Numeri complessi

(richiami)

Definizioni

MQ <=> onde => numeri complessi

Unità immaginaria $\mathrm{i} = \sqrt{-1}$

numero complesso $z = x + \mathrm{i}y = |z|\mathrm{e}^{\mathrm{i}\varphi}$

rappresentazione cartesiana ($x + \mathrm{i}y$) — rappresentazione polare ($|z|\mathrm{e}^{\mathrm{i}\varphi}$)

rappresentazione cartesiana: parte reale+ i parte immaginaria

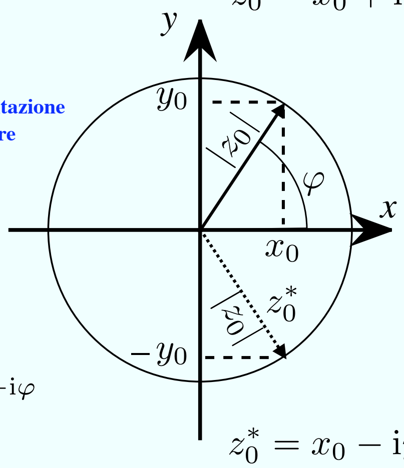
rappresentazione polare: modulo e fase $|z| = \sqrt{x^2 + y^2}$

$\varphi = \mathrm{atan}(y/x)$

complesso coniugato (cc) $z^* = x - \mathrm{i}y = |z|\mathrm{e}^{-\mathrm{i}\varphi}$

$\boxed{\text{regola per trovare il cc: i} \rightarrow \text{–i}}$

$z_0 = x_0 + \mathrm{i}y_0$



$z_0^* = x_0 - \mathrm{i}y_0$

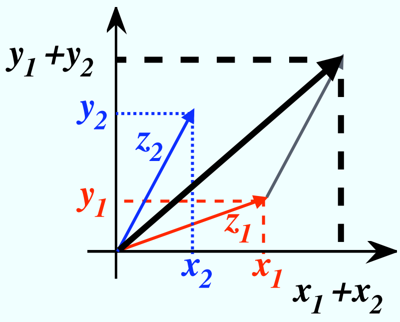
Operazioni elementari

modulo quadro $|z|^2 = z^* z = x^2 + y^2$

prodotto $z_1 z_2 = |z_1||z_2| \mathrm{e}^{\mathrm{i}(\varphi_1+\varphi_2)}$

somma $z_1 + z_2 = (x_1 + x_2) + \mathrm{i}(y_1 + y_2)$

(richiamo: fasori)



Esponenziali

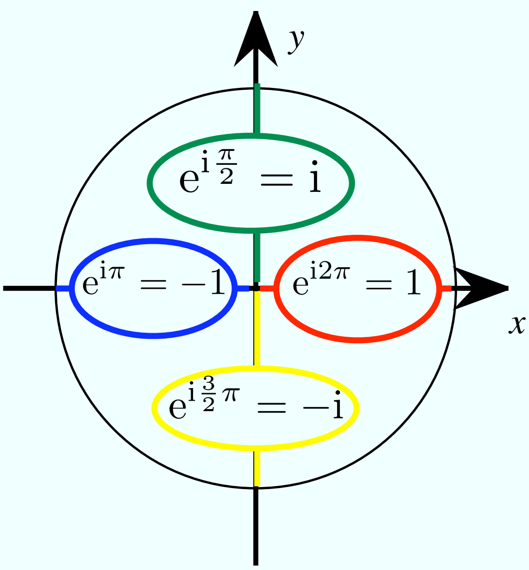
$$\boxed{\mathrm{e}^{\mathrm{i}\varphi} = \cos\varphi + \mathrm{i}\sin\varphi}$$

da cui: $|\mathrm{e}^{\mathrm{i}\varphi}| = 1$ per φ reale

$$\cos\varphi = \frac{\mathrm{e}^{\mathrm{i}\varphi} + \mathrm{e}^{-\mathrm{i}\varphi}}{2}$$

$$\sin\varphi = \frac{\mathrm{e}^{\mathrm{i}\varphi} - \mathrm{e}^{-\mathrm{i}\varphi}}{2\mathrm{i}}$$

argomenti notevoli:



N.B.:

$$z + z^* = 2x = 2|z|\cos\varphi$$

$$z - z^* = 2\mathrm{i}y = 2\mathrm{i}|z|\sin\varphi$$

Numeri complessi: relazioni utili

$$\boxed{\begin{aligned} e^{i\varphi} + 1 &= e^{i\frac{\varphi}{2}}\left(e^{i\frac{\varphi}{2}} + e^{-i\frac{\varphi}{2}}\right) = e^{i\frac{\varphi}{2}}\, 2\cos\frac{\varphi}{2} \\ e^{i\varphi} - 1 &= e^{i\frac{\varphi}{2}}\left(e^{i\frac{\varphi}{2}} - e^{-i\frac{\varphi}{2}}\right) = e^{i\frac{\varphi}{2}}\, 2i\sin\frac{\varphi}{2} \end{aligned}}$$

$$e^{iz} = e^{-y}\left(\cos x + i\sin x\right)$$

$$|e^{iz}|^2 = \left(e^{-y}\right)^2 = e^{-2y}$$

$$\boxed{\frac{1}{i} = -i}$$