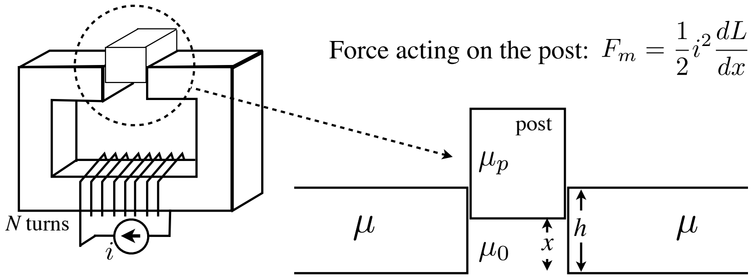
$B_{post} = \mu_p \frac{Ni}{w} \qquad B_{gap} = \mu_0 \frac{Ni}{w}$

$$\Phi = \Phi_{gap} + \Phi_{post} = N \times [B_{gap} x d + B_{post}(h-x)d] = \underbrace{\boxed{N^2 \frac{d}{w}[\mu_0 x + \mu_p (h-x)]}}_{\boxed{L}} i$$

<u>N.B.</u> ∇·**B** = 0 ⇒ flux Φ is conserved through a section of the magnetic circuit.

Post in the gap: force and levitation

Force acting on the post: $F_m = \frac{1}{2} i^2 \frac{dL}{dx}$



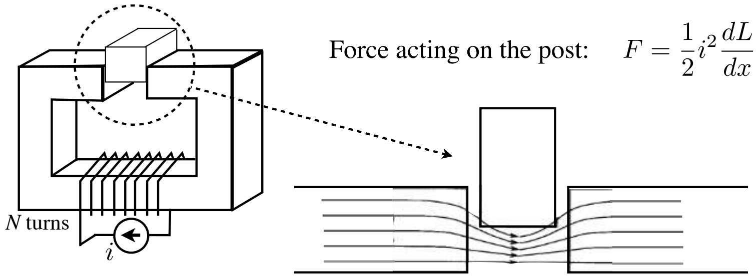
$$F_m = [\mu_0 - \mu_p] \frac{1}{2} N^2 \frac{d}{w} i^2$$

- Does not depend on sign (polarity) of *i*.
- Diamagnet: *F>0*, the post is repelled by the gap.
- Paramagnet: *F<0*, the post is sinked into the gap

In a diamagnet, F_m must equal *mg* in order to "levitate" the post.

Superconducting post

Force acting on the post: $F = \frac{1}{2} i^2 \frac{dL}{dx}$



Expulsion of flux lines

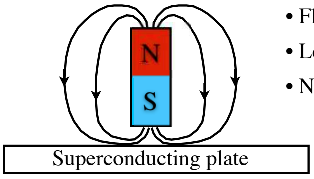
$$F_m = [\mu_0 - \mu_p] \frac{1}{2} N^2 \frac{d}{w} i^2 \longrightarrow \mu_0 \frac{1}{2} N^2 \frac{d}{w} i^2$$

B = 0 in the superconducting post

Magnet levitating above a superconductor.

Key: no current sources.

General aspects



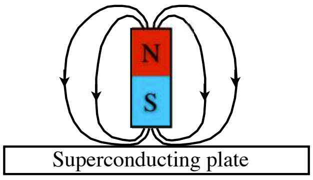
- Flux expulsion causes a magnet levitating
- Levitation is stable if plate is large with respect to magnet
- No free currents (no circuit)

if all the flux generated by the magnetic bar stays above the superconducting surface

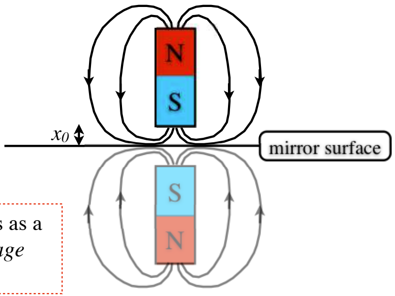
the induced superconducting currents on the plate can be assumed to decay to zero at ∞

the superconductor behaves as a mirror surface for the *image magnet bar*.

Image method



$\Longrightarrow$

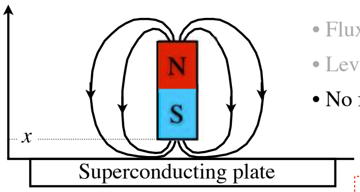


the superconductor behaves as a mirror surface for the *image magnet bar*.

The magnetic repulsion, F_m directed upwards, decreases with height x, and it is balanced by mg at a critical height x_0.

The levitation is stable: the adjustment of the mirror magnet for any displacement of the real magnet produces a restoring force on the real magnet. This is possible when the mirror surface is infinitely extended.

Levitating force: semiquantitative analysis



- Flux expulsion cause a magnet levitating
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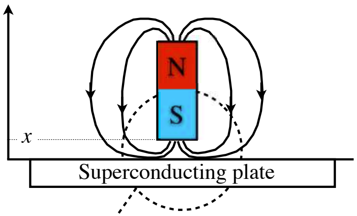
The only source of energy is the field (no external sources, such as generators).

We express the energy coming from field energy density: $W_{mag} = \frac{1}{2}\int_V \mathbf{B}\cdot\mathbf{H}dV$

We derive the energy with respect to the height to obtain the magnetic ("levitating") force: $F_m = -\frac{\partial W_{mag}}{\partial x}$

Problems to be solved: find useful approximations for **B** and the integration volume *V*.

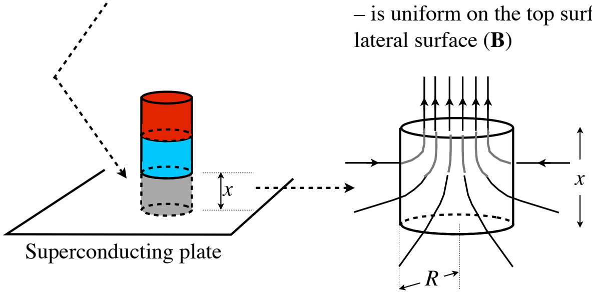
Approximations



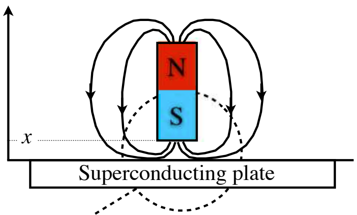
$$W_{mag} = \frac{1}{2}\int_V \mathbf{B}\cdot\mathbf{H}dV$$

Crude approximations:
- *V:* the cylinder below the magnet bar
- the field **B** originating from the magnet bar:
  - – exits *V* from the top surface only
  - – enters *V* from the lateral surface only
  - – is uniform on the top surface ($\mathbf{B}_0$) and on the lateral surface (**B**)

Note: flux is fixed



Flux



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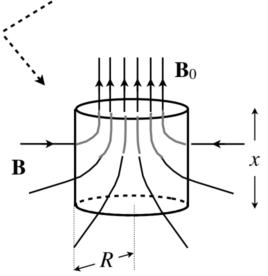


Flux (absolute value):

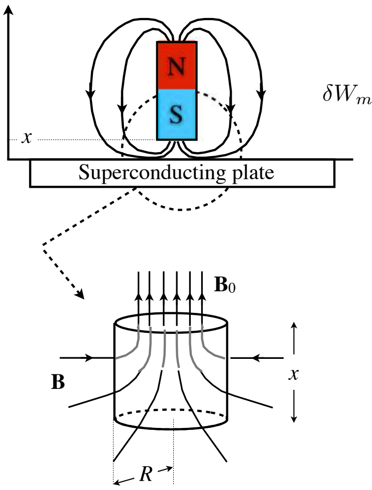
– across top surface $\Phi = B_0\pi R^2$

– across lateral surface $\Phi = B2\pi Rx$

Lifting the magnet bar by δx, **B** changes to **B´**, but flux **Φ** stays constant (be careful in doing calculations of the energy!):

$$\Phi = B_0\pi R^2 = B2\pi Rx = B'2\pi R(x+\delta x)$$

Energy and (levitating) force



$$W_{mag} = \frac{1}{2}\int_V \mathbf{B}\cdot\mathbf{H}dV$$

$$\delta W_{mag}(\delta x) = \{W_{mag}(x+\delta x) - W_{mag}(x)\}_{\Phi=const}$$

$$\simeq \frac{1}{2\mu_0}B'^2\underbrace{\pi R^2(x+\delta x)}_{V+\delta V} - \frac{1}{2\mu_0}B^2\underbrace{\pi R^2 x}_{V}$$

$$\simeq \frac{\Phi^2}{8\pi\mu_0}\left(-\frac{\delta x}{x^2}\right)$$

to first order in δx, and using (exercise)

$$\Phi = B_0\pi R^2 = B2\pi R x = B'2\pi R(x+\delta x)$$

whence the levitating force (upward):

$$F_m = -\frac{\partial W_{mag}}{\partial x} \simeq \frac{\Phi^2}{8\pi\mu_0}\frac{1}{x^2}$$

Some estimate

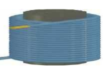
Levitating force (upward): $F_m = -\frac{\partial W_{mag}}{\partial x} \simeq \frac{\Phi^2}{8\pi\mu_0}\frac{1}{x^2}$

Gravity: $F_g = mg$

Nd-Fe-B magnet
e.g.: www.supermagnete.de
typical:
area ≈ 1 cm^2
B$_0$ ≈ 1.3 T
at x = 10 cm

F_m ≈ 50 mN

Fe-core electromagnet
e.g.:
area ≈ 100 cm^2
B$_0$ ≈ 2 T
at x = 10 cm



F_m ≈ 1.25 kN

Heavy !
Power consumption!

Superconducting solenoid
e.g.:
area ≈ 100 cm^2
B$_0$ ≈ 3.5 T
at x = 10 cm

F_m ≈ 3 kN

(relatively) lightweight.
Persistent mode.

But:

- a superconducting surface is impractical (a little bit...) for real world levitation
(what about a 200 km-long superconducting rail, liquid He-cooled?)
- levitation requires large fields at the "mirror" surface, *larger* than H_c or H_{c1} !

?

Exercise

Nd-Fe-B magnet
e.g.: www.supermagnete.de
typical:
area ≈ 1 cm^2
B$_0$ ≈ 1.3 T

Assume the Nd-Fe-B magnet has a mass of m = 10 g.
Is it possible that it levitates above Mercury, Tin, Lead and Indium, at 1.2 K and 2.7 K?
At which height?
Motivate.

Would the results change for a magnet mass m=1g?

[You will need some knowledge about superconductors. Feel free to search the internet]

Do we really need Meissner effect (to levitate a train)?

The requirement for levitation (see past slides) is a (nearly) perfect magnetic field screening.

Meissner effect is fine for fields at any frequency (well below superconducting gap)

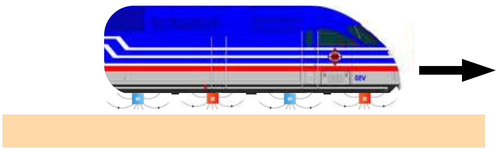
but

normal conductors do screen ac fields (skin depth)!

Your mobile phone does not work inside a screened room!

Basic idea:
- mount several magnets with opposite polarities on the floor of a railcar
- thrust the railcar over a *normal conducting* rail: the rail will see an alternating field
- (good old) Faraday-Lenz law will provide levitation force!

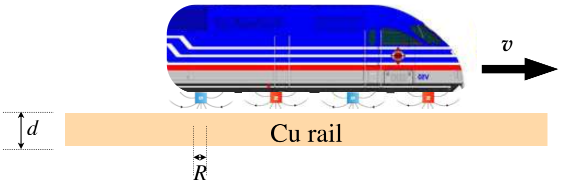
(yes, you'll need an additional engine to start the railcar)
(and yes, the rail will dissipate energy by Joule losses)



Can we levitate a train?

Rough estimate.

Requirements for nearly perfect screening:



The skin depth $\delta \ll d, R \quad \Longrightarrow \quad \delta \lessapprox (Rd)^{1/2}$ with $\delta = \sqrt{\dfrac{2}{\mu\omega\sigma_0}}$

The frequency represents the time rate of variation of the field: $\omega \sim v/R$

$$\Longrightarrow \quad v \gg \frac{2}{\mu\sigma_0 d}$$

with (e.g.):
$d \approx 10$ cm (rail)
Cu: $\frac{1}{\sigma_0} \approx 2\ \mu\Omega cm,\ \mu \simeq \mu_0$

lift-off for $v >> 0.6$ km/h

(say, $v \sim 50$ km/h)

Maglev and superconductors?

Superconducting Maglev are wonderful technology.
Superconductors only provide strong magnetic fields.

Yamanashi Superconducting Maglev tested at 581 km/h.
part of the Chūō Shinkansen
Expected to operate Tokio-Nagoya (40 min) by 2027,
Tokyo-Osaka (60 min) by 2045.